Design of a PM Wind Generator, Optimised for Energy Capture over a Wide Operating Range

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***Abstract*-- This paper outlines the design procedure for a permanent-magnet (PM) wind generator, which is optimized for energy capture over a wide operating range (i.e. a range of speeds and loads). This is achieved by using an optimisation routine to find the permutation of generator design variables, which maximises its energy capture when operated through the normal operating regions of a variable speed wind turbine. In so doing, the efficiency vs. speed curve of the generator is shaped to produce a high average efficiency over the operating range of the turbine. This is in direct contrast to conventional machine design methods of producing a high absolute efficiency at a single operating point (i.e. rated speed and load).**

***Index Terms*--Permanent magnet generators, synchronous generators, wind energy**

I. Introduction

Wind energy conversion systems (WECSs) are becoming increasingly popular as the demand for stand-alone renewable electricity generation grows globally. Permanent-magnet (PM) machines are ideally suited for these applications, as they are inherently more efficient than wound-field machines. Moreover, PM machine rotors are easy to manufacture with the large number of poles required by low-speed, direct-drive WECSs.

A direct-drive WECS with fixed-pitch blades is typically operated at variable speeds, which offers the advantage of maximising the energy capture from prevailing winds. However, a consequence of variable speed operation of a WECS is that the generator hardly operates at its rated operating point (i.e. rated speed and load). With conventional machine design procedures producing high absolute efficiencies at the rated operating point, the energy capture of a variable speed WECS is therefore naturally reduced due to low generator efficiency at operation away from the designed operating point.

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In this paper, the design procedure is outlined for a permanent-magnet (PM) wind generator, which optimizes the energy capture from a variable speed WECS, over a wide range of operating speeds and loads. This is achieved by using an optimisation routine to find the permutation of generator design variables, which maximises its energy capture when operated through the normal operating regions of a variable speed wind turbine. In so doing, the efficiency vs. speed curve of the generator is shaped to produce a high average efficiency over the operating range of the turbine.

In designing the PM wind generator, this paper starts by sizing the turbine rotor blades and shaft speed for a range of WECS output powers and rated wind speeds. The sizing of a radial-flux PM wind generator is then addressed. The normal operating region of a variable speed WECS is then considered. The optimisation procedure for the wind generator is then discussed.

II. Turbine Blade Sizing and Shaft Speed

In general, most modern 3-bladed HAWTs are designed to operate at maximum aerodynamic efficiency at tip speed ratios between $\lambda_{Popt}=5\text{–}7$ [2]. Furthermore, the maximum aerodynamic efficiencies (C_{Pmax}) typically vary between 0.35 and 0.45 [2]. The radius of the turbine blades can be determined as a function of the rated output electrical power of the WECS and its operating wind speed. This is illustrated in Fig. 1 for 3-bladed HAWTs with $C_{Pmax}=0.4$, $\lambda_{Popt}=6$ and $\eta_{gen}=80\%$.

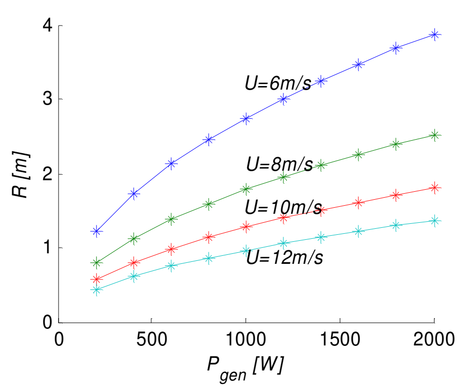

Fig. 1. Turbine blade radius as a function of P_{gen}, at various rated wind speeds

number of poles required and hence the resulting pole pitch of a design. The diameter should however be restricted in order to limit the proportion of inactive copper in the overhang [4].

The airgap diameter of generators for various WECS output powers and rated wind speeds are illustrated in Fig. 3

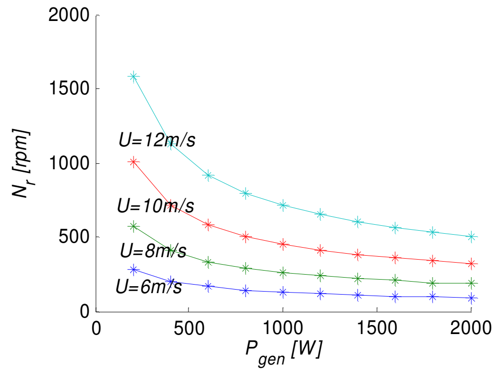

Fig. 2. Shaft speed as a function of P_{gen}, at various rated wind speeds

The operating shaft speed as a function of the WECS output power and rated wind speed is illustrated in Fig. 2, for the same 3-bladed HAWTs. Fig. 1 and Fig. 2 can be used as a guideline when designing direct-drive WECSs with varying output power and rated wind speed requirements. The rated speed and output power of the WECS can now be used to facilitate the design of the PM wind generator.

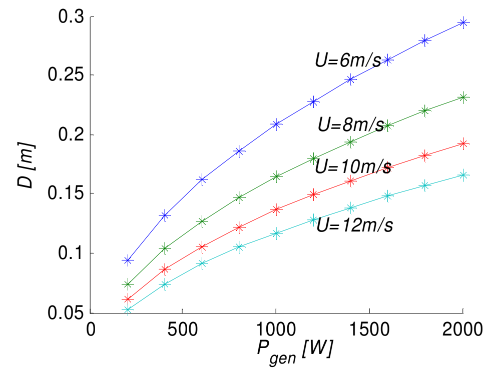

Fig. 3. Airgap diameter of PM wind generators as a function of P_{gen}, at various rated wind speeds

III. PM Wind Generator Sizing

The sizing and main dimensions of a radial-flux PM wind generator is determined in this section. The machine under investigation in this paper has radially magnetised NdFeB PMs mounted on the surface of a solid mild-steel rotor core. The stator core consists of silicon sheet steel laminations with semi-closed rectangular slots.

The product D^2L, which is proportional to the rotor volume of a radial-flux machine, determines its output torque capability [3]. This product can be expressed as:

$$D^2L = \frac{\varepsilon P_{gen}}{0.5\pi^2 \cdot K_{w1} \cdot n_s \cdot SML_{pk} \cdot SEL_{pk} \cdot \cos\phi} \qquad (1)$$

where P_{gen} is the output electrical power of the generator, K_{w1} is the fundamental winding factor, n_s is the rotational speed in rev/sec, SML_{pk} and SEL_{pk} are the peak values of the specific magnetic and electric loadings of the machine.

With the D^2L product determined from (1), the relative apportionment of D and L in a conventional radial-flux machine design is purely based upon practical requirements of the machine application [5], [9]. This is facilitated by the choice of a suitable aspect ratio coefficient, defined as $K_L = L/D$, for each application. Typical values of K_L reported in the literature vary widely from 0.14 to 0.5 for direct-drive PM wind generator applications [10],[13],[18],[19].

In choosing the airgap diameter in a radial-flux PM wind generator design, due consideration should be given to the

IV. Magnetic Circuit Design

The magnetic circuit design of the PM wind generator is considered in this section. In particular, the required number of generator poles, PM material sizing, and stator and rotor core dimensions are discussed.

A. *Airgap flux density*

The airgap flux density level in a PM machine is constrained by saturation of the stator teeth [6]. The SML_{pk} of the machine considered was therefore chosen to be 0.9T. This flux density level provides a good compromise between adequate torque production and saturation of the magnetic circuit of PM machines [3].

In conventional PM machine designs with only a few pole pairs, a pole pitch is generally much larger than a stator slot pitch. The ratio α can therefore be selected independently, as it does not affect the saturation of the stator teeth. This may not be the case in PM wind generator designs with many pole pairs, where a pole pitch may be comparable to a slot pitch. The prescribed range for α is: 0.67-0.77 [3].

B. *Number of Generator Poles*

The operating frequency range for small PM wind generators is reported as typically: 30-80Hz [10] and 10-70Hz [11]. Thus, for the purposes of this paper, the nominal frequency of the generator at rated wind speed is chosen to be within the range 45-65Hz. The required number of generator poles for various WECS output powers and rated wind speeds are illustrated in Fig. 4.

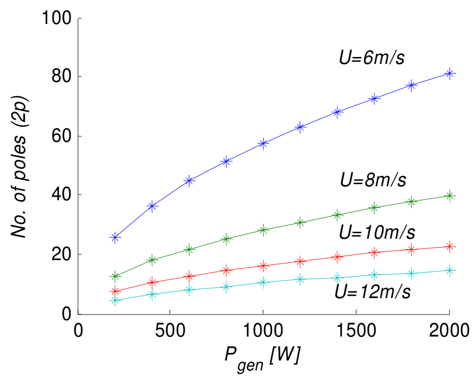

Fig. 4. Number poles for generators operating at a nominal frequency of 60Hz, as a function of P_{gen}, at various rated wind speeds

C. PM Material Sizing

The operating point of a magnetic circuit can be determined by considering the demagnetisation curve of the PM material and the load-line representing the magnetic circuit. Furthermore, slope of the open circuit load-line, defined as the permeance coefficient (*PC*), provides a measure of the withstand capability of the PMs to demagnetisation by external applied magnetic fields. In PM machines with surface mounted PMs, adequate margin against demagnetisation is ensured with a *PC* of 6 or more [3],[12]. The length of the PM material required for a particular machine design can be expressed in terms of the *PC* as [3]:

$$l_m = \left(PC - p_{r1}\mu_r\right) \cdot C_\phi K_c l_g \tag{2}$$

where C_ϕ is the flux focusing factor, K_c is Carter's coefficient and p_{r1} is the normalised rotor leakage permeance. The range of values for p_{r1} is typically 0.05 - 0.2 [3].

It can be seen that the length of the PM material required to provide adequate excitation in a PM machine is largely dependant on the airgap length in the machine.

The airgap length in the PM wind generator design considered in this paper was chosen to be 1% of *D* [13].

D. Stator and Rotor Sizing

The steel yokes of the stator and rotor cores provide the return paths for flux between poles. Saturation of these flux paths are avoided if the respective yoke cross-sectional areas are adequate. For a given axial length of a radial-flux machine, this is ensured by choosing the yoke heights as follows [9],[12]:

$$h_{sy} > \frac{\alpha\pi D}{4pK_s} \cdot \frac{B_g}{B_{sat\_stator}} \tag{3}$$

$$h_{ry} > \frac{\alpha\pi\left(D-2l_m\right)}{4p} \cdot \frac{B_g}{B_{sat\_rotor}} \tag{4}$$

where K_s is the stacking factor of the stator laminations, h_{sy} and h_{ry} are the heights of the stator and rotor yokes respectively, B_{sat_stator} and B_{sat_rotor} are the saturation flux densities of stator and rotor steels.

Saturation of the stator teeth is avoided if adequate tooth area is provided. For parallel-sided teeth and approximately rectangular stator slots in a radial-flux machine design, this is ensured by choosing widths of the stator teeth as follows:

$$w_t > \frac{\tau_s}{K_s} \cdot \frac{B_g}{B_{sat\_stator}} \tag{5}$$

where $\tau_s = \pi D / S$ is the stator slot pitch.

The corresponding widths of the stator slots can be expressed as: $w_s = \tau_s - w_t$. In most machine applications, the ratio of slot width to slot pitch is usually within the range: $0.5 < w_s/\tau_s < 0.6$ [12]. However, if $w_t = w_s = 0.5\tau_s$, the product of specific electric and magnetic loadings of the machine is maximised, which in turn maximises the torque capability of the machine [6]. Thus, for PM wind generator designs with equal tooth and slot widths, (5) establishes a constraint on the airgap flux density which will ensure that saturation of the stator teeth is avoided. This can be expressed as:

$$B_g < \frac{1}{2} K_s B_{sat\_stator} \tag{6}$$

V. Electrical Design

The specific electric loading of a machine is the circumferential current density of the stator. It is limited by the slot-fill factor, slot height and current density of stator conductors [3]. The range of peak specific electric loadings (SEL_{pk}) for small PM machines is typically: 10,000 - 40,000A/m [20].

The slot-fill factor (K_{sf}) is defined as the ratio of copper to slot area and is usually in the range: 0.3-0.5 for small low-voltage machines [5],[6]. This range ensures that the stator cores can be easily wound.

The current density (*J*) of the stator conductors in low voltage machines range between: 3-8A/mm^2 [4].

The required height of the stator slots for the specific electric loading, slot-fill factor and current density chosen for a radial-flux machine with parallel-sided stator teeth can be expressed as:

$$h_s = \frac{SEL_{pk}}{\sqrt{2} \cdot J \cdot K_{sf}\left(1 - w_t/\tau_s\right)} \tag{7}$$

VI. Validation of Turbine Sizing and Machine Design Equations

The validation of the turbine sizing and machine design equations is provided by means of comparing the main specifications of a designed WECS to that of a commercial WECS. This is illustrated in Table I for a 1kW WECS intended for operation at a rated wind speed of 8m/s.

TABLE I
COMPARISON OF MAIN SPECIFICATIONS OF A DESIGNED WECS TO THAT OF A COMMERCIAL WECS

Design Specification	Designed System	Commercial system
Turbine blade diameter ($2R$)	3.56m	3.6m
Rated speed (N_r)	206pm	200rpm
Airgap diameter (D)	0.3158m	0.3238m
Axial length of stator core (l)	0.0632m	0.0647m
No. of poles ($2p$)	30	30
Length of magnets (l_m)	0.0171m	0.0201m
Height of stator yoke (h_{sy})	0.00512m*	0.0212m
Height of rotor yoke (h_{ry})	0.00434m*	0.0495m
Width of stator tooth (w_t)	0.0034m*	0.0042m
Number of Stator Slots (S)	90	90
Conductor diameter	0.00174	0.00134m

* Note that these values are the minimum values tolerable for the machine design, as indicated by equations (3), (4) and (5).

From the results presented in this section it can be seen that good agreement exists between the calculated and measured design specifications of an actual machine, thereby verifying the turbine sizing and machine design equations.

VII. PM Generator Control and Performance

In general, three regions of operating tip speed ratio are defined for the operation of variable speed wind turbines. These regions are defined in relation to the power coefficient curve and include [14],[15],[16]:

- Region I, where $\lambda = \lambda_{Popt}, C_P(\lambda_{Popt}) = C_{P\max}$
- Region II, where $\lambda < \lambda_{Popt}, C_P(\lambda) < C_{P\max}$
- Region III, where $\lambda \ll \lambda_{Popt}, C_P(\lambda) \ll C_{P\max}$

Operation of a wind turbine through each of the three operating regions can be achieved by controlling the generator to which it is coupled. Aspects relating to the control of a PM wind generator and its performance are assessed in the regions of turbine operation.

A. Generator Control

It is assumed that the PM wind generator is controlled by a Field Orientated Control (FOC) loop of a three-phase converter, implemented in the rotor reference frame. The speed reference for the FOC loop can be generated as a function of wind speed. The *q-axis* current reference is determined from the electromagnetic and applied shaft torques, in order to assess the machine losses. With negligible saliency present in a surface-mounted PM rotor construction $(L_d = L_q)$, maximum torque can be developed per ampere of stator current if i_d is maintained to be zero [17]. The *q-axis* current can therefore be expressed as a function of wind speed, as:

$$i_q = \frac{1}{3}\frac{\rho_{air}\pi R^3}{p\lambda_{pm}} C_T U^2 \tag{8}$$

B. Generator losses

The losses in the PM generator are assessed in an effort to accurately predict the performance of the WECS when operated in each of the three regions of operation. The dominant losses considered were the copper and rotational losses.

The rotational losses include core losses and friction & windage losses. The rotational losses are speed dependent and can be fully characterised at no-load. This is due to the presence of core losses even at no-load, which result from the rapid change in flux density in the stator teeth and yoke, as the edges of the PMs rotate through the airgap [6],[8]. The rotational losses of the generator are calculated using the models presented in [5],[6] and [8].

The stator copper losses of the generator can be written in terms of the *q-axis* current as:

$$P_{Cu} = \frac{3}{2} i_q^2 R_a \tag{9}$$

The variation of the generator copper and rotational losses with wind speed and turbine shaft speed are determined for the three regions of turbine operation.

C. Generator Efficiency

The efficiency of the PM generator can be expressed as $\eta = P_{gen}/P_{shaft}$, where P_{shaft} is the input mechanical power applied to the shaft of the generator by the turbine and P_{gen} is the real power output of the generator. The input mechanical and output electrical power of the generator can further be related by:

$$P_{gen} = P_{shaft} - P_{Cu} - P_{rot} \tag{10}$$

Equation (10) is applicable for steady-state conditions, as the rotational energy stored in the drive train of the WECS has been omitted. Steady-state conditions are of interest in order to simplify the performance analysis of the machine.

VIII. Annual Energy Capture

In general, the wind speed information for an area can be modelled by a statistical Rayleigh distribution, as [5],[15]:

$$f\left(U_i\right)=\frac{\pi}{2}\frac{U_i}{\overline{U}^2}e^{-\frac{\pi}{4}\left(\frac{U_i}{\overline{U}}\right)^2} \quad (11)$$

where $\overline{U}$ is the average wind speed for the area and U_i is the likelihood of a prevailing wind having a particular speed.

The total number of hours per year for a particular wind speed *(U_i)* can be determined by simply multiplying the probability of its occurrence by the duration of one year. This can be expressed as [15]:

$$H\left(U_i\right)=365\times 24\times f(U_i)\Delta U \quad (12)$$

The energy contribution at each wind speed can be determined by the product of the power output of the WECS at a specific wind speed and the duration that the wind speed occurs annually. The annual energy output (AEO) of the WECS can be determined by summing the incremental energy contributions at each wind speed. This can be expressed as [15]:

$$AEO=\sum_{i=1}^{n}P\left(U_i\right)\cdot H\left(U_i\right) \quad (13)$$

where $P(U_i)$ is the power output of the WECS at wind speed U_i.

The energy capture of the WECS, in regions I, II and III can be now determined at various average wind speeds. The total AEO represents the sum total of energies from the three regions of operation at each average wind speed.

IX. Optimisation of the Wind Generator Design

A direct-drive WECS is operated at variable shaft speeds as the wind speed varies. This offers the advantage of maximising the energy capture from prevailing winds. However, a consequence of variable speed operation of a WECS is that the generator hardly operates at its rated operating point (i.e. rated speed and load). With conventional machine design procedures producing high absolute efficiencies at the rated operating point, a wind generator designed using this methodology will therefore result in a WECS with reduced energy capture. This is due to the low generator efficiency at operation away from the designed operating point.

The design procedure proposed in this paper optimizes the energy capture from a variable speed WECS, over a wide range of operating speeds and loads. This is achieved by using an optimisation routine to find the permutation of generator design variables, which maximises its energy capture when operated through the normal operating regions of a variable speed wind turbine.

A. Optimisation Routine

The Population Based Incremental Learning (PBIL) optimisation algorithm was used to optimise the generator design [6],[7],[21],[22]. This was accomplished by means of the optimisation algorithm generating trial wind generator designs, then evaluating the fitness of each design and then regenerating trial designs based on the recombination of best previous designs. The fitness of each trial design is determined on the basis of maximum energy capture over the regions of turbine operation.

B. Generator Design Procedure

Suitable ranges were specified for ten variables in the machine design. The variables included: the remanent flux density of the PMs (B_r), the rated terminal voltage, frequency and power factor of the generator, the aspect ratio coefficient (K_L), the slot-fill factor (K_{sf}), the pole-arc to pole-pitch ratio (α), the stator current density (J), the peak specific electric loading SEL_{pk} and the permeance coefficient (PC). The optimisation algorithm then searched the design space for a suitable permutation of the design variables, which would produce a WECS with the highest energy capture over the operating speed range.

C. Comparison of PM Wind Generator Designs

The effect of optimising the wind generator design for energy capture can be clearly noticed in Fig. 5. The efficiency vs. speed curve of the generator is shaped to produce a high average efficiency over the operating speed range of the turbine. The detailed design specifications of the two machine designs are illustrated in Table III. Most of the design specifications of the two machines compare favourably. The energy capture of the design optimised for AEO exceeds that of the design optimised for efficiency by 8%. The total mass of active material used in the former design is however almost double that of the latter design.

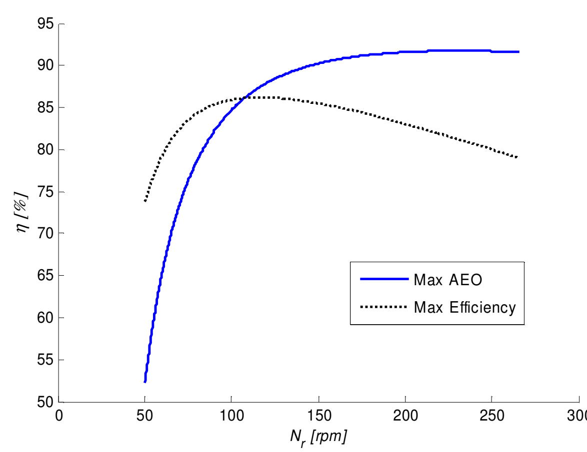

Fig. 5. Comparision of efficiency vs speed curves for generator designs optimised for energy capture (Max AEO) and efficiency (Max Efficiency)

TABLE II
COMPARISON OF PM WIND GENERATOR DESIGNS OPTIMISED FOR ENERGY CAPTURE (MAX AEO) AND FFFICIENCY (MAX EFFICIENCY)

Parameter	Max Efficiency Design	Max AEO Design
D [m]	0.2057	0.2702
l [m]	0.0366	0.0507
Poles (2p)	28	20
Stator Slots (S)	84	60
l_m *[m]*	0.0060	0.0067
R_a *[Ω]*	8.2651	8.282
w_s *[m]*	0.00385	0.00707
B_g *[T]*	0.767	0.767
SML_{pk} *[T]*	0.917	0.917
B_r *[T]*	1	1.15
V_{a_rated} *[V]*	170.23	215.08
f_{rated} *[Hz]*	60	46.4
α	0.776	0.77604
K_L	0.178	0.1875
SEL [A-Cond./m]	32,266	13,516
J [A/mm²]	5.461	3.0
AEO [Wh]	$1.4454x10^6$	$1.5603x10^6$
Total mass [kg]	6.544	12.789

X. CONCLUSIONS

This paper introduced an alternative design procedure for PM wind generators, which selects an optimised permutation of design variables based on maximising the energy yield from the machine. This is in direct contrast to the classical machine design methodology, which optimises for maximum efficiency. It was shown that the former design procedure results in an 8% increase in the annual energy yield compared to a conventionally designed machine.

XI. REFERENCES

[1] J. F. Manwell, J. G. McGowan, A. L. Rogers, *Wind Energy Explained - Theory, Design and Application*, 1st ed., New York: John Wiley & Sons, 2002.

[2] M. M. Duquette, "The Effect of Solidity and Blade Number on the Aerodynamic Performance of Small Horizontal Axis Wind Turbines," M.Sc. dissertation, Dept. Mech. & Aero. Eng., Clarkson Univ., Potsdam, New York USA, 2002.

[3] T. J. E. Miller, *Brushless Permanent-Magnet and Reluctance Motor Drives*, New York: Oxford University Press, 1989.

[4] M. G. Say, *The Performance and Design of Alternating Current Machines*, 3rd ed., London: Pitman & Sons, 1965.

[5] J. F. Gieras, M. Wing, *Permanent Magnet Motor Technology: Design and Applications*, 1st ed., New York: Marcel Dekker, 1997.

[6] G. R. Slemon, "Design of Permanent Magnet AC Motors for Variable Speed Drives," in *Tutorial Course IEEE-IAS Annual Meeting*, P. Pillay, Ed. Michigan: IEEE, 1991, pp. 3-1 - 3-35.

[7] M. Wing, "Analysis of an Energy Efficient Permanent Magnet Brushless Universal Motor," Ph.D. dissertation, Dept. Elec. Eng., Univ. Cape Town, South Africa, 1996.

[8] G. R. Slemon, and X. Liu, "Core Losses in Permanent Magnet Motors," *IEEE Trans. Magnetics*, vol. 26, pp. 1653-1655, Sep. 1990.

[9] S. Huang, J. Luo, F. Leonardi, and T. A. Lipo, "A General Approach to Sizing and Power Density Equations for Comparison of Electrical Machines," *IEEE Trans. Ind. Applications*, vol. 34, pp. 92-97, Jan. / Feb. 1998.

[10] J. Chen, C. V. Nayar, and L. Xu, "Design and Finite-Element Analysis of an Outer-Rotor Permanent-Magnet Generator for Directly Coupled Wind Turbines," *IEEE Trans. Magnetics*, vol. 36, pp. 3802-3809, Sep. 2000.

[11] A. M. De Broe', S. Drouilhet, and V. Gevorgian, "A Peak Power Tracker for Small Wind Turbines in Battery Charging Applications," *IEEE Trans. Energy Conversion*, vol. 14, No. 4, pp. 1630-1635, Dec. 1999.

[12] M. Comanescu, A. Keyhani, and M. Dai, "Design and Analysis of 42-V Permanent-Magnet Generator for Automotive Applications," *IEEE Trans. Energy Conversion*, vol. 18, No. 1, pp. 107-112, Mar. 2003.

[13] A. Grauers, "Design of Direct-Driven Permanent-Magnet Generator for Wind Turbines," Ph.D. dissertation, School Elec. and Comp. Eng., Chalmers Univ., Sweden, 1996.

[14] T. Thiringer, J. Linders, "Control by Variable Rotor Speed of a Fixed-Pitch Wind Turbine Operating in a Wide Speed Range," *IEEE Trans. Energy Conversion*, vol. 8, No. 3, Sep. 1993, pp. 520-526.

[15] D. S. Zinger, E. Muljadi, "Annualized Wind Energy Improvement Using Variable Speeds," *IEEE Trans. Industry Applications*, vol. 33, No. 6, Nov./Dec. 1997, pp. 1444-1447.

[16] A. Miller, E. Muljadi, D. S. Zinger, "A Variable Speed Wind Turbine Power Control," *IEEE Trans. Energy Conversion*, vol. 12, No. 2, Jun. 1997, pp. 181-186.

[17] P. Pillay, R. Krishnan, "Modelling, Simulation, and Analysis of Permanent-Magnet Motor Drives, Part I: The Permanent-Magnet Synchronous Motor Drive," *IEEE Trans. Industry Applications*, vol. 25, Mar./Apr. 1989, pp. 265-273.

[18] P. Lampola and J. Perho, "Electromagnetic Analysis of a Low-Speed Permanent-Magnet Wind Generator," in *Proc. 1996 IEE Opportunity and Advances in International Power Generation Conf.*, pp. 55-58.

[19] N. Bianchi and A. Lorenzoni, "Permanent Magnet Generators for Wind Power Industry: An Overall Comparison with Traditional Generators," in *Proc. 1996 IEE Opportunity and Advances in International Power Generation Conf.*, pp. 49-54.

[20] S. A. Nasar, I. Boldea, *Permanent Magnet, Reluctance, and Self-Synchronous Motors*, Florida: CRC Press, 1993.

[21] S. Baluja, "Population-based incremental learning: A method for integrating genetic search based function optimization and competitive learning," Carnegie Mellon Univ., Pittsburgh, PA, Tech. Rep.CMU-CS-94-163, June 1994

[22] R. Wang, M. J. Kamper, K. vd. Westhuizen and J. F. Gieras, "Optimal Design of a Coreless Stator Axial Flux Permanent-Magnet Generator," *IEEE Trans. Magnetics*, vol. 41, pp. 55-64, Jan. 2005.