Geometric Distribution: A series of independent identical trials is performed. Each trial can either succeed or fail, and the trials are repeated until the first success. The parameter $p$ represents the probability of success on a single trial and the random variable $X$ represents the number of trials performed. $X \in \{1, 2, 3, \ldots\}$

The Probability Mass Function (pmf) for $X$ is given by

$$p(x) = p(1 - p)^{x-1}, \quad x = 1, 2, 3, \ldots$$

and the Cumulative Distribution Function (cdf) for $X$ is given by

$$F(x) = \sum_{i=1}^{x} p(1 - p)^{i-1}, \quad x = 1, 2, 3, \ldots$$

The expectation and variance of $X$ are given by

$$E(X) = \frac{1}{p}, \quad \text{and} \quad V(X) = \frac{1-p}{p^2}$$

The moment generating function is

$$m(t) = \frac{pe^t}{1 - e^t(1-p)}, \quad t < -\ln(1-p).$$
**Negative Binomial Distribution:** A series of independent identical trials is performed. Each trial can either succeed or fail, and the trials are repeated until $k$ successes occur. The parameter $p$ represents the probability of success on a single trial and the random variable $X$ represents the number of trials performed. $X \in \{k, k+1, k+2, \ldots\}$

The Probability Mass Function (pmf) for $X$ is given by

$$p(x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}, \quad x = k, k+1, k+2, \ldots$$

and the Cumulative Distribution Function (cdf) for $X$ is given by

$$F(x) = \sum_{i=k}^{x} \binom{i-1}{k-1} p^k (1-p)^{i-k}, \quad x = k, k+1, k+2, \ldots$$

The expectation and variance of $X$ are given by

$$E(X) = \frac{k}{p}, \quad \text{and} \quad V(X) = \frac{k(1-p)}{p^2}$$
**Binomial Distribution:** A series of \( n \) independent identical trials is performed. Each trial can either succeed or fail. The parameter \( p \) represents the probability of success on a single trial and the random variable \( X \) represents the number of successes. \( X \in \{0, 1, 2, \ldots , n\} \)

The Probability Mass Function (pmf) for \( X \) is given by

\[
p(x) = \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, 2, \ldots, n
\]

and the Cumulative Distribution Function (cdf) for \( X \) is given by

\[
F(x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1 - p)^{n-i}, \quad x = 0, 1, 2, \ldots, n
\]

The expectation and variance of \( X \) are given by

\[
E(X) = np, \quad \text{and} \quad V(X) = np(1 - p)
\]

The moment generating function is

\[
m(t) = (pe^t + (1 - p))^n, \quad -\infty < t < \infty.
\]
Hypergeometric Distribution: A sample of size $n$ is taken from a population of size $N$ without replacement. A characteristic which exists in $k$ elements of the population is identified as existing or not existing in each element of the sample. The random variable $X$ is the number of elements in the sample with the characteristic. $X \in \{0, 1, 2, \ldots, \min\{n, k\}\}$

The Probability Mass Function (pmf) for $X$ is given by

$$p(x) = \binom{k}{x} \binom{N-k}{n-x} \binom{N}{n}, \quad x \in \{0, 1, 2, \ldots, \min\{n, k\}\}$$

and the Cumulative Distribution Function (cdf) for $X$ is given by

$$F(x) = \sum_{i=0}^{x} \binom{k}{i} \binom{N-k}{n-i} \binom{N}{n}, \quad x \in \{0, 1, 2, \ldots, \min\{n, k\}\}$$

The expectation and variance of $X$ are given by

$$E(X) = n \left( \frac{k}{N} \right), \quad \text{and} \quad V(X) = n \left( \frac{k}{N} \right) \left( \frac{N-k}{N} \right) \left( \frac{N-n}{N-1} \right)$$
**Poisson Distribution:** A count of rare occurrences within a specified time frame. The parameter $\lambda$ represents the average number of occurrences within the specified length of time and the random variable $X$ represents the actual number of occurrences counted. $X \in \{0, 1, 2, \ldots\}$

The Probability Mass Function (pmf) for $X$ is given by

$$p(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x \in \{0, 1, 2, \ldots\}$$

and the Cumulative Distribution Function (cdf) for $X$ is given by

$$F(x) = \sum_{i=0}^{x} \frac{e^{-\lambda} \lambda^i}{i!}, \quad x \in \{0, 1, 2, \ldots\}$$

The expectation and variance of $X$ are both $\lambda$.

The moment generating function is

$$m(t) = e^{-\lambda(1-e^t)}, \quad -\infty < t < \infty.$$
Pareto Distribution: Also called the Zeta or Zipf Distribution, it was used by the Italian economist Pareto to describe the distribution of family incomes in a given country. The random variable $X$ has the positive integers as a sample space. The parameter $\alpha$ is always positive.

The Probability Mass Function (pmf) for $X$ is given by

$$p(x) = \frac{C}{x^{\alpha+1}}, \quad x = 1, 2, 3, \ldots$$

and the Cumulative Distribution Function (cdf) for $X$ is given by

$$F(x) = \sum_{i=1}^{x} \frac{C'}{i^{\alpha+1}}, \quad x = 1, 2, 3, \ldots$$

The normalizing constant $C'$ is given by

$$C' = \left( \sum_{k=1}^{\infty} \left( \frac{1}{k} \right)^{\alpha+1} \right)^{-1}.$$
Continuous Distributions

**Exponential Distribution:** If the number of occurrences in a given time interval follows a Poisson Distribution with parameter $\lambda$, then the waiting time between occurrences is exponentially distributed with parameter $\beta = 1/\lambda$. The random variable $X$ represents this waiting time. $X \in (0, \infty)$

The Probability Density Function (pdf) for $X$ is given by

$$f(x) = \frac{1}{\beta}e^{-x/\beta}, \quad x > 0$$

and the Cumulative Distribution Function (cdf) for $X$ is given by

$$F(x) = 1 - e^{-x/\beta}, \quad x > 0.$$  

The expectation and variance of $X$ are given by

$$E(X) = \beta, \quad \text{and} \quad V(X) = \beta^2.$$  

The moment generating function is

$$m(t) = \frac{1}{1 - \beta t}, \quad t < \frac{1}{\beta}.$$
**Gamma Distribution:** As the Negative Binomial Distribution can be thought of as a sum of Geometric Distributions, the Gamma Distribution is a sum of Exponential Distributions. Specifically, if the number of occurrences in a given time interval follows a Poisson Distribution with parameter \( \lambda \), then the waiting time before the \( \nu \) occurrence is a random variable \( X \) with a Gamma Distribution and parameters \( \nu \) and \( \beta = 1/\lambda \). \( X \in (0, \infty) \)

The Probability Density Function (pdf) for \( X \) is given by

\[
f(x) = \frac{x^{\nu-1}}{\beta^\nu \Gamma(\nu)} e^{-x/\beta}, \quad x > 0.
\]

The expectation and variance of \( X \) are given by

\[
E(X) = \nu \beta, \quad \text{and} \quad V(X) = \nu \beta^2.
\]

**The Gamma Function:** An extrapolation of the factorial function \((n!)\) to all positive real numbers, the Gamma Function is used in the pdfs of the Gamma Distribution, the Weibull Distribution and the Beta Distribution. The Gamma Function is defined by the integral

\[
\Gamma(\nu) = \int_0^\infty x^{\nu-1} e^{-x} \, dx, \quad \nu > 0.
\]

As with the factorial function, the Gamma Function has the property that for every \( \nu > 1 \), \( \Gamma(\nu) = (\nu - 1)\Gamma(\nu - 1) \). Further, for every positive integer \( n \), \( \Gamma(n) = (n - 1)! \).
**Weibull Distribution:** When an exponential lifetime is coupled with the possibility of startup failure the result is a Weibull Distribution. The parameters are $\alpha$ and $\beta$ and the random variable $X$ is the time until failure. $X \in (0, \infty)$

The Probability Density Function (pdf) for $X$ is given by

$$f(x) = \frac{\alpha x^{\alpha-1}}{\beta^\alpha} e^{-(x/\beta)^\alpha}, \quad x > 0$$

and the Cumulative Distribution Function (cdf) for $X$ is given by

$$F(x) = 1 - e^{-(x/\beta)^\alpha}, \quad x > 0.$$ 

The expectation and variance of $X$ are given by

$$E(X) = \beta \Gamma \left( 1 + \frac{1}{\alpha} \right), \quad \text{and}$$

$$V(X) = \beta^2 \left[ \Gamma \left( 1 + \frac{2}{\alpha} \right) - \Gamma^2 \left( 1 + \frac{1}{\alpha} \right) \right].$$
**Beta Distribution:** The Beta Distribution is often used as a model for proportions. The parameters are $\nu_1$ and $\nu_2$ and the random variable $X$ can take on only values between 0 and 1.

$0 \leq X \leq 1$

The Probability Density Function (pdf) for $X$ is given by

$$f(x) = \frac{\Gamma(\nu_1 + \nu_2)}{\Gamma(\nu_1)\Gamma(\nu_2)} x^{\nu_1-1}(1-x)^{\nu_2-1}, \quad x \in [0, 1].$$

The expectation and variance of $X$ are given by

$$E(X) = \frac{\nu_1}{\nu_1 + \nu_2}, \quad \text{and} \quad V(X) = \frac{\nu_1\nu_2}{(\nu_1 + \nu_2)^2(\nu_1 + \nu_2 + 1)}.$$
**Uniform Distribution:** When a random variable $X$ can assume any value in the interval $(a, b)$ with equal probability, it has a Uniform Distribution with parameters $a$ and $b$. $X \in (a, b)$

The Probability Density Function (pdf) for $X$ is given by

$$f(x) = \frac{1}{b - a}, \quad a \leq x \leq b.$$ 

The expectation and variance of $X$ are given by

$$E(X) = \frac{a + b}{2}, \quad \text{and} \quad V(X) = \frac{(b - a)^2}{12}.$$
**Normal Distribution:** The most frequently used distribution is the Normal Distribution with parameters $\mu$ and $\sigma$, otherwise known as the Bell Curve due to the bell-shaped graph of its probability density function. A random variable $X$ which has a Normal($\mu, \sigma$) distribution has a sample space consisting of the entire real line $\mathbb{R}$. $X \in \mathbb{R}$

The Probability Density Function (pdf) for $X$ is given by

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}, \quad x \in \mathbb{R}.$$ 

The expectation and variance of $X$ are given by

$$E(X) = \mu, \quad \text{and} \quad V(X) = \sigma^2.$$ 

The moment generating function is

$$m(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}, \quad -\infty < t < \infty.$$
**Log-Normal Distribution:** A random variable $X$ is said to have a Log-Normal Distribution with parameters $\mu$ and $\sigma$ if the random variable $Y = \log X$ has a Normal Distribution with mean $\mu$ and variance $\sigma^2$. $X > 0$

The Probability Density Function (pdf) for $X$ is given by

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-(\log x - \mu)^2/(2\sigma^2)}, \quad x > 0.$$ 

The expectation and variance of $X$ are given by

$$E(X) = e^{\mu+\frac{\sigma^2}{2}}, \quad \text{and} \quad V(X) = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1).$$