

# Experimental Study of Vibrations in the Switched Reluctance Motor

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## ABSTRACT:

This paper describes an experimental investigation into SRM vibrations and in particular, a method for determining the transfer function from the normal force to the stator vibrations. An indirect method for calculating the normal force is presented.

## I. INTRODUCTION

It is widely accepted [1,2] that the radial vibration in the SRM with radial air-gap and the axial vibration in the SRM with axial air-gap, due to magnetic forces in the normal direction to the SRM pole surfaces, are the dominant source of vibration and acoustic noise in the SRM. This is especially true when the waveform of the magnetic force is similar to a mode of the SRM stator and the frequency of the magnetic force is near or at the corresponding stator resonant modal frequency [2,3].

The magnetic forces and stator mechanical structure can be modelled by either analytical or numerical methods [3,4], which includes computation of the magnetic field and force, identification of the stator resonant parameters and mechanical vibration estimation under magnetic excitation. However, the simulation becomes complicated even for simple structures and intractable for complicated structures with cooling fins for example. For example, the stator assembly of an SRM is composed of a stator stack, phase windings, frame and end-bells etc. The vibration behavior of this assembly depends not only on the uniformity of the materials but also on the stacking, machining and mounting conditions. It is impossible to include all these factors in the numerical computation model of the SRM stator without the help of measurements because of manufacturing variations. Accurate determination of the vibration parameters, such as damping coefficient etc, has to be performed through vibration tests. Besides providing direct results on vibration research, the measurements (especially modal tests) are also used for verifying the prediction/simulation and for refining the system model.

Early applications of modal testing were used mainly to identify resonant frequencies [5]. The modal test was still called "resonance testing" until the 1960s when it was applied to the determination of mode shapes [6]. As new excitations, FFT, data-analysis techniques and microcomputers were introduced, modal testing has experienced a rapid development from analog to digital. At present, there are many methods to perform each of the four stages in modal testing: excitation, data acquisition, signal processing and parameter extraction. A comparison of the advantages and disadvantages of different modal testing techniques was done in [7]. No single excitation, acquisition, processing and analysis method is superior to the rest in every situation. The best method depends on the particular application and available facilities, although there are many papers describing the preferences of their authors.

This paper will concentrate on the modal tests of the SRM. Several simple but practical methods including both time and frequency domain methods are presented for vibration measurements and parameter extraction. This includes the performance of vibration tests with minimum available facilities. A couple of low order mode shapes of a 5-hp 8/6 pole switched reluctance motor have been identified. The frequency response function of a single degree of freedom (SDOF) system for superposition analysis of multi-degree of freedom (MDOF) systems is presented. The resonant frequencies measured by different methods are compared to each other, which shows the validity of the methods. Finally, the effects of the SRM end-bells on stator resonant frequencies are found through tests, which will be beneficial to future vibration analysis and control of rotating electrical machines.

## II. MEASUREMENT TECHNIQUES AND PROCEDURES

The dynamic modal testing procedure includes excitation of the measured object and identification of the modal parameters, such as resonant frequencies, damping coefficients and corresponding mode shapes. Theoretically, there are two basic methods for modal tests of an object, i.e., the normal mode method and the transfer function method. Whichever is used to perform the modal testing in the SRM,

the selection of methods in four basic stages has to be done in advance: (1) excitation in the SRM; (2) data acquisition; (3) signal processing; (4) parameter extraction.

#### (1) Excitation methods

Excitation methods can be categorized into single frequency and multi-frequency. Single frequency or sine wave vibration attempts to excite only one mode at a time while multi-frequency excitation is employed for exciting several modes simultaneously. Single-frequency excitation includes the sinusoidal dwell and sinusoidal sweep, which can provide broadband spectral information, one frequency at a time. Multi-frequency excitation includes ambient (wind, seismic, wave action etc.), transient (impulse or step relaxation/"twang"), random (continuous and non-repetitive signals, pseudorandom or burst), chirp and fast sine sweeps etc. Random and fast sweep techniques have to be analyzed by Fourier analysis. The "twang" by a hammer and random vibration is the most widely used form of excitation. The advantages and disadvantages of the different excitation methods are given in [9]. Selection of excitation method depends largely on the availability of the test equipment.

The SRM stator can be excited by an impulse hammer, a simple vibration test tool. This multi-frequency method can excite a broadband vibration spectrum from which it is difficult to figure out the resonant frequencies corresponding to the low order mode shapes of the SRM stator. This situation is especially true for a stator with ribbed frame since the ribs on the frame produce numerous complicated modes [2,3], and care must be taken with signal acquisition, accelerometer location etc.

Phase current excitation and shaker excitation are applied in this paper. The details of each method are described and the advantages and disadvantages are compared between the two methods. The results compare favorably.

#### (2) Data Acquisition

Data acquisition can be done by either acquiring a history of excitation and response for subsequent analysis (time domain or frequency domain) or obtaining the spectra directly through real time FFT using for example the sine wave integration Fourier transform (SWIFT) algorithm. Of course, current commercial instruments for vibration measurement have integrated data-acquisition and analysis so that the users are only required to choose proper transducers and their mounting locations. In fact, data acquisition for SRM vibration tests, can be performed by using transducers and a digital oscilloscope, if there are no specialized vibration instrumentation available.

Direct force signals and indirect magnetic force (calculated from current) acquisition are described in this paper, which shows that the SRM vibration can be measured without professional vibration test equipment.

#### (3) Data Processing

Data processing, especially of digital signals, has developed rapidly since the 1980s. The acquired data includes time domain and frequency domain data, which can be converted to each other by the FFT or the inverse FFT. Data processing is used to generate the refined frequency response function (FRF), and contains averaging, windowing, wild-point editing, smoothing, refinement algorithms, and even error analysis. The core of data processing lies in constructing filters which can distinguish the real tested signals and remove the noise or unwanted signals. A simple noise-removing algorithm is to transform the real time signal to the frequency domain first, rank the frequency domain signals and reduce the signal corresponding to noise signals (usually with characteristic low amplitude in the frequency domain). The real signals are obtained through transforming the frequency domain signals back to the time domain if required. These processes are critical to a successful measurement.

#### (4) Parameter Extraction Methods

There are three analysis techniques to extract the modal parameters.

(a) *Frequency domain analysis* is based on the frequency response function (FRF) to estimate the resonant frequency, damping factors and corresponding mode shapes. The most frequently used strategies include frequency-domain curve fitting (FDC) and simultaneous frequency domain (SFD) analysis.

An indirect transfer function method is introduced in this paper, in which the excitation force in the frequency domain is indirectly calculated from measured excitation currents by the finite element method. Therefore, modal parameters are identified without directly measuring the excitation force.

(b) *Time domain analysis* includes (i) The state-variable method, and (ii) The complex exponential algorithm (CEA). The state-variable method and its alternative method are seldom used in modal analysis although they lead to a successful formulation of the simultaneous least-squares method. The CEA in the paper is based on the linear superposition principle and exponential curve fitting algorithm. It is performed by equating the polynomial coefficients to the measured response history, so that the resonant frequency and damping factor can be extracted, and the complex modal amplitude determined. This method is especially useful for determining the resonant frequency and damping factor of a mode shape, which can approximately be treated as a single degree of freedom, although it can be extended to multi-dimensions [9].

(c) *Tuned-sinusoid analysis* consists mainly of (i) Tuned sine sweep and (ii) Multi-exciter tuned dwell (MTD) method. The tuned-sine sweep is the traditional method of frequency response measurement. In this method, a sweep oscillator is used to provide a sinusoidal command signal whose frequency sweeps continuously through the range of

interest. The frequency variation has to be so slow that the response can be treated as steady state. The sweep rate will dramatically affect the measuring accuracy [7]. The selection of the excitation location and an automatic tuning procedure were given in [10] and [11], respectively. The MTD method [12] was proposed to excite a pure natural mode of interest while suppressing the others. The single mode response is obtained by varying the ratio of the exciting force to frequency of two or more shakers. It is straightforward since the resonant frequency is the excited frequency and the measured motion is the mode shape, and the damping can be estimated by cutting off the tuned excitation and measuring the transient decay history. The tuning is a skillful task for all tuned sinusoidal analysis methods.

### III. EXPERIMENTAL SYSTEMS

In order to determine the modal parameters and the corresponding mode shapes of a vibration model, two measurement systems are used in this paper in terms of excitation methods: phase current excitation and magnetic shaker excitation.

#### (1) Experimental system with phase current excitation

In Fig.1, the power drive is a PWM voltage source, which provides rectangular voltage to the stator phase windings in the aligned position of the measured SRM. There are two accelerometers screwed to the SRM. They lie right behind the excited stator poles on the case of the motor. The acceleration signals are amplified by the coupler, and then sent to a multi-channel digital oscilloscope. The voltage and current waveforms of the SRM are also recorded by the oscilloscope. The processing can be performed by the oscilloscope or transferred to a computer and processed by the data processing toolbox in Matlab. If the modal equivalent parameters ( $m$ ,  $c$ ,  $k$ ) are required, the magnetic force between the stator and rotor poles can be calculated by finite element methods according to the phase current. Then the frequency domain method or transfer function algorithm can be used to extract the system parameters.

It should be noticed that this excitation method could only be used to measure the 2<sup>nd</sup> order modal parameters since the phase current can only produce a force-pair which excites the vibration with oval deformation. The following frequency spectra and mode shape results verify this theoretical analysis.

#### (2) Experimental system with the permanent magnet shaker

The diagram of this testing system is given in Fig.2. The signal acquisition side of the system is the same as that in diagram of Fig.1. Seven accelerometers (Kistler

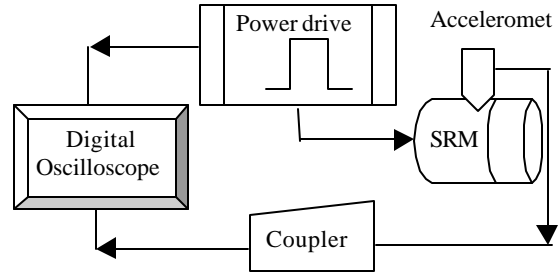


Fig.1 Diagram of the test system with a phase current excitation

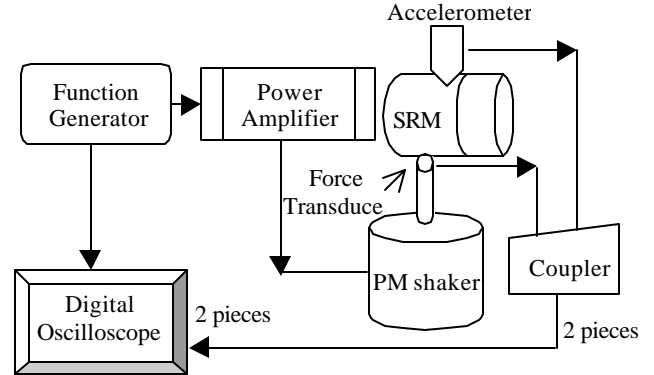


Fig.2 Diagram of vibration testing system with PM shaker excitation

8730A500) are screwed into the SRM surface at equidistances in the circumferential direction, except for the position of terminal box. The force transducer is installed between the push rod of the PM shaker and the SRM. The response signals from the accelerometers and the excitation signal from the force transducer are transferred to two oscilloscopes (100MHz 20Ms/s 12 bits Nicolet and 200MHz 100Ms/s LeCroy 9304AM) through two multi-channel couplers. The permanent magnet shaker is driven by a power amplifier. In any case, the mass of the exciter should not affect the system vibrations. The waveform of the force excitation is generated by a function generator. The signal from the function generator is also connected to one of the oscilloscopes in order to monitor the force exerted on the SRM. This system can also be used to identify the MDOF vibration.

### IV. EXPERIMENTAL RESULTS

#### (1) Real time method to identify modal parameters through phase current excitation

Firstly, the vibration test system with phase current excitation is used to measure a 5hp, 8/6 SRM with 4 phases. The accelerometer is located just behind the excited phase

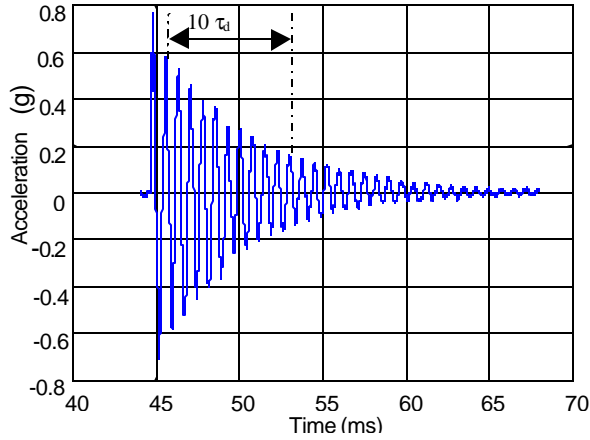


Fig.3. Vibration acceleration curve in time domain

pole on the top of the SRM. The test results of the current, voltage and acceleration are recorded in the time domain.

The acceleration curve of the measurement results can be used directly to determine the resonant frequency and damping ratio. One group of the acceleration waveforms of the SRM vibration is enlarged and redrawn in Fig.3. The damping oscillation period  $\tau_d$  can be obtained by measuring the time interval between two adjacent peaks, or by measuring the time interval between  $N$  peaks, and dividing by  $N$ . The damping ratio is obtained through measuring peak values. Two accelerations,  $a_k$  and  $a_{k+N}$ , separated by  $N$  complete cycles give the logarithmic decrement, i.e.,

$$d = \frac{1}{N} \ln \frac{a_k}{a_{k+N}} \quad (1)$$

Furthermore, the damping ratio  $Z$  can be found by

$$Z = \frac{d}{\sqrt{(2p)^2 + d^2}} \approx \frac{d}{2p} \quad (\text{if } d \ll 2p) \quad (2)$$

The results are given in Table 1. It is not difficult to find that the damped resonant frequency is almost the resonant frequency (or natural frequency) when the damping ratio is small (here it is about 2%).

### (2) Parameter Identification by Indirect Inertance Frequency Response Function (FRF)

Transfer function methods have become more and more popular since the 1970s. The indirect method to determine the resonant frequency is introduced in this paper. Firstly, the magnetic forces corresponding to phase current excitation (repetition = 25ms or 40ms) are obtained through a lookup table of the magnetic force and phase current which are computed by the FE method, as shown in Fig.4. Secondly, the FFT of the force excitation and the FFT of the acceleration response are found, then the inertance FRF (i.e. ratio of response/excitation in the

frequency domain) is obtained. The resonant frequency (1360Hz) will correspond to the zero real part and the minimum imaginary part of inertance FRF. The maximum displacement is about  $4.3 \times 10^{-8}$ m. Another peak is found in

Table 1 Measured results of resonant frequency and damping ratio

Peak No.2 (g)	0.5750
Peak No.12 (g)	0.0625
Time interval $T_{10}$ (ms)	7.418
Period $\hat{\sigma}_d$ (ms)	0.7418
Damped $f_d$ (Hz)	1348
Logarithm decrement <sup>TM</sup>	0.12637
Damping ratio	0.0201
Natural frequency (Hz)	1348.3

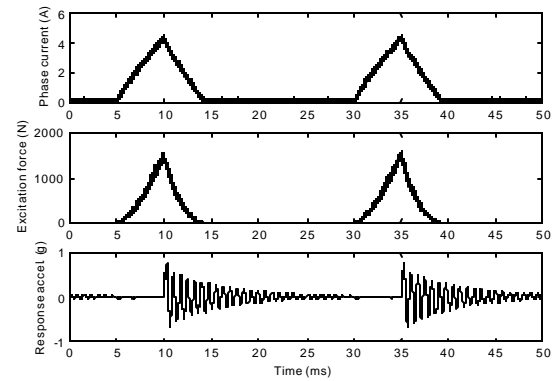


Fig.4 Current, magnetic force, and acceleration vs. time

the imaginary part of inertance response, which corresponds to the 3<sup>d</sup> order mode at 2540Hz. The calculated resonant frequency for the 3<sup>d</sup> mode shape is 2589Hz (error is less than 2%). There is a small error of about 1~2% with the indirect measurement, compared to the result of the direct method. The accuracy of this method depends on the accuracy of the magnetic force computed by the FE method, i.e., the error comes from numerical field computations.

### (3) Measurement results from normal mode method

The normal mode method was used traditionally for modal tests before the use of the transfer function method. The main objective of the normal mode method is to excite the undamped modes of the measured SRM, one at a time. Shaker excitation, with one or more shakers driven by sinusoidal signals at the same time, is one of the principal features. The testing procedure consists of five steps. (1) Wide frequency band sweep: change the frequency of the function generator output from a low to high range, resulting in a variation of the excitation force applied to the SRM. After the amplitude response of the accelerometers is determined with an oscilloscope, the testing enters the

second step. (2) Narrow frequency band sweep: A smaller frequency change is used to locate the modal frequency, corresponding to the maximum response in that frequency region. (3) Modal tuning: the amplitude, polarity and frequency of the sinusoidal excitation from the shakers are adjusted to excite the resonant mode shape. To identify a single mode (or a pure mode) of vibration, multiple shakers are highly recommended. (4) Modal dwell: the vibration amplitudes at many points on the SRM, and the corresponding excitation force are recorded once a mode is well tuned. (5) Damping measurements: the damped sinusoidal response histories of a free vibration are recorded at all points from the moment all shakers are simultaneously shut off. For a pure mode, all responses should exhibit the same sinusoidal decay feature, and the damping at the resonant frequency of the mode can be measured from the envelope of the damped sinusoidal response. Typically the impulse responses will show a beating since more than one mode is often involved.

The oscilloscope output for the 2<sup>nd</sup> order mode shape of the SRM stator with end-bells is given in Fig.5. Channel one shows the force excitation signal, while channels 2~4 are the acceleration responses at the locations with 90° mechanical span in the circumferential direction of the SRM stator. Fig.5 shows in channels 2 and 4, that the response of two accelerometers with 180° span, are in phase. The response of channel 3, which is connected to the accelerometer located 90° from the accelerometers displayed in channels 2 and 4, has a phase difference of 180° from that of channels 2&4. All responses have the same amplitude at 1346Hz. The responses from channels 5~8 have low amplitudes. These show the features of the 2<sup>nd</sup> mode shapes.

Similarly, the 3<sup>rd</sup> and 4<sup>th</sup> order mode shapes of the SRM stator with end-bells are also measured, shown in Table 2.

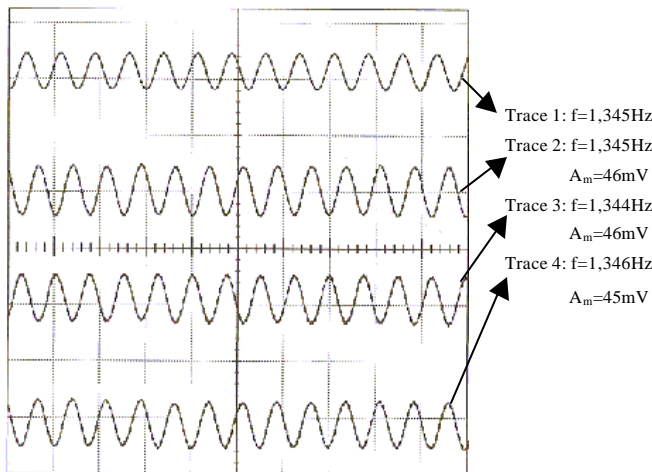


Fig.5 Oscilloscope output at the 2<sup>nd</sup> order mode of the SRM stator with end-bells

#### (4) Experimental Results of the End-bells Effect on the SRM Resonant Frequencies

To verify the effects of the end-bells on the SRM stator vibration, normal mode testing method is repeated after the end-bells are removed. The power spectrum density of the second mode shape is recorded. Similarly, the measured resonant frequencies for the 3<sup>rd</sup> and the 4<sup>th</sup> order mode shape are 2450Hz and 5280Hz, respectively.

To compare the test results with and without end-bells, the resonant frequencies are listed in Table 2.

Table 2 Tested resonant frequencies for the SRM with and without end-bells

Models	2 <sup>nd</sup> mode	3 <sup>rd</sup> mode	4 <sup>th</sup> mode
Testing model without end-bells	1060 Hz	2450 Hz	4850 Hz
Testing model with end-bells	1346 Hz	2680 Hz	4987 Hz
Errors (%)	21.25	8.58	2.75

Obviously, the resonant frequencies increase after installing the end-bells. The error due to neglecting the end-bells is not tolerable in the prediction of the SRM resonant frequency, which is 21.25% for the 2<sup>nd</sup> mode shape. Furthermore, the effect of ribs and end-bells on the resonant frequency should be included in order to improve the accuracy of calculation.

## V. CONCLUSIONS

The study of vibrations in the SRM is important for them to reach their full potential in industrial applications. A real time method through phase current excitation is applied for identification of the vibration parameters of the SRM. An indirect frequency transfer function method for modal tests is introduced in the paper. This method can be used to perform an experimental study of SRM vibrations without the help of force transducer. The test results are very good, compared to the traditional modal test method. The methods are used for modal testing of a 5hp 8/6 SRM stator with and without end-bells. The effects of the end-bells on the resonant frequencies of the SRM are presented, which shows that the end-bells cause a significant increase of resonant frequencies of the SRM, especially for the 2<sup>nd</sup> order mode shape.

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