MATH 383 SAMPLE EXAM ONE
ANSWER KEY

Part One: True/False

Circle **T** for true and **F** for false.

1. **T** If the five summary statistics of a data set are 51,123,130,147,and 151 then the data probably contains an outlier.
2. **F** The function $f(x) = 15x^2 - 12x^3, \quad 0 < X < 2$ is a probability density function.
3. **T** The set of possible values a variable can take is countable for a discrete variable and uncountable for a continuous variable.
4. **T** When constructing a Histogram, the difference between using a frequency scale and using a density scale is that the density scale makes the total area sum to one.
5. **T** The Variance is the average of the squared deviations from the mean.
6. **T** A Histogram is termed "negatively skewed" if it has a long left-hand tail.
7. **F** A Histogram is termed "negatively skewed" if it has a long right-hand tail.

Part Two: Problem Solving

Solve each of the following problems. Show all work.

8. If the random variable X has a Weibull Distribution with parameters $\alpha = 4$ and $\beta = 10$ then the probability of X being greater than 15 is

$$\begin{aligned} \mathbf{F(x)} &= \mathbf{1 - e^{-(x/\beta)^{\alpha}} = 1 - e^{-(x/10)^4}}, \quad \mathbf{x \geq 0}. \\ \mathbf{1 - F(15)} &= \mathbf{e^{-(15/10)^4} = .0063} \end{aligned}$$

9. If X is a random variable with a Uniform distribution between 7 and 20 then the probability that X is less than 12 is

$$\begin{aligned} \mathbf{F(x)} &= \mathbf{\frac{x - a}{b - a} = \frac{x - 7}{20 - 7}}, \quad \mathbf{7 \leq x \leq 20}. \\ \mathbf{F(12)} &= \mathbf{\frac{12 - 7}{13} = 5/13}. \end{aligned}$$

10. If X is a random variable with a Binomial Distribution and parameters n=10 and p=.4 then the probability that X is at least 4 is

$$\begin{aligned} \mathbf{p(x)} &= \frac{\mathbf{n!}}{\mathbf{x!(n-x)!}}\mathbf{p^x(1-p)^{n-x}} = \frac{\mathbf{10!}}{\mathbf{x!(10-x)!}}\mathbf{(.4)^x(.6)^{10-x}}, \quad \mathbf{x = 0, 1, \ldots, 10.} \\ \mathbf{P(X \geq 4)} &= \mathbf{p(4) + p(5) + p(6) + p(7) + p(8) + p(9) + p(10)} \\ &= \mathbf{210(.4)^4(.6)^6 + 252(.4)^5(.6)^5 + 210(.4)^6(.6)^4 + 120(.4)^7(.6)^3 + 45(.4)^8(.6)^2} \\ &\quad \mathbf{+10(.4)^9(.6)^1 + (.4)^{10} = .61772.} \end{aligned}$$

11. If X has a Lognormal Distribution with parameters $\mu = 21$ and $\sigma = 5$ then the probability that X is less than e^{28} is

$$\mathbf{P\left(X \leq e^{28}\right) = P\left(\ln X \leq 28\right) = P\left(Z \leq \frac{28-21}{5}\right) = P\left(Z \leq 1.4\right) = .9192.}$$

12. If X is a Normal random variable with mean 82 and standard deviation 32 then

(a) the probability that X is less than 82 is

$$\mathbf{P\left(X \leq 82\right) = P\left(Z \leq \frac{82-82}{32}\right) = P\left(Z \leq 0\right) = .5}$$

(b) the probability that X is less than 66 is

$$\mathbf{P\left(X \leq 66\right) = P\left(Z \leq \frac{66-82}{32}\right) = P\left(Z \leq -.5\right) = .3085}$$

(c) the probability that X is less than 90 is

$$\mathbf{P\left(X \leq 90\right) = P\left(Z \leq \frac{90-82}{32}\right) = P\left(Z \leq .25\right) = .5987}$$

(d) the first quartile is
Since $\mathbf{P(Z \leq -.67) = .25}$, we get

$$\mathbf{X = (-.67) \times 32 + 82 = 60.56}$$

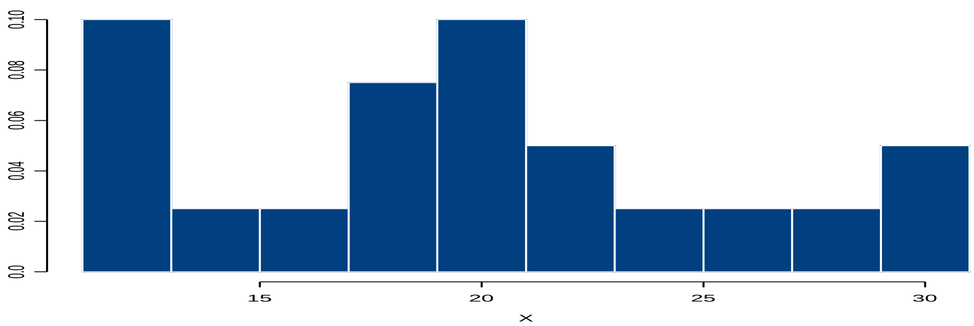
(e) the 95^{th} percentile is
Since $\mathbf{P(Z \leq 1.65) = .95}$, we get

$$\mathbf{X = (1.65) \times 32 + 82 = 134.8}$$

(f) the .025 quantile is
Since $\mathbf{P(Z \leq -1.96) = .025}$, we get

$$\mathbf{X = (-1.96) \times 32 + 82 = 19.28}$$

13. Consider the following density histogram.



(a) The probability that X is between 21 and 23 is $\mathbf{.05 \times 2 = .1}$.

(b) The probability that X is between 23 and 27 is $\mathbf{.025 \times 4 = .1}$.

(c) The probability that X is between 21 and 25 is $\mathbf{.05 \times 2 + .025 \times 2 = .1 + .05 = .15}$.

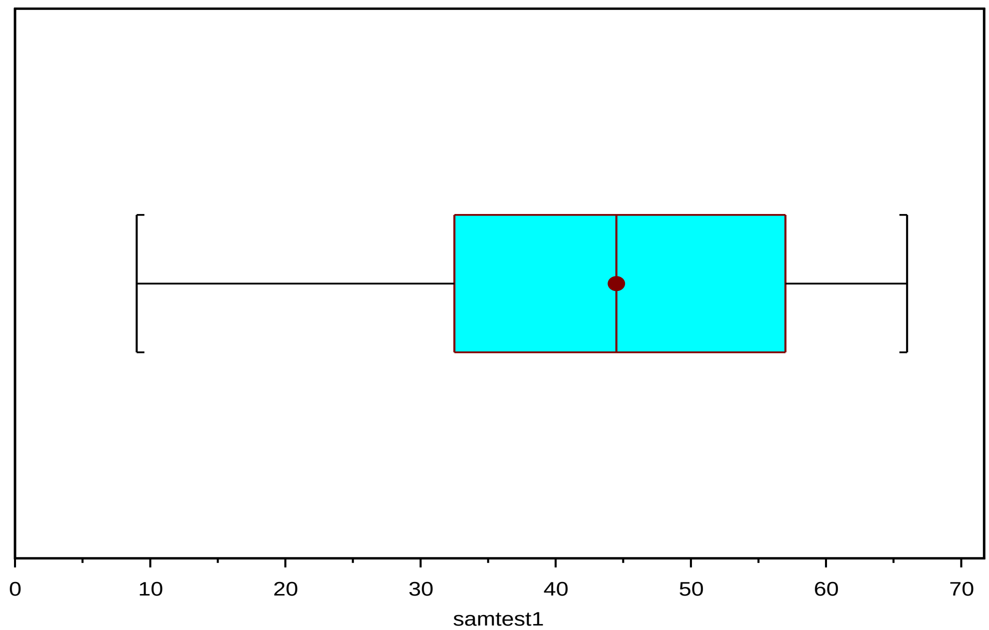
14. Consider the following data: 43, 42, 20, 64, 30, 44, 9, 57, 57, 57, 16, 30, 56, 66, 57, 45, 35, 56, 40, and 47. **We construct the following table, placing the data in order from smallest to largest before squaring them:**

X	$\mathbf{X^2}$
9	**81**
16	**256**
20	**400**
30	**900**
30	**900**
35	**1225**
40	**1600**
42	**1764**
43	**1849**
44	**1936**
45	**2025**
47	**2209**
56	**3136**
56	**3136**
57	**3249**
57	**3249**
57	**3249**
57	**3249**
64	**4096**
66	**4356**
871	**42865**

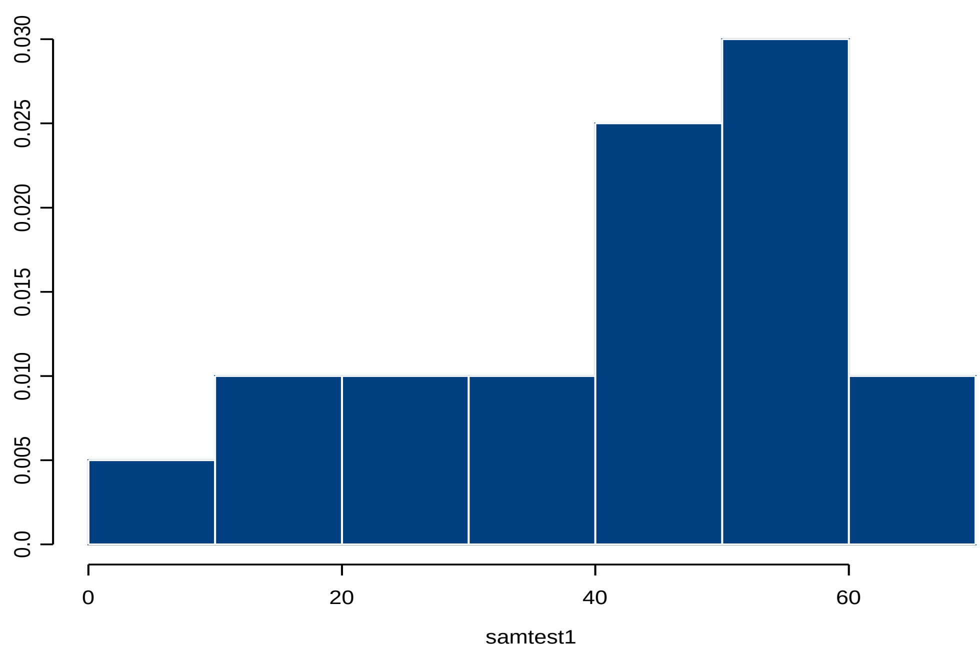
(a) The five summary statistics for the data set are **9**, **32.5**, **44.5**, **57**, and **66**.

(b) The mean of the data set is $\mathbf{871/20 = 43.55}$.

(c) Construct a *Box-and-Whiskers Plot* for the data.



(d) Construct a *Density Histogram* for the data.



(e) The standard deviation of the data set is

$$\sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2\right)} = \sqrt{\frac{1}{19}\left(42865 - \frac{1}{20}(871)^2\right)} = 16.113$$

15. Suppose that your morning waiting time for a bus has a uniform distribution on the interval from 0 to 5 minutes, and your afternoon waiting time also has this distribution. Let X denote the total waiting time on any particular day.
Recall that the density function is

$$f(x) = \begin{cases} x/25 & 0 \le x \le 5 \\ 2/5 - x/25 & 5 \le x \le 10 \end{cases}$$

(a) The probability that X exceeds 1 minute is

$$\int_1^5 \frac{x}{25}dx + \int_5^{10} \frac{2}{5} - \frac{x}{25}dx = \frac{24}{50} + \frac{25}{50} = \frac{49}{50} = .98$$

(b) The probability that X exceeds 5 minutes is

$$\int_5^{10} \frac{2}{5} - \frac{x}{25}dx = \frac{25}{50} = .5$$

(c) The probability that X exceeds 9 minutes is

$$\int_9^{10} \frac{2}{5} - \frac{x}{25}dx = \frac{1}{50} = .02$$

(d) What value separates the longest 50% of your daily waiting times from the remaining 50%?
Since

$$\int_1^5 \frac{x}{25}dx = \int_5^{10} \frac{2}{5} - \frac{x}{25}dx,$$

the answer is 5 minutes.

(e) What value separates the longest 25% of your daily waiting times from the remaining 75%?

$$\int_q^{10} \frac{2}{5} - \frac{x}{25}dx = 2 - \frac{2q}{5} + \frac{q^2}{50} = .25$$

$$q^2 - 20q + 87.5 = 0$$

$$q = \frac{20 - \sqrt{400 - 4 \times 87.5}}{2} = 10 - \sqrt{12.5} = 6.46 \text{ minutes.}$$

(f) What value separates the longest 10% of your daily waiting times from the remaining 90%?

$$\int_q^{10} \frac{2}{5} - \frac{x}{25} dx = 2 - \frac{2q}{5} + \frac{q^2}{50} = .1$$

$$q^2 - 20q + 95 = 0$$

$$q = \frac{20 - \sqrt{400 - 4 \times 95}}{2} = 10 - \sqrt{5} = 7.76 \text{ minutes.}$$

16. The time in seconds that it takes a librarian to locate an entry in a file of records on checked-out books has an exponential distribution with mean 20 seconds.
Recall that the cumulative distribution function is

$$F(x) = 1 - e^{-x/20}, \quad x \geq 0.$$

(a) What proportion of all location times is less than 10 seconds?

$$F(10) = 1 - e^{-10/20} = .3935.$$

(b) What proportion of all location times is less than 20 seconds?

$$F(20) = 1 - e^{-20/20} = .6321.$$

(c) What proportion of all location times is less than 30 seconds?

$$F(30) = 1 - e^{-30/20} = .7769.$$

(d) What proportion of all location times is greater than 30 seconds?

$$1 - F(30) = e^{-30/20} = .2231.$$

(e) What is the median of the distribution?

$$F(x) = 1 - e^{-x/20} = .5$$

$$x/20 = \ln 2$$

$$x = 20 \ln 2 = 13.863.$$

(f) What is the 90^{th} percentile of the distribution?

$$F(x) = 1 - e^{-x/20} = .9$$

$$x/20 = \ln 10$$

$$x = 20 \ln 10 = 46.052.$$