Instructions:
- Show your work.
- Answers without sufficient justification may not receive full credit.
- No books, notes, or calculators.
- Time limit: 90 minutes.
- Some integrals which might help:
  \[ \int \sec(x) \, dx = \ln |\sec(x) + \tan(x)| + C \]
  \[ \int \csc(x) \, dx = \ln |\csc(x) - \cot(x)| + C \]

Do not write below this line

Problem 1: _______ 12 points
Problem 2: _______ 8 points
Problem 3: _______ 12 points
Problem 4: _______ 8 points
Problem 5: _______ 10 points
Problem 6: _______ 10 points
Problem 7: _______ 8 points
Problem 8: _______ 12 points
Problem 9: _______ 12 points
Problem 10: _______ 8 points

Score = [ ] 100 points total
(12) **Problem 1.** Evaluate the integral: \( \int \tan^2(x) \sec^4(x) \, dx \)
Problem 2. For each of the following, give the form of the partial fraction decomposition. Do not solve for the unknown constants. Hint: Factor where possible.

(a) \( \frac{x^3 + 3x^2 - 5}{(x - 2)(x + 5)^2(x^2 + 3)^2} \)

(b) \( \frac{x^4 - 2x^2 + 17}{x^3(x^2 - 1)(x^2 + x - 6)} \)
Problem 3. Evaluate the integral: \[ \int \frac{1}{\sqrt{x^2 - 2x + 5}} \, dx \]
(8) **Problem 4.** The speed $v(t)$ of a crawling slug\(^1\) for certain values of time $t$ is given in the table:

<table>
<thead>
<tr>
<th>$t$ (minutes)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v(t)$ (cm/minute)</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Estimate the distance traveled by the slug from $t = 0$ to $t = 8$ minutes.

(a) Use the Trapezoidal rule with $n = 4$ subintervals.

(b) Use the Midpoint rule with $n = 2$ subintervals.

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\(^1\)measured with a radar gun
Problem 5. Evaluate the integral: \[ \int \frac{x}{x + 3} \, dx \]
Problem 6. Evaluate the integral: $\int \theta^2 \sin(5\theta) \, d\theta$
Problem 7. Evaluate the following limits. If you use L’Hospital’s Rule, show where you used it and that it applies.

(a) \( \lim_{x \to \infty} x \sin \left( \frac{1}{x} \right) \)

(b) \( \lim_{x \to \infty} x^{2/x} \)
Problem 8. Evaluate the integral: \[ \int \frac{x - 6}{x^3 - 2x^2} \, dx \]
(12) **Problem 9.** Determine whether the following integral is convergent or divergent. If it is convergent, evaluate it.

\[ \int_1^\infty \frac{1}{(2x + 1)^2} \, dx \]
Problem 10. The integral $\int_{-1}^{2} f(x) \, dx$ of the function $f(x)$ shown in the graph below is to be estimated by numerical integration. Rank the following five numbers in order of size from smallest to largest:

- $L_n$: left endpoint (left sum) approximation
- $R_n$: right endpoint (right sum) approximation
- $M_n$: midpoint approximation
- $T_n$: trapezoidal approximation
- $I$: the exact value of the integral $\int_{-1}^{2} f(x) \, dx$

Note: each approximation is computed using the same number $n$ of subintervals.