PROBLEM 1.3

KNOWN: Inner surface temperature and thermal conductivity of a concrete wall.

FIND: Heat loss by conduction through the wall as a function of outer surface temperatures ranging from -15 to 38°C.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: From Fourier’s law, if \( q_x^* \) and \( k \) are each constant it is evident that the gradient, \( \frac{dT}{dx} = -\frac{q_x^*}{k} \), is a constant, and hence the temperature distribution is linear. The heat flux must be constant under one-dimensional, steady-state conditions; and \( k \) is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is \( T_2 = -15°C \) are

\[
q_x^* = -k \frac{dT}{dx} = k \frac{T_1 - T_2}{L} = 1 \text{W/m·K} \frac{25°C - (-15°C)}{0.30 \text{m}} = 133.3 \text{W/m}^2 .
\]

\[
q_x = q_x^* \times A = 133.3 \text{W/m}^2 \times 20 \text{m}^2 = 2667 \text{W}.
\]

Combining Eqs. (1) and (2), the heat rate \( q_x \) can be determined for the range of outer surface temperature, \(-15 \leq T_2 \leq 38°C\), with different wall thermal conductivities, \( k \).

For the concrete wall, \( k = 1 \text{ W/m·K} \), the heat loss varies linearly from +2667 W to -867 W and is zero when the inside and outer surface temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

COMMENTS: Without steady-state conditions and constant \( k \), the temperature distribution in a plane wall would not be linear.
PROBLEM 1.18

KNOWN: Hand experiencing convection heat transfer with moving air and water.

FIND: Determine which condition feels colder. Contrast these results with a heat loss of 30 W/m² under normal room conditions.

SCHEMATIC:

ASSUMPTIONS: (1) Temperature is uniform over the hand’s surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

ANALYSIS: The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton’s law of cooling, Eq. 1.3a, written as

\[ q^* = h(T_S - T_\infty) \]

For the air stream:

\[ q_{air}^* = 40 \text{ W/m}^2 \cdot \text{K} \left[ 30 - (-5) \right] \text{K} = 1,400 \text{ W/m}^2 < \]

For the water stream:

\[ q_{water}^* = 900 \text{ W/m}^2 \cdot \text{K} \left( 30 - 10 \right) \text{K} = 18,000 \text{ W/m}^2 < \]

COMMENTS: The heat loss for the hand in the water stream is an order of magnitude larger than when in the air stream for the given temperature and convection coefficient conditions. In contrast, the heat loss in a normal room environment is only 30 W/m² which is a factor of 400 times less than the loss in the air stream. In the room environment, the hand would feel comfortable; in the air and water streams, as you probably know from experience, the hand would feel uncomfortably cold since the heat loss is excessively high.
PROBLEM 1.27

KNOWN: Upper temperature set point, $T_{set}$, of a bimetallic switch and convection heat transfer coefficient between clothes dryer air and exposed surface of switch.

FIND: Electrical power for heater to maintain $T_{set}$ when air temperature is $T_\infty = 50^\circ C$.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Electrical heater is perfectly insulated from dryer wall, (3) Heater and switch are isothermal at $T_{set}$, (4) Negligible heat transfer from sides of heater or switch, (5) Switch surface, $A_s$, loses heat only by convection.

ANALYSIS: Define a control volume around the bimetallic switch which experiences heat input from the heater and convection heat transfer to the dryer air. That is,

$$ \dot{E}_{in} - \dot{E}_{out} = 0 $$

$$ q_{elec} - hA_s (T_{set} - T_\infty) = 0. $$

The electrical power required is,

$$ q_{elec} = hA_s (T_{set} - T_\infty) $$

$$ q_{elec} = 25 \text{ W/m}^2 \cdot \text{K} \times 30 \times 10^{-6} \text{ m}^2 (70 - 50) \text{ K} = 15 \text{ mW}. $$

COMMENTS: (1) This type of controller can achieve variable operating air temperatures with a single set-point, inexpensive, bimetallic-thermostatic switch by adjusting power levels to the heater.

(2) Will the heater power requirement increase or decrease if the insulation pad is other than perfect?
PROBLEM 1.53

KNOWN: Speed, width, thickness and initial and final temperatures of 304 stainless steel in an annealing process. Dimensions of annealing oven and temperature, emissivity and convection coefficient of surfaces exposed to ambient air and large surroundings of equivalent temperatures. Thickness of pad on which oven rests and pad surface temperatures.

FIND: Oven operating power.

SCHEMATIC:

ASSUMPTIONS: (1) steady-state, (2) Constant properties, (3) Negligible changes in kinetic and potential energy.

PROPERTIES: Table A.1, St.St.304 \((\bar{T} = (T_i + T_o)/2 = 775\text{K})\): \(\rho = 7900\text{kg/m}^3, c_p = 578\text{J/kg-K}\);
Table A.3, Concrete, \(T = 300\text{K}\): \(k_c = 1.4\text{W/m-K}\).

ANALYSIS: The rate of energy addition to the oven must balance the rate of energy transfer to the steel sheet and the rate of heat loss from the oven. Viewing the oven as an open system, Equation (1.12e) yields

\[
P_{\text{elec}} - q = \dot{m}c_p(T_o - T_i)
\]

where \(q\) is the heat transferred from the oven. With \(\dot{m} = \rho V_s (W_st_s)\) and

\[
q = (2H_oL_o + 2H_oW_o + W_oL_o) \times \left[ h(T_s - T_{\infty}) + \varepsilon_s \sigma(T_s^4 - T_{s,\text{sur}}^4) \right] + k_c(W_oL_o)(T_s - T_b)/t_c,
\]

it follows that

\[
P_{\text{elec}} = \rho V_s (W_st_s)c_p(T_o - T_i) + (2H_oL_o + 2H_oW_o + W_oL_o) \times \\
\left[ h(T_s - T_{\infty}) + \varepsilon_s \sigma(T_s^4 - T_{s,\text{sur}}^4) \right] + k_c(W_oL_o)(T_s - T_b)/t_c
\]

\[
P_{\text{elec}} = 7900\text{kg/m}^3 \times 0.01\text{m/s} (2\text{m} \times 0.008\text{m}) 578\text{J/kg-K} (1250 - 300)\text{K} + (2 \times 2 \times 2 \times 2 \times 2 \times 2) \times 10\text{W/m}^2 \cdot \text{K} (350 - 300)\text{K}
\]

\[
+ 0.8 \times 5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4 (350^4 - 300^4)\text{K}^4 + 1.4\text{W/m-K} (2.4\text{m} \times 25\text{m})(350 - 300)\text{K}/0.5\text{m}
\]

\[
P_{\text{elec}} = 694,000\text{W} + 169.6\text{m}^2 (500 + 313)\text{W/m}^2 + 8400\text{W}
\]

\[
= (694,000 + 84,800 + 53,100 + 8400)\text{W} = 840\text{kW}
\]

COMMENTS: Of the total energy input, 83% is transferred to the steel while approximately 10%, 6% and 1% are lost by convection, radiation and conduction from the oven. The convection and radiation losses can both be reduced by adding insulation to the side and top surfaces, which would reduce the corresponding value of \(T_s\).
PROBLEM 1.57

KNOWN: Silicon wafer positioned in furnace with top and bottom surfaces exposed to hot and cool zones, respectively.

FIND: (a) Initial rate of change of the wafer temperature corresponding to the wafer temperature \( T_{w,i} = 300 \text{ K} \), and (b) Steady-state temperature reached if the wafer remains in this position. How significant is convection for this situation? Sketch how you’d expect the wafer temperature to vary as a function of vertical distance.

SCHEMATIC:

ASSUMPTIONS: (1) Wafer temperature is uniform, (2) Transient conditions when wafer is initially positioned, (3) Hot and cool zones have uniform temperatures, (3) Radiation exchange is between small surface (wafer) and large enclosure (chamber, hot or cold zone), and (4) Negligible heat losses from wafer to mounting pin holder.

ANALYSIS: The energy balance on the wafer illustrated in the schematic above includes convection from the upper (u) and lower (l) surfaces with the ambient gas, radiation exchange with the hot- and cool-zone (chamber) surroundings, and the rate of energy storage term for the transient condition.

\[
\dot{E}_{in}^* - \dot{E}_{out}^* = \dot{E}_{st}
\]

\[
q_{rad,h}^* + q_{rad,c}^* - q_{conv,u}^* - q_{conv,l}^* = \rho c d \frac{dT_w}{dt}
\]

\[
\varepsilon \sigma (T_{sur,h}^4 - T_w^4) + \varepsilon \sigma (T_{sur,c}^4 - T_w^4) - h_u (T_w - T_{\infty}) - h_l (T_w - T_{\infty}) = \rho c d \frac{dT_w}{dt}
\]

(a) For the initial condition, the time rate of temperature change of the wafer is determined using the energy balance above with \( T_w = T_{w,i} = 300 \text{ K} \),

\[
0.65 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \left( 1500^4 - 300^4 \right) \text{K}^4 + 0.65 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \left( 330^4 - 300^4 \right) \text{K}^4
\]

\[-8 \text{ W/m}^2 \cdot \text{K} (300 - 700) \text{K} - 4 \text{ W/m}^2 \cdot \text{K} (300 - 700) \text{K} =
\]

\[
2700 \text{ kg/m}^3 \times 875 \text{ J/kg} \cdot \text{K} \times 0.00078 \text{ m} \left( \frac{dT_w}{dt} \right)
\]

\[
\left( \frac{dT_w}{dt} \right)_i = 104 \text{ K/s}
\]

(b) For the steady-state condition, the energy storage term is zero, and the energy balance can be solved for the steady-state wafer temperature, \( T_w = T_{w,ss} \).

Continued …..
0.65 \sigma \left(1500^4 - T_{w,ss}^4\right) K^4 + 0.65 \sigma \left(330^4 - T_{w,ss}^4\right) K^4

-8 \text{ W/m}^2 \cdot \text{K} \left(T_{w,ss} - 700\right) K - 4 \text{ W/m}^2 \cdot \text{K} \left(T_{w,ss} - 700\right) K = 0

T_{w,ss} = 1251 K

To determine the relative importance of the convection processes, re-solve the energy balance above ignoring those processes to find \( (d T_w / dt)_i = 101 \text{ K/s} \) and \( T_{w,ss} = 1262 \text{ K} \). We conclude that the radiation exchange processes control the initial time rate of temperature change and the steady-state temperature.

If the wafer were elevated above the present operating position, its temperature would increase, since the lower surface would begin to experience radiant exchange with progressively more of the hot zone chamber. Conversely, by lowering the wafer, the upper surface would experience less radiant exchange with the hot zone chamber, and its temperature would decrease. The temperature-distance trend might appear as shown in the sketch.
PROBLEM 1.80

KNOWN: Uninsulated pipe of prescribed diameter, emissivity, and surface temperature in a room with fixed wall and air temperatures. See Example 1.2.

FIND: (a) Which option to reduce heat loss to the room is more effective: reduce by a factor of two the convection coefficient (from 15 to 7.5 W/m²·K) or the emissivity (from 0.8 to 0.4) and (b) Show graphically the heat loss as a function of the convection coefficient for the range $5 \leq h \leq 20$ W/m²·K for emissivities of 0.2, 0.4 and 0.8. Comment on the relative efficacy of reducing heat losses associated with the convection and radiation processes.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between pipe and the room is between a small surface in a much larger enclosure, (3) The surface emissivity and absorptivity are equal, and (4) Restriction of the air flow does not alter the radiation exchange process between the pipe and the room.

ANALYSIS: (a) The heat rate from the pipe to the room per unit length is

$$q' = q'_L = q'_{\text{conv}} + q'_{\text{rad}} = h\left(\pi D\right)\left(T_s - T_\infty\right) + \varepsilon\left(\pi D\right)\sigma\left(T_s^4 - T_{\text{sur}}^4\right)$$

Substituting numerical values for the two options, the resulting heat rates are calculated and compared with those for the conditions of Example 1.2. We conclude that both options are comparably effective.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>$h$ (W/m²·K)</th>
<th>$\varepsilon$</th>
<th>$q'$ (W/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case, Example 1.2</td>
<td>15</td>
<td>0.8</td>
<td>998</td>
</tr>
<tr>
<td>Reducing $h$ by factor of 2</td>
<td>7.5</td>
<td>0.8</td>
<td>788</td>
</tr>
<tr>
<td>Reducing $\varepsilon$ by factor of 2</td>
<td>15</td>
<td>0.4</td>
<td>709</td>
</tr>
</tbody>
</table>

(b) Using IHT, the heat loss can be calculated as a function of the convection coefficient for selected values of the surface emissivity.

Continued ...
PROBLEM 1.80 (Cont.)

COMMENTS: (1) In Example 1.2, Comment 3, we read that the heat rates by convection and radiation exchange were comparable for the base case conditions (577 vs. 421 W/m). It follows that reducing the key transport parameter (h or $\varepsilon$) by a factor of two yields comparable reductions in the heat loss. Coating the pipe to reduce the emissivity might be the more practical option as it may be difficult to control air movement.

(2) For this pipe size and thermal conditions ($T_s$ and $T_\infty$), the minimum possible convection coefficient is approximately 7.5 W/m$^2$K, corresponding to free convection heat transfer to quiescent ambient air. Larger values of $h$ are a consequence of forced air flow conditions.

(3) The Workspace for the IHT program to calculate the heat loss and generate the graph for the heat loss as a function of the convection coefficient for selected emissivities is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

```c
// Heat loss per unit pipe length; rate equation from Ex. 1.2
q' = q'cv + q'rad
q'cv = \pi*D*h*(T_s - T_\infty)
q'rad = \pi*D*\varepsilon*sigma*(T_s^4 - T_sur^4)
sigma = 5.67e-8

// Input parameters
D = 0.07
Ts_C = 200          // Representing temperatures in Celsius units using _C subscripting
Ts = Ts_C + 273
Tinf_C = 25
Tinf = Tinf_C + 273
h = 15              // For graph, sweep over range from 5 to 20
Tsur_C = 25
Tsur = Tsur_C + 273
\varepsilon = 0.8    // Values of emissivity for parameter study
//\varepsilon = 0.4
//\varepsilon = 0.2

/* Base case results
Tinf Ts Tsur q' q'cv q'rad D Tinf_C Ts_C Tsur_C 
\varepsilon h sigma
298 473 298 997.9 577.3 420.6 0.07 25 200 25
0.8 15 5.67E-8 */
```