We begin with the concept of **Equilibrium (Trim)**. *Equilibrium is a state of an object when it is at rest or in steady uniform motion,* (i.e., with constant linear and angular momenta).

- The resultant of all forces and moment about the CG must both be equal to zero.

**Stability** is defined as the ability of an aircraft to return to a given equilibrium state after a disturbance (it is a property of the equilibrium state)

**STATICALLY STABLE** when

- if it is disturbed from its equilibrium state by a small displacement, then
- the set of forces and moments so caused initially tend to return the aircraft to its original state
Trimmed Flight
(or steady unaccelerated flight)

- Trimmed flight when all the forces and moments are balanced
  (thrust = drag; lift = weight; pitching moment = 0; yawing moment = 0; rolling moment = 0)
  \[ \sum \text{Forces} = 0; \quad \sum \text{Moments} = 0 \]

- The steady flight condition may involve a steady acceleration e.g. a correctly banked turn, or a steady dive or climb.
- Pitch trim would be accomplished by deflecting the horizontal stabilizer, the elevator, or the elevator trim tab.

\[ M_G = 0 \quad \text{for trim} \]

- Trimmed state IS NOT NECESSARILY A STABLE STATE
  - i.e. all the forces and moments may be balanced, but as soon as the state is perturbed the aircraft departs from equilibrium.

Types of Stability

Figure 17-22 Types of Stability
Static Stability

Static stability of a body is an initial tendency of that body to return to its equilibrium state after a disturbance.

Static longitudinal instability
In this case there is no tendency to return to equilibrium.
Any disturbance from equilibrium leads to a larger disturbance, the motion is said to be divergent.

Neutral static stability is the boundary between stability and instability, there is still no tendency to return to equilibrium, the motion is therefore not stable.
But, the motion does not diverge.

Energy is being dissipated
Positive damping
Energy is added to the system
Negative damping

Artificial damping is needed → Stability Augmentation System (SAS)

Static Stability

Lift = weight
Thrust = drag
No net moments

(a) Equilibrium flight.

Equilibrium
Disturbed moments increase disturbed condition
Statically unstable divergent

(b) Statically unstable airplane.

Equilibrium
Disrupted
No moments - airplane holds disturbed condition

(c) Neutral static stability.
Dynamic Stability

DYNAMIC STABILITY characterizes the time history of motion after a disturbance from equilibrium.

An aircraft is said to be dynamically stable if, after a disturbance, it eventually returns to its equilibrium state and remains there.

ABSOLUTE dynamic stability is not concerned with how long this return takes.

RELATIVE dynamic stability examines how long it takes and what the behavior of that return motion is.

To be dynamically stable, a system must first be statically stable.

A system can be dynamically unstable and be statically stable -- but not vice versa.

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Dynamic Stability

![Graphs showing dynamic stability](image)

(a) Dynamically Stable

- aperiodic response
- constant

(b) Dynamically Neutrally Stable

- aperiodic divergence
- undamped oscillation

(c) Dynamically Unstable

- damped oscillation
- divergent oscillation
Dynamic Stability

Pilot-Induced Oscillation

- PIOs occur when a pilot over-controls an aircraft and a sustained oscillation results.
- Pilot-induced oscillations occur when the pilot of an aircraft inadvertently commands an often increasing series of corrections in opposite directions, each an attempt to correct for the previous overcorrection with an overcorrection in the opposite direction. The physics of flight make such oscillations more probable for pilots than for automobile drivers. An attempt to cause the aircraft to climb, say by applying up elevator will also result in a reduction in airspeed.
- Another factor is the response rate of flight instruments in comparison to the response rate of the aircraft itself. An increase in power will not result in an immediate increase in airspeed. An increase in climb rate will not show up immediately on the vertical speed indicator.
- A pilot aiming for a 500 foot per minute descent, for example, may find himself descending too rapidly. He begins to apply up elevator until the vertical speed indicator shows 500 feet per minute. However, because the vertical speed indicator lags the actual vertical speed, he is actually descending at much less than 500 feet per minute. He then begins applying down elevator until the vertical speed indicator reads 500 feet per minute, starting the cycle over. It’s harder than it might seem to stabilize the vertical speed because the airspeed also constantly changes.
- The most dangerous pilot-induced oscillations can occur during landing. A bit too much up elevator during the flare can result in the plane getting dangerously slow and threatening to stall. A natural reaction to this is to push the nose down harder than one pulled it up, but then the pilot finds himself staring at the ground. An even larger amount of up elevator starts the cycle over again.

http://www.dfrc.nasa.gov/Gallery/Movie/F-8DFBW/HTML/EM-0044-01.html
**Statically Stable Response**

- Balanced – positive pitch stiffness (restoring moment)

Other necessary condition to trim at positive angle of attack, \( \alpha \)

\[
C_{m} = \frac{dC_{m}}{d\alpha} = \frac{dC_{L}}{d\alpha} \quad \text{For static stability} \quad \frac{dC_{m}}{dC_{L}} < 0
\]

**Longitudinal Static Stability**

Longitudinal static stability moments as a function of angle of attack. The curve is a composite of all the moment curves caused by the different components of the airplane, (the wing, fuselage, tail, thrust, etc).
Stable, neutral, and unstable static stability

DC-9. Note the contributions from the various components and the highly nonlinear post-stall characteristics
There are different degrees of stability
Some aircraft tend to return to equilibrium faster
An aircraft can be stable at lower angles of attack but may be unstable at higher angles of attack
\[ \sum \text{Moments} = M_{cgw} \]

\[ M_{cgw} = L_w \cos(\alpha_w - i_w) \left[ x_{cg} - x_{ac} \right] + D_w \sin(\alpha_w - i_w) \left[ x_{cg} - x_{ac} \right] \\
+ L_w \sin(\alpha_w - i_w) \left[ z_{cg} \right] - D_w \cos(\alpha_w - i_w) \left[ z_{cg} \right] + M_{aw} \]

Dividing for \( \frac{1}{2} \rho V^2 \sigma \):

\[ C_{m cgw} = C_{Lw} \cos(\alpha_w - i_w) \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right] + C_{Dw} \sin(\alpha_w - i_w) \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right] \\
+ C_{Lw} \sin(\alpha_w - i_w) \left[ \frac{z_{cg}}{c} \right] - C_{Dw} \cos(\alpha_w - i_w) \left[ \frac{z_{cg}}{c} \right] + C_{macw} \]

\[ \cos(\alpha_w - i_w) = 1; \quad \sin(\alpha_w - i_w) = \alpha_w - i_w \]

\[ C_{Lw} \gg C_{Dw} \]

\[ C_{m cgw} = C_{Lw} \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right] + C_{Lw} (\alpha_w - i_w) \left[ \frac{z_{cg}}{c} \right] + C_{macw} \]

\[ C_{Lw} (\alpha_w - i_w) \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right] \text{ negligible} \]

\[ C_{m cgw} = C_{macw} + C_{Lw} \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right]; \quad C_{m cgw} = C_{macw} + \left( C_{L0w} + C_{Lsw \alpha_w} \right) \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right] \]

\[ C_{Lw} = C_{L0w} + C_{Lsw \alpha_w} \quad \text{Lift Coefficient} \]
Nonlinear contributions

\[ C_{mcg_w} = C_{m_w} + C_{L_w} \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right] + C_{D_w} (\alpha_w - i_w) \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right] + \left[ C_{L_w} \alpha_w - i_w \right] - C_{D_w} \left[ \frac{z_{cg}}{c} \right] \]

\[ C_{L_w} = C_{L0_w} + C_{L_w} \alpha_w \]

\[ C_{D_w} = C_{d_w} + \frac{C_{L_w}^2 \alpha_w^2}{\pi eAR} \]

“Wind drag turn”

Wing Contribution

\[ C_{mcg_w} = C_{m_0_w} + C_{m_w} \alpha_w \]

\[ C_{m_0_w} = C_{m_0_w} + C_{L_w} \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right] \]

\[ C_{m_w} = C_{L_w} \left[ \frac{x_{cg}}{c} - \frac{x_{ac}}{c} \right] \]

To have a wing alone statically stable \( C_{m_w} < 0 \) \( x_{cg} < x_{ac} \)

To be able to trim the aircraft at positive angle of attach \( C_{m_0_w} > 0 \)
Wing Contribution

- **Positive camber** give nose-down pitching moment
- **Negatively cambered airfoil** gives nose-up pitching moment and cancels nose-down moment caused by lift and weight vectors
- For straight-winged, tailless airplane, negative camber satisfies conditions for stable, balanced flight
- Not in general use
  - Dynamic characteristics poor
  - Drag and $C_{\text{max}}$ poor

Swept back wing with twisted tips
Conventional and forward tail arrangement

A) Conventional arrangement

B) Forward tail arrangement

Tailless Aircraft

One example of a tailless aircraft that trims using a positive $C_{m_0}$ airfoil section: the AeroVironment Pathfinder, solar-powered aircraft on a flight to over 50,000 ft (15.2 km).
Example # 1

- For a given wing-body combination, the aerodynamic center lies 0.03 chord length ahead of the center of gravity. The moment coefficient about the center of gravity is 0.0050, and the lift coefficient is 0.50.
- Calculate the moment coefficient about the aerodynamic center.

\[
C_{M_{ac,w}} = C_{M_{cg,w}} + C_{L_{w}} \left( \frac{x_{cg}}{c} - \frac{x_{ac,w}}{c} \right)
\]

\[
C_{M_{ac,w}} = C_{M_{cg,w}} - C_{L_{w}} \left( \frac{x_{cg}}{c} - \frac{x_{ac,w}}{c} \right)
\]

\[
C_{M_{ac,w}} = 0.005 - 0.5(0.03) = -0.01
\]

Example # 2

- Consider a model of a wing-body shape mounted in a wind tunnel. The flow conditions in the test section are standard sea-level properties with a velocity of 100 m/s. The wing area and chord are 1.5 m\(^2\) and 0.45 m, respectively.
- Using the wind tunnel force and moment-measuring balance, the moment about the center of gravity when the lift is zero is found to be -12.4 N • m.
- When the model is pitched to another angle of attack, the lift and moment about the center of gravity are measured to be 3675 N and 20.67 N • m, respectively.
- Calculate the value of the moment coefficient about the aerodynamic center and the location of the aerodynamic center.
Example # 2 Cont’d

\[ q_{\infty} = \frac{1}{2} \rho_{\infty} V_{\infty}^2 = \frac{1}{2} 0.225 * 100^2 = 6125 N/m^2 \]

\[ C_{m_{cg,w}} = \frac{M_{cg,w}}{q_{\infty} S c} = \frac{-12.4}{6125 * 1.5 * 0.45} = -0.003 \]

\[ C_{m_{cg,w}} = C_{m_{ac,w}} = -0.003 \text{ at zero lift} \]

Example # 2 Cont’d

\[ C_{Lw} = \frac{L}{q_{\infty} S} = \frac{3675}{6125 * 1.5} = 0.4 \]

\[ C_{m_{cg,w}} = \frac{M_{cg,w}}{q_{\infty} S c} = \frac{20.67}{6125 * 1.5 * 0.45} = 0.005 \]

\[ C_{m_{cg,w}} = C_{m_{ac,w}} + C_{Lw} \left( \frac{x_{cg}}{c} - \frac{x_{ac,w}}{c} \right) \]

\[ \frac{x_{cg}}{c} - \frac{x_{ac,w}}{c} = \frac{C_{m_{cg,w}} - C_{m_{ac,w}}}{C_{Lw}} = \frac{0.005 - (-0.003)}{0.4} = 0.02 \]