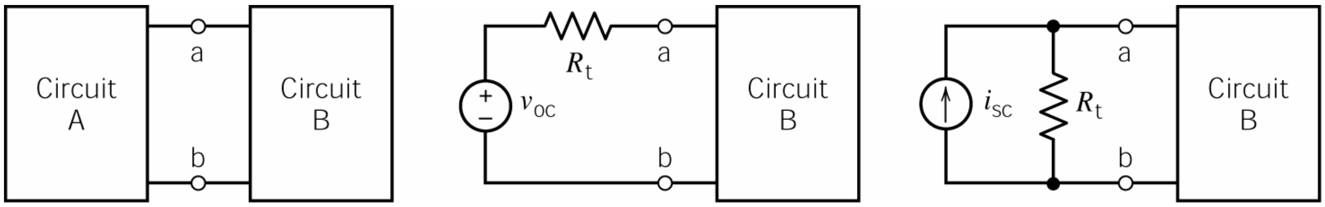
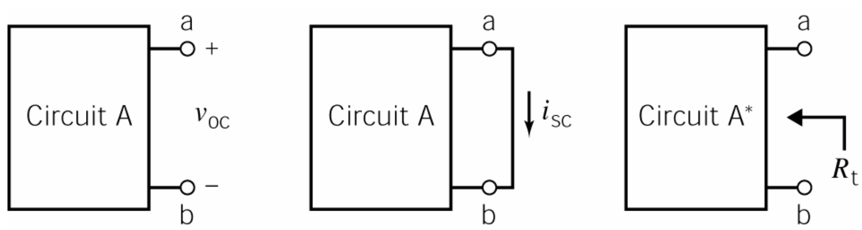
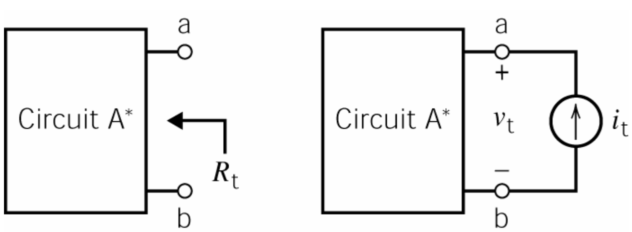






Circuit A* is obtained from Circuit A by

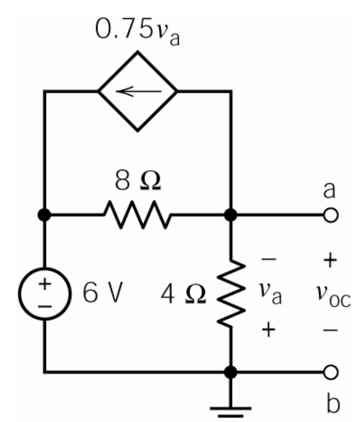
- replacing all independent voltage sources by short circuits

and

- replacing all independent current sources by open circuits.

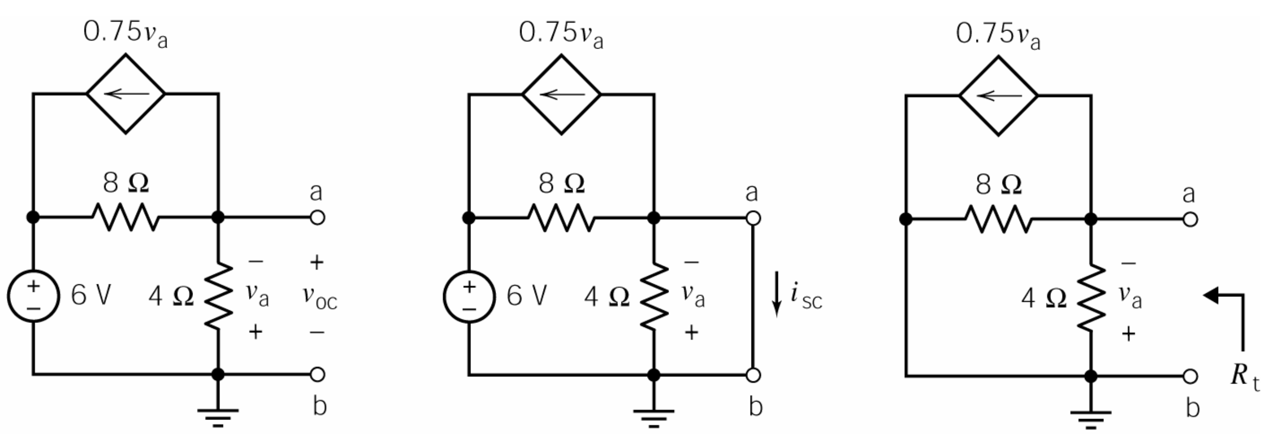
Problem 5.4-5

We want to find the Thevenin equivalent circuit for this circuit:



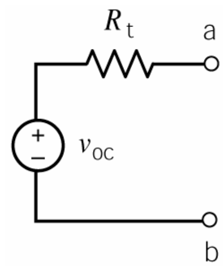
There is a dependent source in this circuit so we will not be able to reduce this circuit to its Thevenin equivalent circuit using source transformations and equivalent resistance.

We will need to find v_{oc}, i_{sc} and R_t. Figure 5.4-3 shows this is done. Replace “Circuit A” in Figure 5.4-3 with the circuit in this problem to get

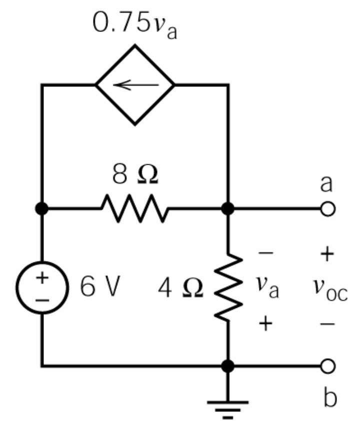


(*a*) (*b*) (*c*)

After we have determined the value of v_{oc}, i_{sc} and R_t, we can draw the Thevenin equivalent circuit as



Let's find v_{oc} using the circuit shown above in (*a*):



Notice that v_{oc} is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:

$$v_a = -v_{oc}$$

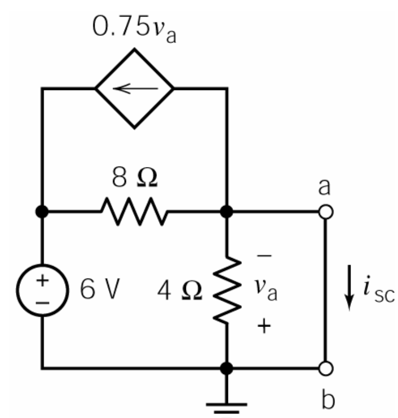
Apply KCL at node a:

$$-\left(\frac{6-v_{oc}}{8}\right)+\frac{v_{oc}}{4}+\left(-\frac{3}{4}v_{oc}\right)=0$$

so

$$-6+v_{oc}+2\,v_{oc}-6\,v_{oc}=0 \quad\Rightarrow\quad v_{oc}=-2\text{ V}$$

Let's find i_{sc} using the circuit shown above in (*b*):



Notice that the short circuit forces

$$v_a = 0$$

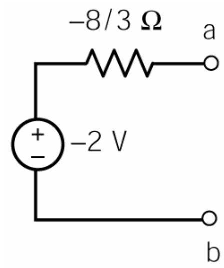
Apply KCL at node a:

$$-\left(\frac{6-0}{8}\right)+\frac{0}{4}+\left(-\frac{3}{4}(0)\right)+i_{sc}=0$$

Solving this equation gives

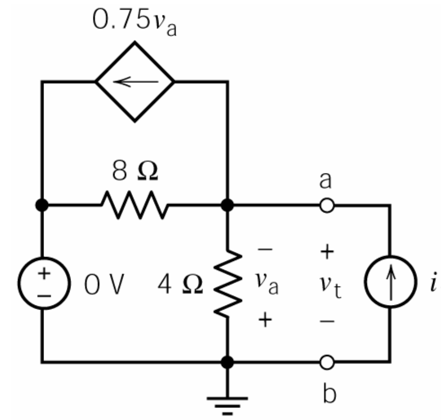
$$i_{sc}=\frac{6}{8}=\frac{3}{4}\text{ A}$$

Once we have found v_{oc} and i_{sc} we can calculate R_t using $R_t=\frac{v_{oc}}{i_{sc}}=\frac{-2}{\frac{3}{4}}=-\frac{8}{3}\ \Omega$. Then the Thevenin equivalent can be drawn as



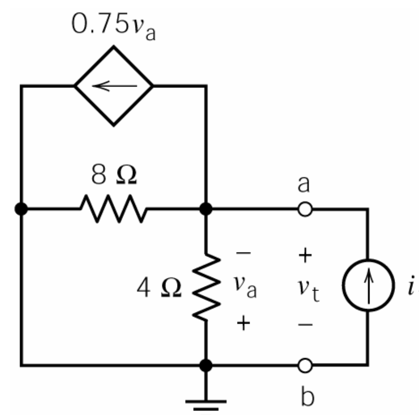
Alternate derivation of R_t:

Figure 5.4-4 illustrates a procedure for calculating R_t by connecting a current source across the terminals of Circuit A* and then calculating the voltage across that current source.



Notice that Figure 5.4-3(c) and Figure 5.4-4 refer to Circuit A* instead of Circuit A. Circuit A* is obtained from Circuit A by setting all of the ***independent*** sources to zero.

There is only one independent source in our circuit, the 6 V voltage source. A zero voltage source is equivalent to a short circuit. First we set that source to zero and the we replaced it with the equivalent short circuit.



We use this circuit to calculate the Thevenin resistance as the ratio

$$R_t = \frac{v_t}{i_t}$$

Let's do that calculation:

First, notice that $v_a = -v_t$. Next apply KCL at node a:

$$0.75\, v_a + \frac{v_t}{8} = \frac{v_a}{4} + i_t$$

so

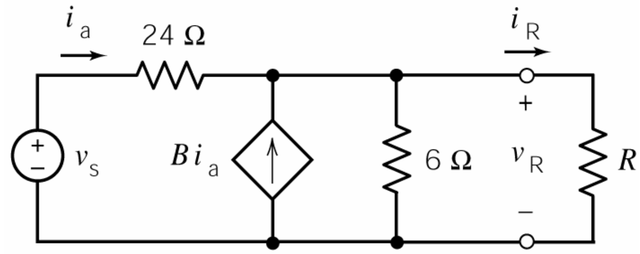
$$i_t = \frac{v_t}{8} + \frac{v_t}{4} - 0.75\, v_t = -\frac{3}{8} v_t$$

Then

$$\frac{v_t}{i_t} = -\frac{8}{3} \quad \Rightarrow \quad R_t = -\frac{8}{3}\ \Omega$$

Remark: We may be bothered by the negative Thevenin resistance. A negative Thevenin resistance may not be desirable, but it is a possibility when the circuit contains a dependent source.

2. Given that $0 \le R \le \infty$ in this circuit, consider these two observations:



When $R = 2\ \Omega$ then $v_R = 4$ V and $i_R = 2$ A.

When $R = 6\ \Omega$ then $v_R = 6$ V and $i_R = 1$ A.

Fill in the blanks in the following statements:

a. The maximum value of i_R is ____4_____A.

b. The maximum value of v_R is ____8_____V.

c. The maximum value of $p_R = i_R v_R$ occurs when R = _____2____Ω.

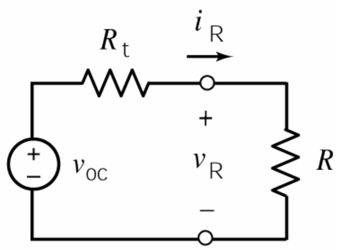
d. The maximum value of $p_R = i_R v_R$ is ____16____W.

e. When $R = 5\ \Omega$ then $v_R =$ __5.714__V.

f. When R = _____8____Ω then $v_R = 6.4$ V.

g. When R = _____14____Ω then $i_R = 500$ mA.

We can replace the part of the circuit to the left of the terminals by its Thevenin equivalent circuit:



Using voltage division $v_R = \dfrac{R}{R+R_t} v_{oc}$ and using Ohm's law $i_R = \dfrac{v_{oc}}{R+R_t}$.

By inspection , $v_R = \dfrac{R}{R+R_t} v_{oc} = \dfrac{v_{oc}}{1+\dfrac{R_t}{R}}$ will be maximum when R = 0. The maximum value of v_R will be v_{oc}. Similarly, $i_R = \dfrac{v_{oc}}{R+R_t}$ will be maximum when R = 0. The maximum value of i_R will be $\dfrac{v_{oc}}{R_t} = i_{sc}$.

The maximum power transfer theorem tells use that $p_R = i_R v_R$ will be maximum when $R = R_t$. Then

$$p_R = i_R v_R = \left(\frac{v_{oc}}{R+R_t}\right)\left(\frac{R}{R+R_t} v_{oc}\right) = R\left(\frac{v_{oc}}{R+R_t}\right)^2 .$$

Let's substitute the given data into the equation $i_R = \dfrac{v_{oc}}{R+R_t}$.

When $R = 2\ \Omega$ we get $2 = \dfrac{v_{oc}}{2+R_t} \quad \Rightarrow \quad 4 + 2R_t = v_{oc}$. When $R = 6\ \Omega$ we get $1 = \dfrac{v_{oc}}{6+R_t} \quad \Rightarrow \quad 6 + R_t = v_{oc}$.

So $6 + R_t = 4 + 2R_t \quad \Rightarrow \quad R_t = 2\ \Omega$ and $v_{oc} = 4 + 2R_t = 8$ V. Also $i_{sc} = \dfrac{v_{oc}}{R_t} = \dfrac{8}{2} = 4$ A.

Now the blanks can be easily filled-in.