In situ mechanical testing of templated carbon nanotubes

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A new microelectromechanical system (MEMS)-based tensile testing stage (with integrated actuator, direct load sensing beam, and electrodes for controlled assembly of an individual nanostructure) was developed and used for in situ tensile loading of a templated carbon nanotube (T-CNT) inside a scanning electron microscope (SEM). Specifically, an increasing tensile load was applied to the T-CNT by actuating the device and high-resolution scanning electron microscopy images were acquired at different loads. The load (from the bending of the direct force-sensing beam), the elongation of the specimen during loading, and the specimen geometry were all obtained from analysis of SEM images. The stress versus strain curve and Young’s modulus were thus obtained. A model is presented for the tensile loading experiment, and the fit value of Young’s modulus from this model is compared to values obtained by an independent method. The results of this experiment on a T-CNT suggest the use of this device for loading other nanostructures and also for designing other MEMS-based systems, such as a compressive testing stage. © 2006 American Institute of Physics. [DOI: 10.1063/1.2400212]

I. INTRODUCTION

The mechanics of nanostructures are of intrinsic and practical interest. Example approaches for studying mechanical properties of nanostructures include nanoindentation, tensile loading experiments with atomic force microscopy (AFM) cantilevers, and bending or rolling on substrate surfaces using scanning tunneling microscopy (STM)/AFM tips. Here, we report a microelectromechanical system (MEMS) testing stage with integrated actuators, force-sensing structures, and electrodes for assembly of nanostructures. A useful feature of MEMS-based testing stages is versatility of design and fabrication, and that they can be mass manufactured. Recently, MEMS have been used as a testing tool for probing mechanics, such as for actuators or force sensors.

Configuring nanostructures on MEMS devices is challenging. Thin Al films have been tested in a MEMS force-sensing device. One typical approach was to use a manipulator to first pick up and then transfer nanostructures onto the MEMS device. Direct growth of nanostructures such as carbon nanotubes (CNTs) on MEMS has also been achieved. However, the chemical vapor deposition process used in the growth limits the types of materials that the MEMS device can be made of. At this time, mounting nanostructures on MEMS devices, particularly those having moving components, remains a significant challenge.

We present a new MEMS device composed of an actuation unit, direct force-sensing beams, and integrated electrodes for dielectrophoretic deposition of nanostructures. The device has been successfully used to conduct tensile testing on templated carbon nanotubes (T-CNTs) in a scanning electron microscope (SEM). Single T-CNTs were assembled onto the device across the testing platforms by electric field guided assembly as will be described further below. A “fountain pen-like” writing technique with a glass micropipette was used to deposit a small amount of paraffin material on top of the suspended platforms at a distance less than 30 \( \mu \text{m} \) from the T-CNT, to speed up the electron beam induced deposition (EBID) process of making clamps. An increasing tensile load was applied to the T-CNTs and the magnitude of the load was obtained from the bending of the force-sensing beam, analyzed from SEM images. By fitting the stress-strain curve a modulus of 14.6±4.6 GPa was obtained for the T-CNT. These T-CNTs are known to be amorphous and probably contain hydrogen.

The experimental methods were confirmed by a theoretical model, and also by measurement of other T-CNTs by another method involving independently manipulated AFM cantilevers.

II. DESIGN AND FABRICATION OF THE SEM TESTING STAGE

Figure 1 shows the SEM image of our MEMS device. This device is designed with three levels of functionality: mechanical structure, electrical circuit, and thermal path.
The mechanical structure includes an actuation unit, two supporting beams [to attach platform A, Fig. 1], and two direct force-sensing beams (with platform B). The actuation unit is composed of thermal expansion beams and a motion amplification mechanism. By running current through the thermal expansion beams, displacement at their free ends [end bar in Fig. 1(a)] is transferred into nanoscale motion along the axis of the platform A [OO’ in Fig. 1(b)] by a V-shaped structure for motion amplification. On the opposite side of platform A, platform B is attached in the middle of the direct force-sensing beam, which is suspended and clamped at both ends to the fixed contact pads (Ω1 and Ω3). In typical use, the specimen (here, a T-CNT) would be mounted and clamped across platforms A and B. When loading the specimen with the actuation unit, the force applied can be measured from the deflection of the direct force-sensing beam. Such an approach has the flexibility to separate the electrical circuits for assembly and testing in the same device.

The electrical circuit has three parts: (i) the contact pads for introducing electrical current into the silicon structure (the contact pads contain the silicon base and metal films), (ii) the silicon structure itself as the Joule heating element (overlaps the thermal expansion beams of the mechanical actuation unit), and (iii) the electrical path for electric-field-assisted deposition of nanowires such as T-CNTs (dielectrophoretic deposition), including the metal electrodes on the two opposing platforms [A and B, Fig. 1(b)] and the conductive silicon structures. The platforms A and B are fabricated such that they are electrically isolated from each other, so that an electric field can be applied between A and B through the large contact pads P2 and Ω2 on the silicon base. The shape of the platforms (and electrodes) is optimized for controlled deposition of nanowires, for a high aspect ratio Si structure. After T-CNT assembly, pads Ω1, Ω2, and Ω3 are electrically connected to achieve equal potential across the T-CNT (Ω3 is in a position symmetric to Ω1, and not shown in Fig. 1). To prevent any short circuit among the silicon structures occurring through the substrate, the device was fabricated in a silicon wafer bonded to a Borofloat® glass wafer. The thermal path is created such that the heat is generated in the thermal expansion beams and dissipated to the large silicon base of the contact pads through the supporting beams (the two suspended thin beams attached on each side of the platform A). Such a layout ensures that the actuator retracts platform A away from platform B.21

Finally, a reservoir for liquid runoff was designed to enclose the entire device, for the electric-field guided assembly.

The device was fabricated in silicon-on-glass made through the anodic bonding process. A 3 in. double side polished (DSP) wafer (P type, 0.02 Ω-cm, 100 µm thick, KMBH Associates) was used as the structure layer, and bonded to a 4 in. DSP Borofloat® glass (Mark Optics) using the EV501 bonder (EV group). Electrodes were first patterned by photolithography (S1818, Shipley), metal evaporation (Cr 10 nm and Au 100 nm) with a CVC SC4500 evaporator, followed by a lift-off process. A second 3.7 µm thick layer of photore sist (SPR 220-3.0, Shipley) was then coated to pattern the silicon structure and also served as an etching mask for the deep reactive ion etching (DRIE) process (Uniliver axis 770). The device layer silicon was etched through until etching stopped at the glass surface. Each individual device was cut off from the bonded wafer, and the freestanding silicon structure was released by timed hydrofluoric acid etching (49%, Fisher Scientific). The remaining photore sist was then removed by Nanostrip (Cyantek Corporation). The device was finally rinsed with absolute alcohol (200 Proof, Pharmco Products, Inc.) to thoroughly clean the device, followed by drying.

III. ASSEMBLY OF TEMPLATED CARBON NANOTUBE (T-CNT) AND INSTRUMENTATION OF THE DEVICE

Due to the challenges and difficulties associated with direct growth of CNTs at high temperature on MEMS
devices, we used the electric field-guided assembly method (dielectrophoretic deposition) to assemble a single T-CNT onto the device from a liquid dispersion, as reported in our previous work. For the current MEMS device, the design layout of the mechanical structure and electrode shape (and size) was optimized to maximize the uniformity of the electric field. The electrical signal was experimentally determined to minimize the interference from the surrounding Si structures around platforms A and B. A composite electrical signal with a 20 \( V_{pp} \) ac (\( f = 5 \) MHz, Agilent 33120A function generator) and a 1.5 V dc (Agilent 6544A dc power supply) bias was applied to the two Au contact pads [P2 and \( \Omega \) in Fig. 1(b)]. Approximately 14 \( \mu l \) of T-CNT/ethanol dispersion (T-CNT in 100% ethanol) was dropped onto the device while the electric field was on. (The volume of the droplets should not exceed the reservoir volume surrounding the device.) The deposition was conducted on a probe station equipped with an optical microscope (Bausch & Lomb MicroZoom™) and a charge-coupled device (CCD) video camera (Hitachi HV-C20M). The deposition process was monitored on a TV screen connected to the CCD camera. Since the silicon is conductive, the electric field is mostly concentrated between the A and B platforms. During the deposition process, the T-CNTs were attracted and aligned across the Au coated surfaces of the platforms. (This was due to the native oxide coating on the exposed silicon surface beyond the gold and remaining polymer on the sidewall from the Bosch etching process.) The power was manually turned off about 30 s after a deposition of one T-CNT. The device was then dried in a critical point dryer (Polaron E3000), as described previously.

The MEMS device with the T-CNT assembled was then glued to a chip carrier (LDCC 28-pin leaded chip carrier, Spectrum Semiconductor Materials, Inc.). The electrical connection between the contact pads of the MEMS device and the chip carrier was made by wire bonding (West Bond Model 7400E). Pads \( \Omega 1 \), \( \Omega 2 \), and \( \Omega 3 \) were also connected with wire bonding. This assembly could then be mounted on a regular SEM sample stub with proper insulation between the pins of the LDCC chip carrier. Electrical signals were introduced through an electrical feedthrough (flange) on the SEM.

To perform tensile loading experiments, it is necessary to first clamp the T-CNT to the two platforms because the interaction between the T-CNT and substrate surface is not strong enough for loading the T-CNT. (We observed T-CNTs sliding on the testing stage without application of any adhesive or other clamps when the testing stage was actuated.) One method for making clamps is via electron beam induced deposition (EBID), which is a process of using an electron beam to decompose molecules and thereby deposit material on the surface region exposed to the e-beam. It is an effective method for fabricating local electrodes for transport measurements or clamps to hold one dimensional nanostructures, such as nanotubes, nanowires, or nanorods, for mechanical or electrical testing. The EBID process in a standard SEM relies on the residual hydrocarbon in the vacuum chamber, thus the deposition is relatively slow. It is challenging to fabricate strong enough clamps for testing (to fracture) T-CNTs of 200–400 nm diameter. To accelerate the clamping process we introduced a solid phase hydrocarbon source (paraffin) near the T-CNT. The greatly increased deposition rate, and the composition and mechanical properties (strength) of the EBID clamp from such a precursor have been discussed in our previous work.

From trial tests we learned that the paraffin should be placed within 20 \( \mu m \) of the desired area for rapid clamp deposition. The amount of paraffin material was limited by the surface size of each platform, thus it must be within an area less than 20 \( \times 40 \) \( \mu m^2 \). We developed a method to make an EBID deposit on the free-standing platform attached to the suspended microbeam that avoids breaking the suspended beam or wiping off the T-CNT. As mentioned, a glass micropipette was fabricated for delivering the paraffin from a liquid solution to the desired location. This is similar to the “nano-dispenser” and dip-pen lithography approaches that have also exploited a capillary bridge between a “fountain pen” writing element and the substrate. The paraffin was first dissolved in a liquid solution [5% weight paraffin dissolved in toluene; the paraffin used here was 99% \( n \)-hexacosane \((CH_{23}CH_{23}CH_{23}CH_{23}Alfa Aesar)]

The glass fountain pen with a tapered and sharp nozzle [at the tip, outside diameter (o.d.) \( \sim 10 \mu m \) and inside diameter (i.d.) \( \sim 5 \mu m \)] was fabricated using a borosilicate glass pipette having uniform diameter (o.d. 1.0 mm, i.d. 0.58 mm, 10 cm length, Warner Instrument Corp., Cat. No. G100-4). The pipette was pulled with a vertical pipette puller (David Kopf Instruments, model 700C) until it broke, which produced a tapered section along the axis of the glass pipette. This section was much more compliant than the beam structure of the MEMS device, so it did not break any silicon structures when touching the platforms and microbeams. The solution was injected into the glass pen (micropipette) from the wider end with a loading pipette (the loading pipette can be made by pulling, for example, the Fisher brand disposable soda-lime glass Pasteur pipette). The liquid solution was then pushed through the narrow orifice of the micropipette by using a FemtoJet device (Eppendorf, parameters: \( p_1 = 880 \) hPa, \( t_1 = 1–3 \) s, \( p_2 = 30 \) hPa). After a small amount of paraffin solution was pushed out of the narrow nozzle of the glass fountain pen, the toluene evaporates immediately, leaving a minute amount of paraffin deposit at the outer surface of the nozzle tip. Once this paraffin deposit is formed, the micropipette is removed from the FemtoJet unit and attached to a probe holder on the probe station to perform the deposition on the MEMS testing stage.

The process was monitored with the same optical microscope mentioned above. The probe holder has three degrees of freedom so that the pipette nozzle could be positioned close to that end of the T-CNT in contact with the platform. Once the nozzle was brought into contact with the top surface of platform A or B, the paraffin solution was drawn out of the pipette by the capillary force (between the paraffin cluster, edge of the pipette nozzle, and the stage surface) and also by wetting by the surface. The toluene rapidly evaporates, leaving paraffin on the surface near the T-CNT.

After the paraffin is deposited on each platform near the ends of the specimen, the device was transferred into the
SEM for clamp fabrication. Figure 2(a) shows an SEM image of such a device, with a T-CNT deposited across the two platforms and with paraffin nearby. The clamps were fabricated by scanning a small region such that the T-CNT end was located at the center of the scanned area, with the SEM electron beam at a magnification in the range of 100 000–120 000. The clamp was typically 1–1.4 μm long and 0.7–1 μm wide. The time needed to fabricate such a (comparatively) large clamp varied from 10 to 30 min, depending on the distance and the quantity of the paraffin nearby. Figure 2(b) is an enlarged view of the T-CNT in Fig. 2(a) after clamping. A close-up view of each clamp is shown in Figs. 2(c) and 2(d).

IV. MECHANICAL TESTING AND ANALYSIS OF THE RESULTS

After deposition of the carbonaceous clamp material by EBID, the T-CNT was then tensile loaded. A device with the same dimensions as shown in Fig. 1 was used. The resistance of the thermal expansion beams on each side of the central electrode was 86.4 Ω [beams 1 and 1’ in Fig. 1(a)]. The resistivity was 0.02 Ω cm, as determined with a four-point probe method and the dimensions measured from SEM images. The total resistance of the entire device including the electrical wires and soldering joints was about 45 Ω. The testing was carried out in the vacuum chamber of a LEO 1525 SEM (−10⁻⁴ Pa) with the dc current applied on the MEMS device and incremented in 10 mA steps. The voltage drop across the device was simultaneously recorded to obtain the supplied power. With the current running through the thermal expansion beams, the actuation unit (as defined above) retracts platform A away from platform B, which applies a tensile load to the T-CNT.

The gauge length of the T-CNT (chosen as the length between the inner edges of the two clamps) was measured from SEM images, usually at a magnification in the range 25 000–36 000. Strain was computed from the original length and the measured length during loading. The force on the T-CNT was calculated by $F = K_f \Delta x_f$, where $\Delta x_f$ is the deflection of the force-sensing beam and $K_f$ is the beam stiffness. The deflection of the force-sensing beam is obtained by measuring the displacement of platform B attached at the center of the force-sensing beam. A spring diagram that represents the force-sensing beam and the specimen (T-CNT) is shown in the inset of Fig. 3. From the equilibrium of forces, the load on the T-CNT from the actuator is equal to the force that the T-CNT applies on the force-sensing beam. The stiffness of the force-sensing beam was calculated from $K_f = \frac{24EI_f}{L_{fb}^3}$, where the silicon modulus $E$ for the (110) direction [169 GPa (Ref. 31)] is used, since the device is fabricated such that the long edge of the beam was along the (110) direction. $I_f$ is the moment of inertia of the beam, and $L_{fb}$ is the length from the center of the beam (at platform B) to one of the fixed ends, as in Fig. 1(a).

The diameters of the T-CNTs were measured in the SEM. As the T-CNT sample was synthesized by pyrolyzing ethylene in an alumina nanopore membrane, the T-CNT diameters mimic the i.d. of each of the pores in the membrane, and we have found that the T-CNTs that we made were not uniform along their entire length. Such T-CNTs also are amorphous in structure. An average diameter was used to
calculate the cross sectional area. For the particular T-CNT tested, the average outer diameter was 344 nm, and the average inner diameter was assumed to be half the outer diameter, as observed from the open ends of the T-CNT projecting past the clamps. With these values chosen for the (assumed average) geometry, a “stress” in the T-CNT can be calculated and plotted against the strain, as shown in Fig. 3. Young’s modulus, based on these assumptions, was obtained from a linear fit of the stress-strain curve, yielding straightforward.

somewhat nonuniform T-CNTs were chosen due to their size, also useful approaches for attaching nanowires and nanoparticles is a report of the development of a new device and future studies will involve testing more uniform nanowires; the somewhat nonuniform T-CNTs were chosen due to their size, which made their observation by standard light microscopy straightforward.

The device was further analyzed with a theoretical model developed by Lu et al. This model was used to obtain the stiffness of the T-CNT from the displacement of platform A with a load from the supporting beam only, versus with a different magnitude of load when a T-CNT is attached to the fixed platform, as

\[ K_m = \frac{(K_d + K_b)(x_1 - x_2)}{x_2} \]  

(1)

Here \( K_m \), \( K_d \), and \( K_b \) are the stiffness of the specimen (T-CNT) unit, the actuation unit, and the supporting beam, respectively. The stiffness of a “unit” is defined as the force required for the unit to have a unit displacement in a particular direction. For the specimen, \( K_m = A \cdot E_s / L_s \) and \( x_1 \) is the displacement of platform A without a T-CNT attached, while \( x_2 \) is the displacement of platform A with a T-CNT attached.

To apply this model for the new device used here, we consider the T-CNT \(( K_s )\) and the direct force-sensing beam \(( K_{fb} )\) as one specimen unit (with stiffness \( K_m \)). We also define a measurement unit composed of the specimen unit and the supporting beam. When actuating the device without a T-CNT assembled, the load applied on the actuation unit is only from the supporting beam \(( K_b )\). With the T-CNT present, the load will be from the entire measurement unit. Hence, Eq. (1) becomes

\[ K_m = \frac{(K_d + K_b)(x_1 - x_2)}{x_2} = \frac{1}{1/K_s + 1/K_{fb}} \]  

(2)

and the stiffness of the specimen (T-CNT) is

\[ \frac{1}{K_s} + \frac{1}{K_{fb}} = \frac{x_2}{(x_1 - x_2) \cdot (K_d + K_b)} \]

\[ \Rightarrow K_s = \frac{(x_1 - x_2)(K_d + K_b)K_{fb}}{x_2K_{fb} - (x_1 - x_2)(K_d + K_b)} \] 

(3)

\( K_{fb} \), \( K_d \), and \( K_b \) can be obtained from finite element analysis (FEA) by using programs such as ABAQUS with the actual beam dimensions as measured in the SEM.

The FEA simulation shows that the theoretical value of \((x_1 - x_2)/x_2\) is 0.0071. From the experimental results presented above, we already have the displacement \( x_2 \) with the load from the sample unit. To obtain the displacement \( x_1 \) with load from the supporting beam only, we cut the T-CNT by focused ion beam milling (Hitachi FIB 2000A) and measured the displacement of platform A \(( x_1 )\) while operating the device at 65.6 mW. The value of \((x_1 - x_2)/x_2\) from this experiment was \(0.0071\). This non-zero difference between the theoretical and measured values may be due to the measurement resolution (\(\sim 4\) nm pixel resolution at 100 000\(\times\) magnification). In this case, to detect \((x_1 - x_2)/x_2\) as small as the theoretical value of 0.0071 with a displacement \( x_1 \) on the order of 300 nm (for example, for the 65.6 mW power input), the difference \((x_1 - x_2)\) should be \(\sim 3.0\) nm, beyond the accuracy of our measurement.

Denoting the displacement of the force-sensing beam as \( x_3 \) (also as \(\Delta x_{fb} \) in the above analysis), the elongation of the T-CNT is \((x_2 - x_3)\). Alternatively, the specimen’s stiffness \( K_s \) can also be obtained from the force and its elongation directly from

\[ K_s = \frac{F}{x_2 - x_3} = \frac{x_2K_{fb}}{x_2 - x_3} \]  

(4)

There are two approaches for obtaining a higher accuracy measurement of \( K_s \): (i) to enlarge the elongation \((x_2 - x_3)\) of the specimen, which requires an increase in the stiffness of the force-sensing beam \((K_{fb})\), since \( x_3 \) should decrease, (ii) to achieve a more accurate reading of the force applied on the specimen (for the same load, a larger deflection \( x_3 \) thus a more compliant force-sensing beam, equivalently less stiffness). As often occurs, there are factors that “trade-off” in trying to find the optimal design. Preferably, the stiffness of the force-sensing beam \((K_{fb})\) should be of the same order as that of the specimen \((K_s)\). (One seems to need to prior knowledge of what one is attempting to measure.) For a nanowire with unknown mechanical properties a device without the direct force-sensing beam can be used for preliminary testing. Based on this preliminary value and the above equations, the device design can be optimized to have a direct force-sensing beam of proper stiffness to provide better results, yielding more accurate stress versus strain data.

The force-displacement response of similar T-CNTs was measured with a home-built nanomanipulator to compare with the results from the MEMS testing stage. The method of using AFM cantilevers for tensile loading was employed, as well as used for testing multiwalled carbon nanotubes and carbon nanocoils. T-CNTs from the same fabrication run were clamped between two opposing AFM cantilever tips (NSC12 \( k = 0.3\) N/m and \( k = 14.0\) N/m, MikroMasch, Inc.) with the EBID method. A tensile force was applied to the T-CNT by displacing the soft cantilever with voltage applied to a piezoelectric bimorph component. The relatively compliant cantilever was used as the force-sensing element and its deflection was obtained from the recorded SEM images, as were the values of elongation of the T-CNT at each separate load. The moduli obtained for two T-CNTs were 16 ± 2 and 30 ± 12 GPa, respectively. As mentioned, these T-CNTs have an amorphous structure, so the modulus obtained from using the MEMS testing device is in the right range for the amorphous carbon that they are comprised of. We note
that the T-CNTs may possibly contain hydrogen, that is, might be composed of hydrogenated amorphous carbon.

V. DISCUSSION

In summary, a MEMS-based tensile testing stage was developed, with integrated actuator, direct load sensing beam, and circuits for assembling an individual nanostructure, and the capability for in situ mechanical testing on individual templated carbon nanotubes (T-CNTs) in SEM was demonstrated. The device provides well-controlled nanoscale displacement and force for loading nanostructures. Novel approaches have been developed to align and mount an individual T-CNT across the two opposing platforms of the testing stage. A fountain- pen writing technique was developed to deposit paraffin for rapid clamping of the T-CNT in the SEM by electron beam induced deposition.

Tensile tests were performed on the T-CNT with the MEMS testing stage. The load was obtained from the deformation of the direct force-sensing beam and the strain was calculated from the recorded SEM image by pixel counting. Young’s modulus was obtained from a linear fit of the stress-strain curve. The fit value for the modulus of the T-CNT is evidently due to its atomic structure and composition. Based on a previous model, an experiment was conducted by cutting off the T-CNT, to verify the testing results. Independent tests of the stiffness of such T-CNTs were performed with a home-built nanomanipulator for comparison.

The experiments suggest the potential use of this MEMS testing stage for tensile testing on a variety of nanowires and nanotubes. Suggestions on optimizing the design and how to use the device, based on the theoretical simulation and the testing results, were presented for future testing of new nanostructures with unknown properties. Finally, the current modular design approach of defining different functional units, such as an actuation unit, measurement unit, and sample unit, provides flexibility, such as for design of a compressive testing stage.

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32. The diameter along the T-CNT was not constant. It was measured from SEM images and averaged (with four measurement points: one near the inner edge of each clamp, one at the thinnest cross section, and one between the thinnest point and one inner edge of clamp). The average cross sectional area $A$, is $6.97 \times 10^{-14}$ m$^2$.