

Example:

The input to a linear circuit is the voltage, v_i . The output is the voltage, v_o . The step response of the circuit is

$$v_o(t) = (40 + 1.03e^{-8t} - 41e^{-320t})u(t) \text{ V}$$

Determine the network function, $\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$, of the circuit. Sketch the asymptotic magnitude Bode plot.

Solution:

$$\frac{H(s)}{s} = \mathcal{L}[(40 + 1.03e^{-8t} - 41e^{-320t})u(t)] = \frac{40}{s} + \frac{1.03}{s+8} - \frac{41}{s+320} = \frac{102400}{s(s+8)(s+320)}$$

so

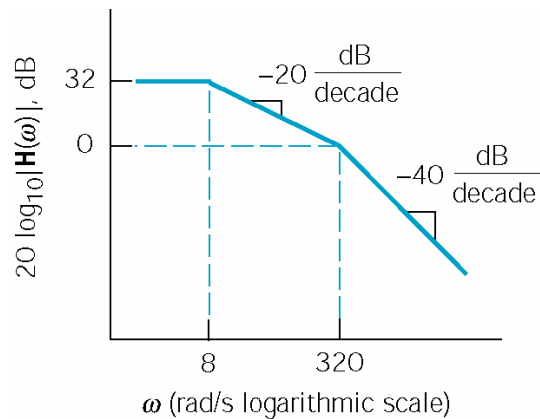
$$H(s) = \frac{102400}{(s+8)(s+320)}$$

The poles of the transfer function are $s_1 = -8 \text{ rad/s}$ and $s_2 = -320 \text{ rad/s}$, so circuit is stable.

Consequently,

$$\mathbf{H}(\omega) = H(s)|_{s=j\omega} = \frac{102400}{(j\omega+8)(j\omega+320)} = \frac{40}{\left(1+j\frac{\omega}{8}\right)\left(1+j\frac{\omega}{320}\right)}$$

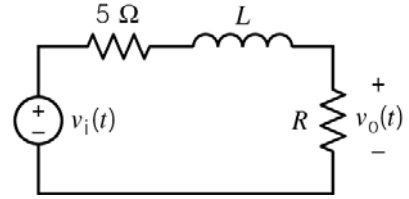
The network function has poles at 8 and 320 rad/s and has a low frequency gain equal to 32 dB = 40. Consequently, the asymptotic magnitude Bode plot is



Example:

The input to a circuit is the voltage source voltage, v_i . The step response of the circuit is

$$v_o(t) = \frac{3}{4}(1 - e^{-100t})u(t) \text{ V}$$



- Determine the value of the inductance, L , and of the resistance, R .
- Determine the impulse response of this circuit.
- Determine the steady-state response of this circuit when the input is

$$v_i(t) = 5 \cos(100t) \text{ V}.$$

Solution:

- From the given step response:

$$\frac{H(s)}{s} = \mathcal{L}\left[\frac{3}{4}(1 - e^{-100t})u(t)\right] = \frac{75}{s(s+100)}$$

From the circuit:

$$H(s) = \frac{R}{R+5+Ls} \Rightarrow \frac{H(s)}{s} = \frac{\frac{R}{L}}{s\left(s + \frac{R+5}{L}\right)}$$

Comparing gives

$$\left. \begin{array}{l} \frac{R}{L} = 75 \\ \frac{R+5}{L} = 100 \end{array} \right\} \Rightarrow \begin{array}{l} R = 15 \Omega \\ L = 0.2 \text{ H} \end{array}$$

- The impulse response is

$$h(t) = \mathcal{L}^{-1}\left[\frac{75}{s+100}\right] = 75 e^{-100t}u(t)$$

- The steady-state response is

$$\mathbf{H}(\omega)\big|_{\omega=100} = \frac{75}{j100+100} = \frac{3}{4\sqrt{2}} \angle -45^\circ$$

$$\mathbf{V}_o(\omega) = \left(\frac{3}{4\sqrt{2}} \angle 45^\circ\right)(5 \angle 0^\circ) = \frac{15}{4\sqrt{2}} \angle -45^\circ \text{ V}$$

$$v_o(t) = 2.652 \cos(100t - 45^\circ) \text{ V}$$

Example:

The input to a circuit is the voltage source voltage, v_i . The step response of the circuit is

$$v_o(t) = 5(1 - (1 + 2t)e^{-2t})u(t) \text{ V}$$

Determine the steady-state response of this circuit when the input is

$$v_i(t) = 5 \cos(2t + 45^\circ) \text{ V}$$

Solution:

The transfer function of this circuit is given by

$$\frac{H(s)}{s} = \mathcal{L}[(5 - 5e^{-2t}(1 + 2t))u(t)] = \frac{5}{s} + \frac{-5}{s+2} + \frac{-10}{(s+2)^2} = \frac{20}{(s+2)^2} \Rightarrow H(s) = \frac{20}{(s+2)^2}$$

This transfer function is stable so we can determine the network function as

$$\mathbf{H}(\omega) = H(s)|_{s=j\omega} = \frac{20}{(s+2)^2}|_{s=j\omega} = \frac{20}{(2+j\omega)^2}$$

The phasor of the output is

$$\mathbf{V}_o(\omega) = \frac{20}{(2+j2)^2} (5 \angle 45^\circ) = \frac{20}{(2\sqrt{2} \angle 45^\circ)^2} (5 \angle 45^\circ) = 12.5 \angle -45^\circ \text{ V}$$

The steady-state response is

$$v_o(t) = 12.5 \cos(2t - 45^\circ) \text{ V}$$

Example

The input to a linear circuit is the voltage, v_i . The output is the voltage, v_o . The impulse response of the circuit is

$$h(t) = 30t e^{-5t} u(t) \text{ V}$$

Determine the steady-state response of this circuit when the input is

$$v_i(t) = 10 \cos(3t) \text{ V}$$

Solution:

The transfer function of the circuit is $H(s) = \mathcal{L}^{-1}[30t e^{-5t} u(t)] = \frac{30}{(s+5)^2}$. The circuit is stable so we can determine the network function as

$$\mathbf{H}(\omega) = H(s) \Big|_{s=j\omega} = \frac{30}{(s+5)^2} \Big|_{s=j\omega} = \frac{30}{(5+j\omega)^2}$$

The phasor of the output is

$$\mathbf{V}_o(\omega) = \frac{30}{(5+j3)^2} (10 \angle 0^\circ) = \frac{30}{(5.83 \angle 31^\circ)^2} (10 \angle 0^\circ) = 8.82 \angle -62^\circ \text{ V}$$

The steady-state response is

$$v_o(t) = 8.82 \cos(3t - 62^\circ) \text{ V}$$