**Air Loads**

**Airfoil Geometry**

- **Camber line** - line joining the midpoints between upper and lower surfaces.
- **Chord line** - straight line joining end points of camber line (length=c)
- **Camber** - max. distance of camber line from chord line (expressed as %c; usually less than 5%c)

\[
Z_u = (Z_c + Z_t) \quad Z_l = (Z_c - Z_t)
\]
Forces and Moments

\[ \alpha = \text{Angle of attack} \]

\[ F = f(\alpha, M, R_N), \rho V^2 S \]

For airfoil, \[ S = c \times 1 \text{(unit span)} \]

Lift co-efficient \[ C_l = \frac{l}{\frac{1}{2} \rho V^2 c} = \frac{l}{q} \]

Drag co-efficient \[ C_d = \frac{d}{q} \]

Pitching moment coefficient \[ C_m = \frac{m}{q c^2} \]

l = lift = \( F \cos \alpha \)

d = drag = \( F \sin \alpha \)

m = pitching moment
Wing Planform

- **Aerodynamic characteristics** generally based on gross wing area (assumed extended up to fuselage centerline)
- Exposed wing (only outside fuselage) area used for skin friction drag

$$S_{wing} = \frac{b}{2}(c_i + c_r) = 2 \int_0^{b/2} c(y)dy = \frac{b}{2}c_r(1 + \lambda)$$

Wing aspect ratio, $$A = \frac{b^2}{S_{wing}}$$

Wing taper ratio, $$\lambda = \frac{c_i}{c_r}$$
Mean Aerodynamic Chord

Mean Aerodynamic Chord (MAC),

$$\overline{c} = \frac{\int_0^{b/2} c^2(y)dy}{\int_0^{b/2} c(y)dy} = \frac{2}{S} \int_0^{b/2} c^2(y)dy = \frac{2}{3} c_r \left( \frac{1 + \lambda + \lambda^2}{1 + \lambda} \right)$$

$$y_{mac} = \frac{2}{S} \int_0^{b/2} c(y) y dy = \frac{b}{6} \left( \frac{1 + 2\lambda}{1 + \lambda} \right)$$

(Aerodynamic center at \((1/4)\overline{c}\) for subsonic \(M\) )
The V-n Diagram (Flight Envelope)

V-n (velocity-load factor) diagram for a typical Jet Trainer (1 knot = 1.15 mph)
V-n Diagram

- Limit load is the safe limit up to which there is no permanent deformation.
- Ultimate load factor – Structural failure occurs when $n>n_{\text{ultimate}}$.
- V-n (velocity-load factor) diagram includes both aerodynamic and structural limitations and establishes maneuver boundaries.
- Curve A-B: aerodynamic limit on load factor, imposed by $(C_L)_{\text{max}}$

$$n_{\text{max}} = \frac{1}{2} \rho V^2 \frac{(C_L)_{\text{max}}}{{W / S}}$$

Pt 1 $C_L < C_{L\text{max}}$, $n < n_{\text{max}}$

Pt 2 $C_L = C_{L\text{max}}$, $n = n_{\text{max}}$

Pt 3 Outside flight envelope

As $V$ increases, $n_{\text{max}}$ possible also increases ($V_4$)
V-n Diagram

- Horizontal line B-C: Positive limit load factor of the structure

\[ AtV > V^*, \text{where} V^* = \left[ \frac{2n_{\text{max}} W}{\rho C_{L_{\text{max}}} S} \right]^{1/2}, C_L < C_{L_{\text{max}}} \]

- Line C-D: high speed limit set by maximum dynamic pressure (design dive speed, \( V_{CD} \))

At higher speeds, undesirable instabilities (like flutter, aileron reversal, divergence, buffeting etc.) may occur.

\( V_{CD} = 1.5 \times V_{\text{max,cruise}} \) (max. cruise velocity) (FAR part 25 airplanes)

For supersonic aircraft, \((V_{\text{max}}/a_{sL}) = \text{Max. Mach no. in level flight} + 0.2\) (a=speed of sound)
V-n Diagram

- Maneuver pt. B: $C_L$ and $n$ are simultaneously at their highest possible values. Highest Instantaneous Turn Rate ($V^* =$corner velocity)

$$\frac{1}{2} \rho_{SL} V_e^2 = \frac{1}{2} \rho V^2$$

$\rho_{SL} =$ Sea level density; $V_e =$ Equivalent air speed (EAS)

$\rho =$ density at flight altitude; $V =$ True air speed (TAS)

- Curve A-E: Negative $C_{L\text{max}}$ limit (flow separation from bottom surface)

- Line E-D: Negative limit load factor (Why different from the positive $n_{\text{max}}$? - skin thickness)
Air Load Distribution on Lifting Surfaces

- Use high $\alpha$ (CL max) limit and max “q” limit points for load calculations on wings.
- Spanwise lift distribution is proportional to the circulation at each span station. For an elliptical planform, lift distribution is elliptical. For non-elliptical wings, use Schrenk’s approximation (semi-empirical) to estimate lift distribution (“Load distribution on a wing is the average of actual planform shape and an elliptic shape of the same span and area.”)

\[
Trapezoidal : c(y) = C_r \left[ 1 - \frac{2y}{b} (1 - \lambda) \right]
\]

\[
Elliptical : c(y) = \frac{4S}{\pi b} \sqrt{1 - \left( \frac{2y}{b} \right)^2}
\]

- Schrenk’s method is not applicable to highly swept wings (such as delta wings) due to vortex flow

AE212 Jha    Loads-9
Shear Forces and Bending Moments

Beam (wing) with distributed load

Ultimate Load on each wing, \( L_w = (W \times n \times 1.5 / 2) \)
Shear Forces and Bending Moments

- For any span station, the shear force is simply the sum of the vertical loads outboard of that station (or, the integral of distributed load).
- Bending moment at any station equals the sum of product of load at each outboard station and its distance from the station.
- For a positive Bending Moment (as shown in the figure), the internal forces produce compression on upper part and tension on lower part.
- Wing weight is proportional to $\frac{1}{\sqrt{t/c}}$. Halving (t/c) increases wing weight by 41%. Wing weight is typically 15% of total empty weight of aircraft.
- Add fuel weight to empty wing weight to obtain gross wing weight.
- Chordwise lift distribution may be approximated as shown below.
SF, BM, Torsion Calculation

(1) Pick load cases from V-n diagram (max AoA, max dynamic pressure, max. negative AoA , etc.)

(2) Calculate total lift force (approx. normal force);

Load on each wing, \( L_w = (W * n*1.5 / 2) \)

(3) Approx. wing as ‘strips’ from center line to tip (e.g., 10 strips of 10% semi-span each)

(4) Distribute lift force on each strip using Schrenk’s approximation

(6) Estimate shear force and bending moment

(7) Use wing center of pressure at 25% chord (subsonic speeds)

(8) Using same strips as in (3), calculate torque about front spar location (say, 15% chord). Then sum torque values from tip to root
Example - SF, BM, Torsion Calculation

Rectangular wing: chord = 0.5 m, span = 4 m, TOGW = 5,000 N, \( n_{\text{max}} = 4 \)

Wing area = 2 sq m, \( \text{AR} = \frac{b}{c} = 8 \)

Calculate total lift force (approx. normal force) on each wing:

\[
L_w = (W \times n \times 1.5 / 2) = 15,000 \text{ N (Ultimate load on each wing)}
\]

Distribute \( L_w \) along wing span using strips of equal width

Use 3 strips for this example problem

Chord for elliptical wing

\[
c(y) = \frac{4S}{\pi b} \sqrt{1 - \left(\frac{2y}{b}\right)^2} = \frac{4(2)}{\pi(4)} \sqrt{1 - \left(\frac{2y}{4}\right)^2} = 0.637 \sqrt{1 - \left(\frac{y}{2}\right)^2}
\]

<table>
<thead>
<tr>
<th>y-station</th>
<th>Wing chord, c</th>
<th>Elliptical c(y)</th>
<th>Average chord</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.637</td>
<td>0.569</td>
</tr>
<tr>
<td>0.66</td>
<td>0.5</td>
<td>0.601</td>
<td>0.550</td>
</tr>
<tr>
<td>1.33</td>
<td>0.5</td>
<td>0.475</td>
<td>0.488</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.250</td>
<td></td>
</tr>
</tbody>
</table>
Example - SF, BM, Torsion Calculation

Distribute lift force on each strip using Schrenk’s approximation

Calculate strip area = (Average of geometric and elliptical chord)*width

= Average chord*0.667

Calculate “factor” for lift distribution: \( L_w = (\text{factor})*(\text{sum of strip areas}) \)

\( 15,000 \text{ N} = (\text{factor})*(0.965 \text{ sq m}) \)

\[ \text{factor} = 15,544 \text{ N/sq m} \]

<table>
<thead>
<tr>
<th>Strip</th>
<th>Strip area</th>
<th>Lift on each strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.373</td>
<td>5798 N</td>
</tr>
<tr>
<td>2</td>
<td>0.346</td>
<td>5378 N</td>
</tr>
<tr>
<td>3</td>
<td>0.246</td>
<td>3824 N</td>
</tr>
</tbody>
</table>
Example - SF, BM, Torsion Calculation

Estimate shear force and bending moment

SF at any y-station = sum of lift force outboard of y-station

BM at any y-station = sum of (lift force * distance) outboard of y-station

For calculating distance, assume lift acting through the center of strip width

<table>
<thead>
<tr>
<th>y-station</th>
<th>Shear Force, N</th>
<th>Bending Moment, N-m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15000</td>
<td>13676 (6361+5381+1934)</td>
</tr>
<tr>
<td>0.667</td>
<td>9202</td>
<td>5620 (3826+1794)</td>
</tr>
<tr>
<td>1.33</td>
<td>3824</td>
<td>1275</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Calculate torque about 15% c using wing center of pressure at 25% c (good approximation at subsonic speeds); sum torque values from tip to root

<table>
<thead>
<tr>
<th>Strip</th>
<th>Torque, N-m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>750 (460.1+289)</td>
</tr>
<tr>
<td>2</td>
<td>460.1 (191.2+268.9)</td>
</tr>
<tr>
<td>3</td>
<td>191.2</td>
</tr>
</tbody>
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