

**INDUCTION MOTOR PERFORMANCE WHEN FED FROM SINGLE TO  
THREE PHASE CONVERTERS**

P. Pillay, MIEEE and J. Brzezinski  
Department of Electrical & Electronic Engineering  
University of Newcastle upon Tyne, NE1 7RU, England

**ABSTRACT**

The most popular single to three phase converter consists of a capacitor connected between the motor's third phase and either the live or neutral of the single phase supply. Proper voltage balance at any one given operating point is obtainable with the autotransformer-capacitor converter (ACC), while proper balance over the entire operating range is obtainable with a variable inductor-variable capacitor converter (LCC). The appropriate vector diagrams of the above mentioned converters are developed to provide insight into their operation. An assessment of the derating necessary in the presence of unbalanced voltages and currents is presented. A technique for the measurement of the negative sequence voltage is discussed. Measured results when an induction machine starts directly off a three phase supply, and off the above converters are included.

**1. INTRODUCTION**

Single to three phase converters are used to drive three phase machines (normally induction motors) from single phase supplies. The need for these converters arise in remote locations like farms or home industries which may only have single phase supplies, but may have three phase induction motors (IMs) driving pumps, grinders, drills, woodworking and textile machinery. Electrification in developing countries is also often based on a single-phase, earth-return system which make these converters necessary if three phase machinery is used.

The first type of single to three phase converter developed consisted of a capacitor (capacitor-converter (CC)) connected to an IM fed from a single phase supply as shown in figure 1. This system has been the subject of a fair amount of analysis, with a heavy reliance on positive and negative sequence theory as a tool. The magnitude of the capacitance needed to provide the best possible balance has been calculated [1] while the transient behavior has also been examined [2-4]. Where several machines are to be fed from a single converter, the rotating or "Ferraris-arno" system [5] has been used as shown in figure 2. This consists of an unloaded IM with a

capacitor single to three phase converter acting as the phase balancer; the hp rating of the so-called "pilot motor" having to be equal to the sum of the hp ratings of all the load motors for proper performance. The calculation of the optimum magnitude of capacitance for a variety of operating conditions has also been presented [6].

When the CC is used, it is not normally possible to obtain a balanced set of motor terminal voltages at the operating power factor of most IMs. Proper balance at any one chosen power factor (except unity), is possible by using an autotransformer in addition to the capacitor, known as a autotransformer-capacitor converter (ACC). A balanced set of terminal voltages at any power factor is obtainable by using a variable inductor/variable capacitor converter (ICC) [7]. The variability in the inductance can be produced by an ac power controller using an anti-parallel set of devices, while that in the capacitor can be produced by a chopper. This increases the cost and complexity while reducing the reliability; the latter being extremely important in remote locations.

Most of the contributions mentioned above use positive and negative sequence theory in the analysis or design of the above. This is a powerful tool and its importance should not be underated. On the other hand, it was found during this investigation that additional insight was obtained by using vector diagrams to obtain a physical understanding of the above systems. For example, using the positive and negative sequence equations of the IM, it is possible to show that perfect balance using the simple capacitor converter can be obtained only if the motor power factor angle is  $60^\circ$ . To obtain a physical understanding of why this should be so is difficult using the sequence equations; the use of appropriate vector diagrams can provide such an insight and is one

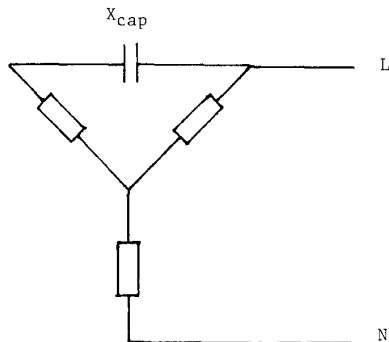


Figure 1. Capacitor single to three phase converter.

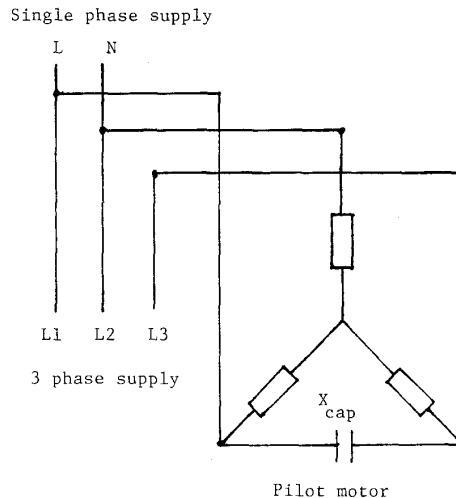


Figure 2. Rotary single to three phase converter.

of the purposes of this paper. The appropriate vector diagrams of the two other types of converter, the capacitor-autotransformer (figure 3) and the inductor-capacitor (figure 4) are developed and presented as well. The vector diagrams show that whereas the capacitor-only converter can obtain perfect balance at only a power factor angle of  $60^\circ$ , that the capacitor-autotransformer converter can obtain balance at any one chosen power factor, except unity, but becomes unbalanced at any other operating power factor. The capacitor-inductor converter on the other hand, can obtain perfect balance at any operating motor power factor provided proper control of the capacitance and inductance is obtained. A comparison of these converters is also made. This is the first aim of the paper.

The magnitude of the negative sequence voltage provides insight into the performance of the converter. A technique for the measurement of this voltage is presented. This circuit can however be used in a more central role in the closed loop control of the system. This is the second aim of the paper.

When the capacitor-only or the autotransformer-capacitor converter is used, some unbalance would exist whenever the motor power factor deviates from the value chosen for balance. This implies a negative sequence voltage which when impressed upon the negative sequence IM network, produces a corresponding negative sequence current and hence torque. The negative sequence current increases the copper loss while the negative sequence torque subtracts from the positive sequence value so as to enforce a torque derating of the machine. The second contribution of this paper is therefore an assessment of the derating necessary in the presence of unbalanced voltages and currents. Normalised curves are produced so that for a given level of voltage or current unbalance, the corresponding torque derating can be determined. Theoretical predictions are supported by practical measurements. This is the third aim of the paper.

The run-up performance of an IM fed from such converters is of course of crucial importance in a practical application. Measured results of the motor's performance is presented when starting directly off a three phase supply, off a CC and a ICC. The performance results when operating off a capacitor-inductor converter has already been presented elsewhere [7]. This is the fourth aim of this paper.

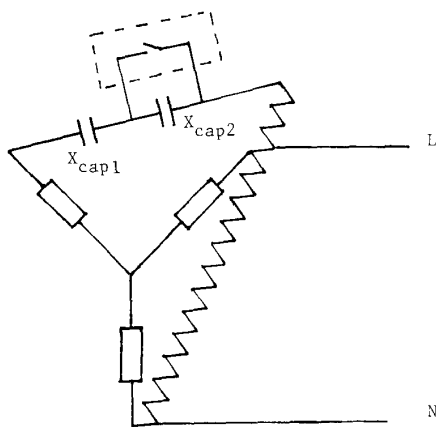


Figure 3. Autotransformer-capacitor converter.

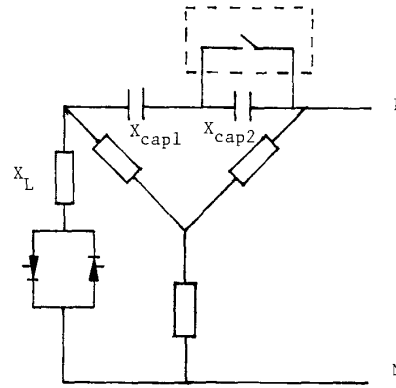


Figure 4. Inductor-capacitor converter.

## 2. VECTOR DIAGRAMS OF SINGLE TO THREE PHASE CONVERTERS

The vector diagram of the converter system shown in figure 1 is drawn in figure 5. From figure 5, it is clear that the vector  $v_a$  is equal to the vector sum of  $v_c$  and  $i_c X_c$ . Alternately, vector  $v_c$  is defined by the vector difference between  $v_a$  and  $i_c X_c$ . Given that  $v_c$  should make an angle of  $120^\circ$  with the horizontal under balanced conditions and that it should be of the same magnitude as  $v_a$ , this forces the vector  $i_c X_c$  to be  $30^\circ$  from the horizontal so that it can terminate on  $v_a$ . This means that  $i_c$  must be  $60^\circ$  from the horizontal since  $i_c X_c$  is perpendicular to  $i_c$ . Of course, most induction motors operate at a power factor angle significantly larger than  $60^\circ$  when running and hence proper balance cannot normally be achieved with this simple converter. Note however that in the Ferraris-arno system the pilot IM is required to be unloaded when the power factor can be quite close to 0.5. This enables the system to achieve a more balanced supply.

For a given magnitude and angle of  $i_c$ , there are an infinite number of  $v_c$ , depending on the magnitude of the  $X_c$  chosen.  $X_c$  may be chosen to satisfy the magnitude criterion of  $v_c$  (ie =  $v_a$ ) or some other criterion. A particular case of the latter is to consider the CC as a special case of the ICC, ie with no inductor.

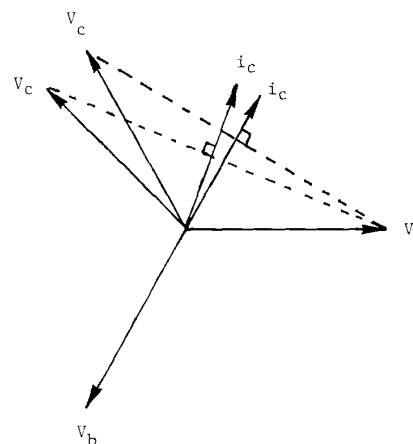


Figure 5. Vector diagram of the capacitor converter.

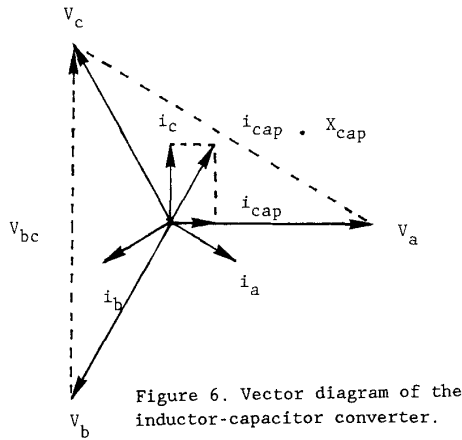


Figure 6. Vector diagram of the inductor-capacitor converter.

Figure 6 shows the vector diagram of the ICC. The idea here is force the capacitor current to operate at a power factor angle of  $60^\circ$  by adding the correct contribution from the inductor. The voltage across the inductor is  $v_{bc}$  and hence  $i_l$ , which must lag  $v_{bc}$  by  $90^\circ$ , lies along  $v_a$ . The magnitude of this contribution is controlled by varying the effective inductance by using an antiparallel pair of devices [7]. The magnitude of the  $i_c X_c$  vector is controlled by variation of the effective capacitance. Both the chopper and the ac power controller generate harmonics into the system, which in addition to the increased complexity is a disadvantage of this scheme.

From symmetrical component analysis [6], the capacitance required for balance as a function of slip is given in figure 7 from which the capacitance used in the CC is taken. Note that ideally, the capacitance should vary continuously as a function of slip. It is also clear however, that between a slip of 1 and 0.3, that the required capacitance is approximately constant. Minimization of the number of capacitances required can be obtained by switching in the capacitance required at full load at some slip greater than 0.3. This limits the number of discrete capacitances to two, although some commercial designs use several.

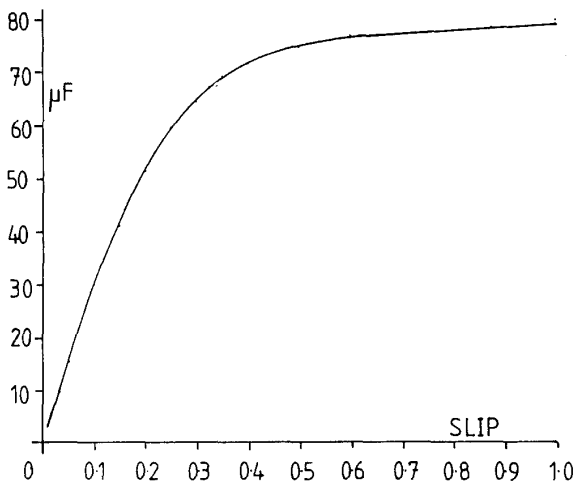


Figure 7. Capacitance vs slip

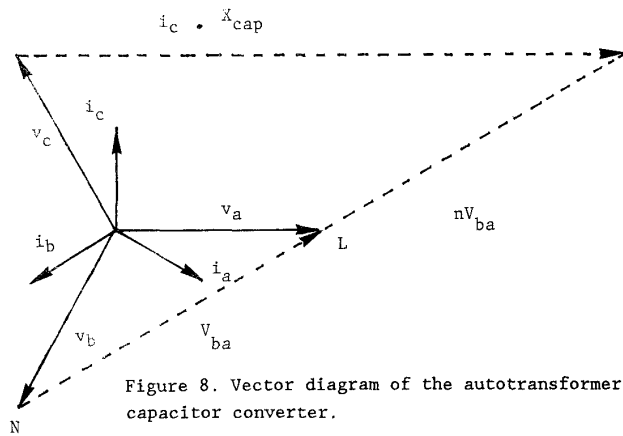


Figure 8. Vector diagram of the autotransformer-capacitor converter.

Figure 8 shows the vector diagram of a capacitor-autotransformer converter for a power factor angle of approximately  $30^\circ$ . The vector  $i_c X_c$  is also shown. While it does not reach  $v_a$ , it does reach an extension of the vector  $v_{ba}$ . But  $v_{ba}$  is the single phase input line voltage and an extension of it can be produced through an autotransformer; this is the basis of the operation of this converter. It is evident that the closer the power factor of the motor is to  $60^\circ$ , the lower is the required autotransformer output voltage. Of course no autotransformer is necessary at  $60^\circ$ . At unity power factor, the vector  $i_c X_c$  is parallel to that of the input supply voltage and hence a balanced terminal voltage set cannot be obtained with this converter. However this is not a practical problem since IMs do not operate at unity power factor.

Figure 9 shows figure 8 with relevant angles included. From simple trigonometry it is easy to show that the angle between  $nV$  and  $i_c X_c$  is the power factor angle  $\phi$ . The magnitude of  $nV$  is calculated using the sine rule as follows:

$$\sin(\phi)/v_{ca} = \sin(60^\circ - \phi)/nV \quad (1)$$

Similarly, the magnitude of  $i_c X_c$  is calculated from

$$\sin(60^\circ)/i_c X_c = \sin(\phi)/v_{bc} \quad (2)$$

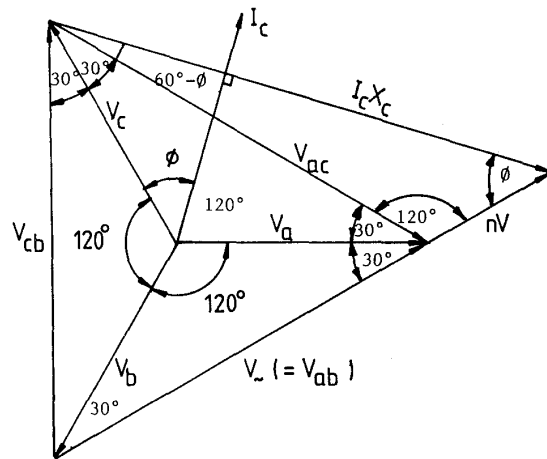


Figure 9. Detailed representation of figure 8.

### 3. MEASUREMENT OF NEGATIVE SEQUENCE VOLTAGE

The magnitude of the negative sequence voltage is of importance in assessing the ability of the converter to obtain balance. It can however, perform a more useful function. The magnitude of the capacitance in the converter should ideally vary as a function of the slip. However, for the machine used, the starting capacitance can remain essentially constant up to slip = 0.5, when the capacitance should then reduce rapidly as the slip reduces.

With the fixed value of starting capacitance, the magnitude of the negative sequence voltage increases as the speed increases. The negative sequence detector can therefore be used to decide when to start decreasing the capacitance, instead of using a more expensive slip detector as proposed in [6]. A schematic of the negative sequence detector used is shown in figure 10 with the corresponding phasor diagram in figure 11. When the line voltages are balanced,  $I_A + I_B = 0$  and no current flows through the ammeter M. It can be shown [8] that this current is directly proportional to the magnitude of the negative

sequence voltage. Also, it is necessary that  $Z_B = aZ_A$ . Also,  $Z_A$  must provide a leading current of  $60^\circ$  while  $Z_B$  must provide a lagging current of  $60^\circ$ . The values chosen were  $R_B = R_A = 185$  Ohms,  $L_B = 1.02$  H and  $C_A = 9.95$   $\mu$ F. In order to obtain a voltage signal from the current flowing in the negative sequence detector, a 1 Ohm resistor was inserted in place of the ammeter

The output of the negative sequence detector during run-up with the starting capacitance only is shown in figure 12. Clearly, the output can be used to control the switching-in of the run capacitance. This is done in figure 13 where at a slip of 0.13, the run capacitance is inserted. The effect is to reduce the magnitude of the negative sequence voltage since the run capacitance creates a more balanced supply near the full load slip. This technique removes the need for a centrifugal switch to control the connection of the run capacitance.

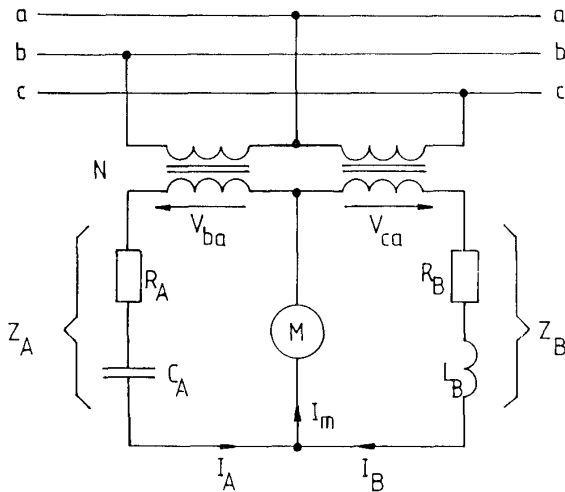


Figure 10. Negative sequence voltage detector.

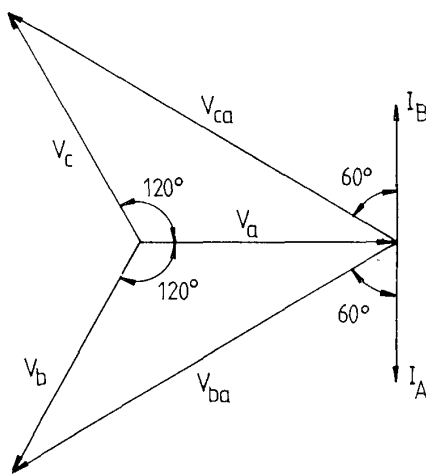


Figure 11. Vector diagram of the negative sequence detector.

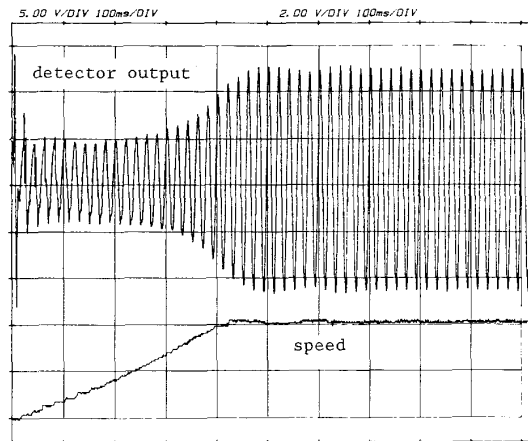


Figure 12. Negative sequence detector output.

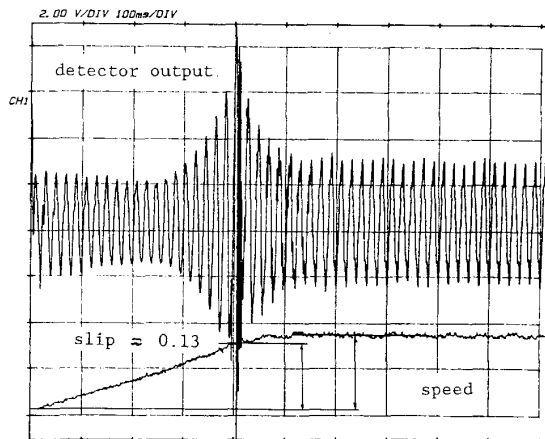


Figure 13. Negative sequence detector output during switching.

#### 4. STEADY-STATE RESULTS

The well known IM equivalent circuit, in the presence of unbalanced supply voltages is used to calculate the derating of the machine used in this investigation. The positive sequence voltage is applied to the positive sequence network (figure 14a) to produce the positive sequence current and hence torque. The negative sequence voltage is applied to the negative sequence network (figure 14b) to produce the negative sequence current and hence torque. The net machine torque is the difference between the positive and negative sequence components. The derating is based on the most conservative approach, i.e. when any one phase current reaches its rating. In practice, there is thermal conduction between the most heavily loaded phase and the more lightly loaded phases which allows the former to be loaded beyond its rated value. The calculation of this degree of overloading is complicated because it depends for example on the thermal conductivity of the slot insulation and the degree of contact between the slot insulation and teeth. It is therefore heavily dependent on the motor's construction and size.

The motor parameters were measured using the locked rotor and zero-slip tests and are shown in Table 1. This was used to calculate the full load performance which was verified experimentally. Unbalanced voltages were also applied to the networks and the performance calculated and verified experimentally. Having verified the model, it was then used to determine the torque derating for a series of unbalanced voltages and currents. Two derating curves were produced, figure 15, assuming the machine is supplied with unbalanced voltages and figure 16 for the derating necessary for a given level of current unbalance. The standard torque equation of an induction motor was used in the calculation.

Table 1.

R1	= 4.58 Ohms
R2	= 5.60 Ohms
X1	= 7.18 Ohms
X2	= 10.78 Ohms
Xm	= 115.78 Ohms
Rm	= 695.65 Ohms

Figure 15 shows the torque derating as a function of  $v_b$ , with  $v_a$  held constant at 1 pu and  $v_c$  varying between 0.2 and 1.2 pu. Hence for any unbalance in the phase voltages between 0.2 and 1.2 pu in two phases, with the third phase at 1 pu, the torque derating can be calculated. Figure 16 can be used correspondingly if the level of unbalanced currents are known.

#### 5. TRANSIENT RESULTS

The run-up performance of IMs is of importance in a practical application. A long run-up time implies longer voltage dips, and more severe operation of the motor. The performance of other equipment on the same bus can also be adversely affected.

Measurements when an IM is fed from a balanced three phase supply, a capacitor only converter and a capacitor-autotransformer converter are presented here. (The performance when fed from a capacitor-inductor converter being already presented in [6].) Figures 17, 18 and 19 show the speed when the IM is started off a balanced supply, a CC and an ACC. In the case of the converter starting, only the start capacitance is used. The motor takes approximately 325ms to run up on a balanced supply, and 400 ms with

the capacitor only or autotransformer converter. This is an increase of 23% when compared to starting on a balanced supply. Although the ACC can provide better balance at full-load, the start-up time is essentially the same as that for a CC, indicating approximately the same level of unbalance during run-up.

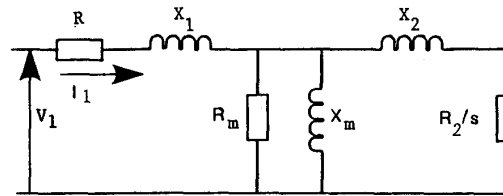


Figure 14a. Positive sequence network of the IM

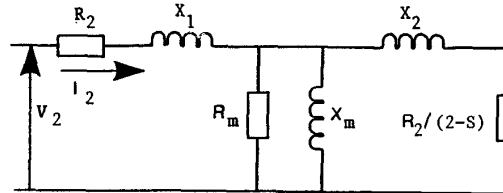


Figure 14b. Negative sequence network of the IM

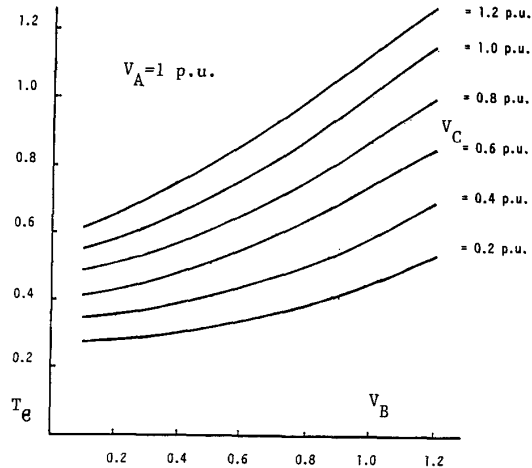


Figure 15. Derating for unbalanced voltages.

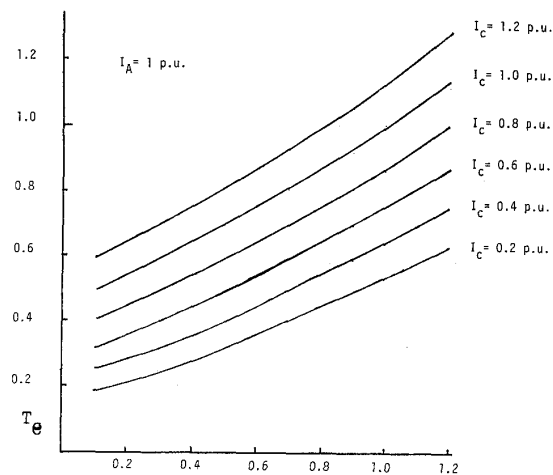


Figure 16. Derating for unbalanced currents.

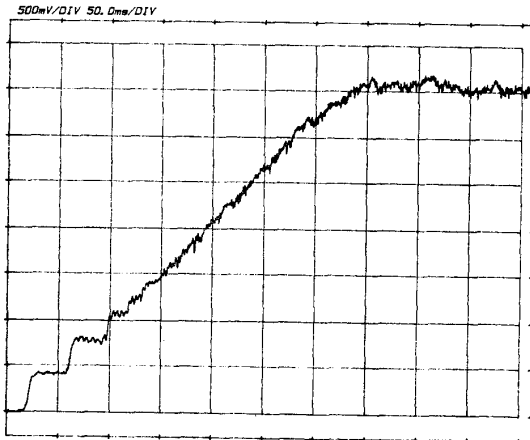


Figure 17. Startup speed from a balanced 3 phase supply.

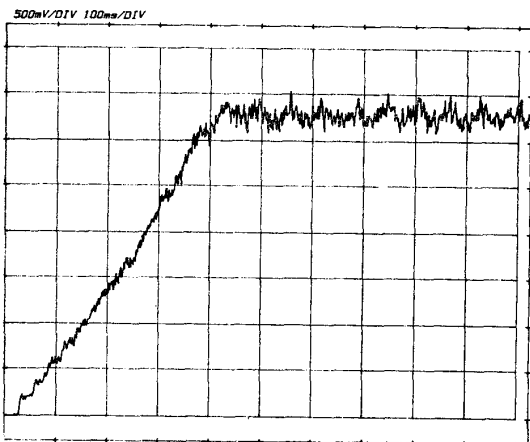


Figure 18. Startup speed from a CC converter.

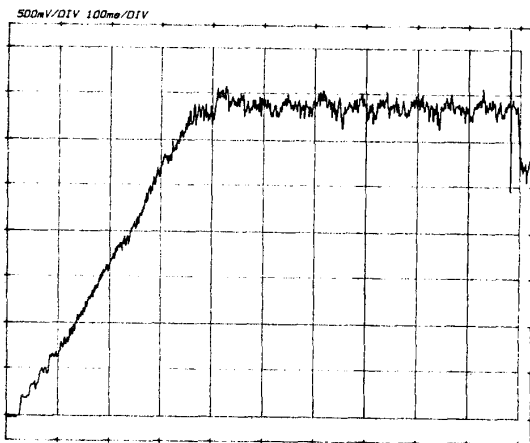


Figure 19. Startup speed from a ACC converter.

## 6. CONCLUSIONS

The vector diagrams of the capacitor-only, the autotransformer-capacitor and the inductor-capacitor converters supplying 3-phase IMs have been presented. Physical insight into the operation and performance of the motor-converter set was obtained.

A circuit for the detection of the negative sequence voltage was presented and used to switch in the run capacitance. This avoided the use of a slip detector or centrifugal switch.

The steady-state derating of the machine was presented when fed from a CC or a ACC. Unbalance in the terminal voltages or currents could be used to determine the derating.

Finally the start-up results of the IM when fed from a 3-phase supply, a CC and an ACC were presented. It was shown that the run-up time was increased when the CC or ACC was used. This indicates a more onerous operation of the motor as well as associated equipment connected to the motor's bus.

## ACKNOWLEDGEMENTS

The authors acknowledge the assistance of D.J.Whitley with measurement of the motor parameters.

## REFERENCES

- [1] R.Habermann, "Single phase operation of a 3-phase motor with a simple static phase converter", AIEE Transactions, August 1954, pp 833-837.
- [2] J.E.Brown and C.S.Jha, "The starting of a three phase induction motor connected to a single phase supply system", Proc.IEE, April 1959, pp 183-190.
- [3] K.A.Ahmed, A.M.Osheiba and M.A.Rahman, "Dynamic performance of a three phase induction motor fed from a single phase supply", IEEE IAS Annual Meeting, 1989, pp 137-146.
- [4] S.S. Murthy, G.J.Berg, B.Singh, J.S.Jha and B.P.Singh, "Transient analysis of a three phase induction motor with single phase supply", IEEE Trans., vol. PAS-102, No.1, January 1983.
- [5] A.H.Maggs, "Single-phase to three-phase conversion by the Ferraris-Arno system", Proc IEE
- [6] A.L. Mohamadein, A.Al-Ohaly and A.Al-Bahrani, "On the choice of phase balancer capacitance for induction motors fed from a single phase supply", IEEE Trans., vol EC-2, No.3, pp 458-464.
- [7] P.G.Holmes, "Single to 3 phase transient phase conversion in induction motor drives", Proc IEE, vol 132, Pt. B, No.5, Sept 1985, pp 289-296.
- [8] C.F.Wagner and R.D.Evans, "Symmetrical components as applied to the analysis of unbalanced circuits".