Vibration Measurements in the Switched Reluctance Motor

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***Abstract*¾ Techniques for the measurement of vibrations in switched reluctance motors (SRMs) are given in this paper, together with experimental results. The measured results are compared to each other as well as with Finite Element calculated results, with good correlation.**

I. INTRODUCTION

The switched reluctance motor (SRM) is attractive to the industry for several reasons, including a mechanically and thermally robust rotor, simple stator windings and ease of manufacture. Design choices force the machine into a highly saturated operation, accompanied by high normal forces causing ovalizing of the stator [1,2]. Deenergizing of the phase current at the position of alignment and maximum radial force and flux generates a vibration, resulting in acoustic noise generally considered worse than a comparable induction motor. This problem is aggravated if the waveform of the magnetic force coincides with a mode shape of the SRM stator and the frequency of the magnetic force is near or at the corresponding stator resonant modal frequency [2,3].

It is very difficult, if not impossible, to model the vibration behaviour of the SRM analytically. The magnetic forces alone require extensive finite element modelling and the stator mechanical structure can quickly become intractable for complicated structures with cooling fins [3,4]. For example, the stator assembly of an SRM is composed of a stator stack, phase windings, frame and end-bells etc. The vibration behavior of this assembly depends not only on the uniformity of the materials but also on the lamination stacking, machining and mounting conditions, not to mention the stiffness and mass contributions of the windings. It is impossible to include all these factors in the numerical computation model of the SRM stator without the help of measurements because of manufacturing variations and difficulties associated with calculating for example the stiffness of a winding. Accurate determination of the vibration parameters, such as damping coefficient etc, has to be performed through vibration tests. Besides providing direct results on vibration research, the measurements (especially modal tests) are also used for verifying the prediction/simulation and for refining the system model.

Early applications of modal testing were used mainly to identify resonant frequencies [5]. The modal test was still called "resonance testing" until the 1960s when it was applied to the determination of mode shapes [6]. As new excitations, FFT, data-analysis techniques and microcomputers were introduced, modal testing has experienced a rapid development from analog to digital. At present, there are many methods to perform each of the four stages in modal testing: excitation, data acquisition, signal processing and parameter extraction. A comparison of the advantages and disadvantages of different modal testing techniques was done in [7]. No single excitation, acquisition, processing and analysis method is superior to the rest in every situation. The best method depends on the particular application and available facilities, although there are many papers describing the preferences of their authors.

This paper concentrates on modal tests of the SRM. Several simple but practical methods including both time and frequency domain methods are presented for vibration measurements and parameter extraction. This includes the performance of vibration tests with minimum available facilities. A few low order mode shapes of a 5-hp 8/6 pole SRM are identified. The frequency response function of a single degree of freedom (SDOF) system for superposition analysis of multi-degree of freedom (MDOF) systems is presented. The resonant frequencies measured by different methods are compared to each other. Finally, the effects of the SRM end-bells on stator resonant frequencies are found through tests, which will be beneficial to future vibration analysis and control of rotating electrical machines.

II. MEASUREMENT TECHNIQUES AND PROCEDURES

The dynamic modal testing procedure includes excitation of the measured object and identification of the modal parameters, such as resonant frequencies, damping coefficients and corresponding mode shapes. Theoretically, there are two basic methods for modal tests of an object, i.e., the normal mode method and the transfer function method.

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Whichever is used to perform the modal testing in the SRM, the selection of the methods in four basic stages has to be done in advance: (1) excitation in the SRM; (2) data acquisition; (3) signal processing; (4) parameter extraction.

(1) Excitation methods

Excitation methods can be categorized into single frequency and multi-frequency. Single frequency or sine wave vibration attempts to excite only one mode at a time while multi-frequency excitation is employed for exciting several modes simultaneously. Single-frequency excitation includes the sinusoidal dwell and sinusoidal sweep, which can provide broadband spectral information, one frequency at a time. Multi-frequency excitation includes ambient (wind, seismic, wave action etc.), transient (impulse or step relaxation/"twang"), random (continuous and non-repetitive signals, pseudorandom or burst), chirp and fast sine sweeps etc. Random and fast sweep techniques have to be analyzed by Fourier analysis. The "twang" by a hammer and random vibration is the most widely used form of excitation. The advantages and disadvantages of the different excitation methods are given in [9]. Selection of excitation method depends largely on the availability of the test equipment.

The SRM stator can be excited by an impulse hammer, a simple vibration test tool. This multi-frequency method can excite a broadband vibration spectrum from which it is difficult to figure out the resonant frequencies corresponding to the low order mode shapes of the SRM stator. This situation is especially true for a stator with ribbed frame since the ribs on the frame produce numerous complicated modes [2,3], and care must be taken with signal acquisition, accelerometer location etc.

Phase current excitation and shaker excitation are applied in this paper. The details of each method are described and the advantages and disadvantages are compared between the two methods. The results compare favorably.

(2) Data Acquisition

Data acquisition can be done by either acquiring a history of excitation and response for subsequent analysis (time domain or frequency domain) or obtaining the spectra directly through real time FFT using for example the sine wave integration Fourier transform (SWIFT) algorithm. Of course, current commercial instruments for vibration measurement have integrated data-acquisition and analysis so that the users are only required to choose proper transducers and their mounting locations. In fact, data acquisition for SRM vibration tests, can be performed by using transducers and a digital oscilloscope, if there are no specialized vibration instrumentation available.

Direct force signals and indirect magnetic force (calculated from current) acquisition are described in this paper, which shows that the SRM vibration can be measured without professional vibration test equipment.

(3) Data Processing

Data processing, especially of digital signals, has developed rapidly since the 1980s. The acquired data includes time domain and frequency domain data, which can be converted to each other by the FFT or the inverse FFT. Data processing is used to generate the refined frequency response function (FRF), and contains averaging, windowing, wild-point editing, smoothing, refinement algorithms, and even error analysis. Data processing requires constructing filters, which can distinguish the wanted signals from noise or unwanted signals. A simple noise-removing algorithm is to transform the real time signal to the frequency domain first, rank the frequency domain signals and reduce the signal corresponding to noise signals (usually with characteristic low amplitude in the frequency domain). The real signals are obtained through transforming the frequency domain signals back to the time domain if required. These processes are critical to a successful measurement.

(4) Parameter Extraction Methods

There are three analysis techniques to extract the modal parameters.

(a) Frequency domain analysis is based on the frequency response function (FRF) to estimate the resonant frequency, damping factors and corresponding mode shapes. The most frequently used strategies include frequency-domain curve fitting (FDC) and simultaneous frequency domain (SFD) analysis.

An indirect transfer function method is introduced in this paper, in which the excitation force in the frequency domain is indirectly calculated from measured excitation currents by the finite element method. Therefore, modal parameters are identified without directly measuring the excitation force.

(b) *Time domain analysis* includes (i) The state-variable method, and (ii) The complex exponential algorithm (CEA). The state-variable method and its alternative method are seldom used in modal analysis although they lead to a successful formulation of the simultaneous least-squares method. The CEA in the paper is based on linear superposition and an exponential curve-fitting algorithm. It is performed by equating the polynomial coefficients to the measured response history, so that the resonant frequency and damping factor can be extracted, and the complex modal amplitude determined. This method is especially useful for determining the resonant frequency and damping factor of a mode shape, which can approximately be treated as a single degree of freedom, although it can be extended to multi-dimensions [9].

(c) Tuned-sinusoid analysis consists mainly of (i) Tuned sine sweep and (ii) Multi-exciter tuned dwell (MTD) method. The tuned-sine sweep is the traditional method of

frequency response measurement. In this method, a sweep oscillator is used to provide a sinusoidal command signal whose frequency sweeps continuously through the range of interest. The frequency variation has to be so slow that the response can be treated as steady state. The sweep rate will dramatically affect the measuring accuracy [7]. The selection of the excitation location and an automatic tuning procedure were given in [10] and [11], respectively. The MTD method [12] was proposed to excite a pure natural mode of interest while suppressing the others. The single mode response is obtained by varying the ratio of the exciting force to frequency of two or more shakers. It is straightforward since the resonant frequency is the excited frequency and the measured motion is the mode shape, and the damping can be estimated by cutting off the tuned excitation and measuring the transient decay history. The tuning is a skillful task for all tuned sinusoidal analysis methods.

III. EXPERIMENTAL SYSTEMS

In order to determine the modal parameters and the corresponding mode shapes of a vibration model, two measurement systems are used in this paper in terms of excitation methods: phase current excitation and magnetic shaker excitation.

(1) *Experimental system with phase current excitation*

In Fig.1, the power drive is a PWM voltage source, which provides rectangular voltage to the stator phase windings in the aligned position of the measured SRM. There are two accelerometers screwed to the SRM. They lie right behind the excited stator poles on the case of the motor. The acceleration signals are amplified by the coupler, and then sent to a multi-channel digital oscilloscope. The voltage and current waveforms of the SRM are recorded by the oscilloscope. The processing can be performed by the oscilloscope or transferred to a computer and processed by the data processing toolbox in Matlab. If the modal equivalent parameters (m, c, k) are required, the magnetic force between the stator and rotor poles can be calculated by finite element methods according to the phase current. Then the frequency domain method or transfer function algorithm can be used to extract the system parameters.

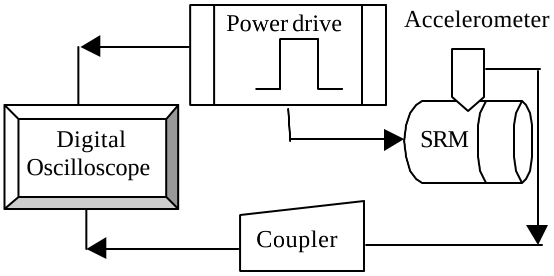


Fig.1 Diagram of the test system with a phase current excitation

It should be noticed that this excitation method could only be used to measure the 2^{nd} order modal parameters since the phase current can only produce a force-pair, which excites the vibration with oval deformation. The following frequency spectra and mode shape results verify this theoretical analysis.

(2) *Experimental system with the permanent magnet shaker*

Two different excitation methods are introduced here: sinusoidal excitation and random (white noise) excitation.

The diagram of this testing system is given in Fig.2. The only difference between these two methods lies is that the excitation signal of the shaker: sinusoidal signal or white noise signal. The signal acquisition side of the system is the same as that in diagram of Fig.1. Seven accelerometers (Kistler 8730A500) are screwed into the SRM surface at equi-distances in the circumferential direction, except for the position of terminal box. The force transducer is installed between the push rod of the PM shaker and the SRM. The response signals from the accelerometers and the excitation signal from the force transducer are transferred to two oscilloscopes (100MHz 20Ms/s 12 bits Nicolet and 200MHz 100Ms/s LeCroy 9304AM) through two multi-channel couplers. A power amplifier drives the permanent magnet shaker. In any case, the mass of the exciter should not affect the system vibrations. A function generator generates a sinusoidal waveform of the sinusoidal force excitation, and a white noise generator generates white noise signal of the random force excitation. The signal from the function generator (white noise generator) is also connected to one of the oscilloscopes in order to monitor the force exerted on the SRM. This system can also be used to identify the MDOF vibration.

(3) *Experimental system with no-load normal running condition*

The no-load normal running condition experiment system is the same as shown in Fig.1. The difference is: during phase current excitation, only one phase is excited and

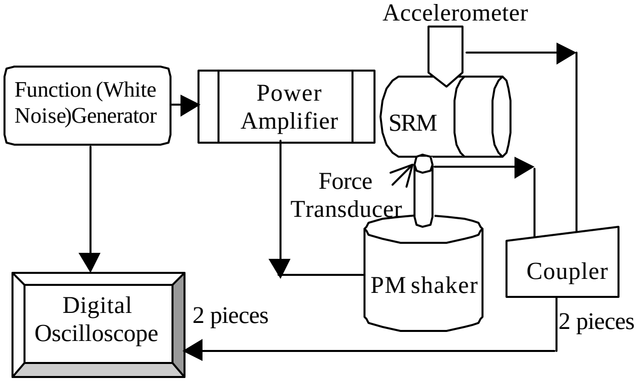


Fig.2 Diagram of vibration testing system with PM shaker excitation

the motor is stationary, while here the SRM is running in the normal condition with no-load at several different speeds.

IV. EXPERIMENTAL RESULTS

(1) Real time method to identify modal parameters through phase current excitation

Firstly, the vibration test system with phase current excitation is used to measure a 5hp, 8/6 SRM with 4 phases. The accelerometer is located just behind the excited phase pole on the top of the SRM. The test results of the current, voltage and acceleration are recorded in the time domain.

The acceleration curve of the measurement results can be used directly to determine the resonant frequency and damping ratio. One group of the acceleration waveforms of the SRM vibration is enlarged and redrawn in Fig.3. The damping oscillation period t_d can be obtained by measuring the time interval between two adjacent peaks, or by measuring the time interval between *N* peaks, and dividing by *N*. The damping ratio is obtained through measuring peak values. Two accelerations, a_k and a_{k+N} , separated by *N* complete cycles give the logarithmic decrement, i.e.,

$$d = \frac{1}{N}\ln\frac{a_k}{a_{k+N}} \qquad (1)$$

Furthermore, the damping ratio z can be found by

$$z = \frac{d}{\sqrt{(2p)^2 + d^2}} \approx \frac{d}{2p} \quad \text{(if } \delta \ll 1\text{)} \qquad (2)$$

The results are given in Table I. It is not difficult to find that the damped resonant frequency is almost the resonant frequency (or natural frequency) when the damping ratio is small (here it is about 2%).

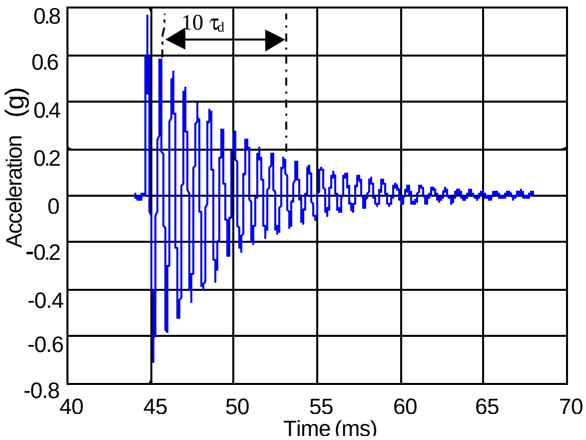


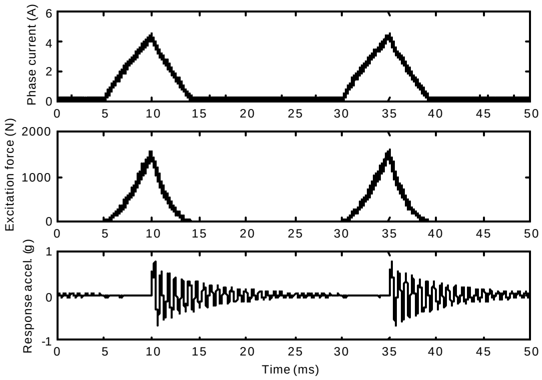
Fig.3. Vibration acceleration curve in time domain

TABLE 1
MEASURED RESULTS OF RESONANT FREQUENCY AND DAMPING RATIO

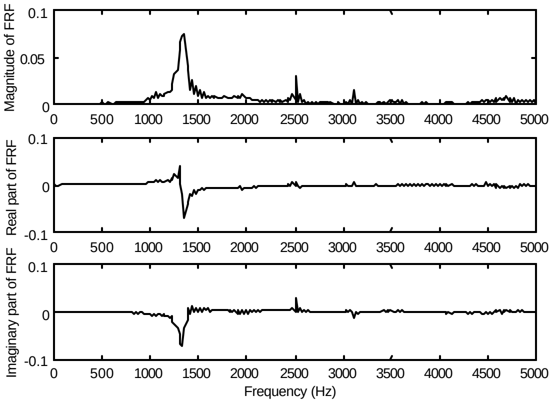
Peak No.2 (g)	0.5750
Peak No.12 (g)	0.0625
Time interval T_{10} (ms)	7.418
Period t_d (ms)	0.7418
Damped f_d (Hz)	1348
Logarithm decrement δ	0.12637
Damping ratio ζ	0.0201
Natural frequency (Hz)	1348.3

(2) *Parameter Identification by Indirect Inertance Frequency Response Function (FRF)*

Transfer function methods have become more and more popular since the 1970s. The indirect method to determine the resonant frequency is introduced in this paper. Firstly, the magnetic forces corresponding to phase current excitation (repetition =25ms or 40ms) are obtained through a lookup table of the magnetic force and phase current which are computed by the FE method, as shown in Fig.4 (a). Secondly, the FFT of the force excitation and the FFT of the acceleration response are found, then the inertance FRF (i.e. ratio of response/excitation in the frequency domain) is obtained. The resonant frequency (1360Hz) will correspond to the zero real part and the minimum imaginary part of inertance FRF, shown in Fig.4 (b) The maximum displacement is about 4.3×10^{-8}m. Another peak is found in the imaginary part of inertance response, which corresponds to the 3rd order mode at 2540Hz. The calculated resonant frequency for the 3rd mode shape is 2589Hz (error is less than 2%). There is a small error of about 1~2% with the indirect measurement, compared to the result of the direct method. The accuracy of this method depends on the accuracy of the magnetic force computed by the FE method, i.e., the error comes from numerical field computations.



(a) Current, magnetic force, and acceleration vs. time



(b) Frequency response function

Fig.4 Results by transfer function technique

(3) Measurement results from normal mode method

The normal mode method was used traditionally for modal tests before the use of the transfer function method. The main objective of the normal mode method is to excite the undamped modes of the measured SRM, one at a time. Shaker excitation, with one or more shakers driven by sinusoidal signals at the same time, is one of the principal features. The testing procedure consists of five steps. (1) Wide frequency band sweep: change the frequency of the function generator output from a low to high range, resulting in a variation of the excitation force applied to the SRM. After the amplitude response of the accelerometers is determined with an oscilloscope, the testing enters the second step. (2) Narrow frequency band sweep: A smaller frequency change is used to locate the modal frequency, corresponding to the maximum response in that frequency region. (3) Modal tuning: the amplitude, polarity and frequency of the sinusoidal excitation from the shakers are adjusted to excite the resonant mode shape. To identify a single mode (or a pure mode) of vibration, multiple shakers are highly recommended. (4) Modal dwell: the vibration amplitudes at many points on the SRM, and the corresponding excitation force are recorded once a mode is well tuned. (5) Damping measurements: the damped sinusoidal response histories of a free vibration are recorded at all points from the moment all shakers are simultaneously shut off. For a pure mode, all responses should exhibit the same sinusoidal decay feature, and the damping at the resonant frequency of the mode can be measured from the envelope of the damped sinusoidal response. Typically the impulse responses will show a beating since more than one mode is often involved.

The oscilloscope output for the 2nd order mode shape of the SRM stator with end-bells is given in Fig.5. Channel one shows the force excitation signal, while channels 2~4 are the acceleration responses at the locations with 90° mechanical span in the circumferential direction of the SRM stator. Fig.5 shows in channels 2 and 4, that the response of two accelerometers with 180° span, are in phase. The response of channel 3, which is connected to the accelerometer located 90° from the accelerometers displayed in channels 2 and 4, has a phase difference of 180° from that of channels 2&4. All responses have the same amplitude at 1346Hz. The responses from channels 5~8 have low amplitudes. These show the features of the 2nd mode shapes.

Similarly, the 3rd and 4th order mode shapes of the SRM stator with end-bells are also measured, shown in Table II.

(4) Experimental Results of the End-bells Effect on the SRM Resonant Frequencies

To verify the effects of the end-bells on the SRM stator vibration (Fig 6), normal mode testing method is repeated after the end-bells are removed. The power spectrum density of the second mode shape is recorded. Similarly, the measured resonant frequencies for the 3rd and the 4th order mode shape are 2450Hz and 4850Hz, respectively.

To compare the test results with and without end-bells, the resonant frequencies are listed in Table II.

Obviously, the resonant frequencies increase after installing the end-bells. The error due to neglecting the end-bells is not tolerable in the prediction of the SRM resonant frequency, which is 21.25% for the 2nd mode shape.

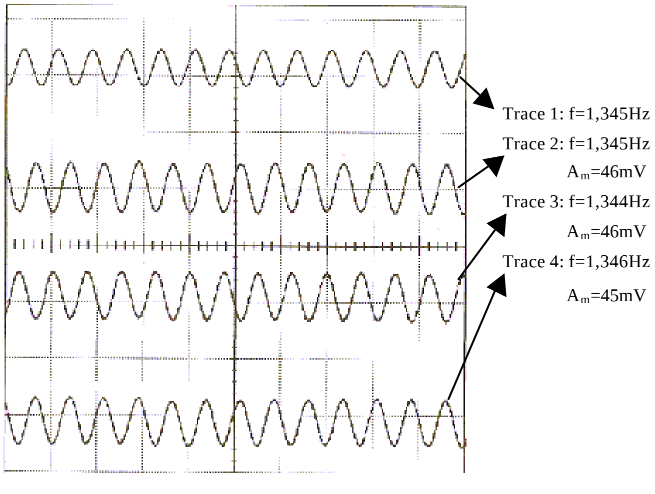


Fig.5 Oscilloscope output at the 2nd order mode of SRM stator with endbells

TABLE II

TESTED RESONANT FREQUENCIES FOR THE SRM WITH AND WITHOUT END-BELLS

Models	2nd mode	3rd mode	4th mode
Testing model without end-bells	1060 Hz	2450 Hz	4850 Hz
Testing model with end-bells	1346 Hz	2680 Hz	4987 Hz
Errors (%)	21.25	8.58	2.75

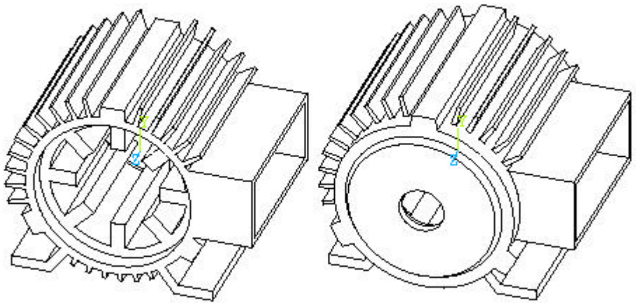


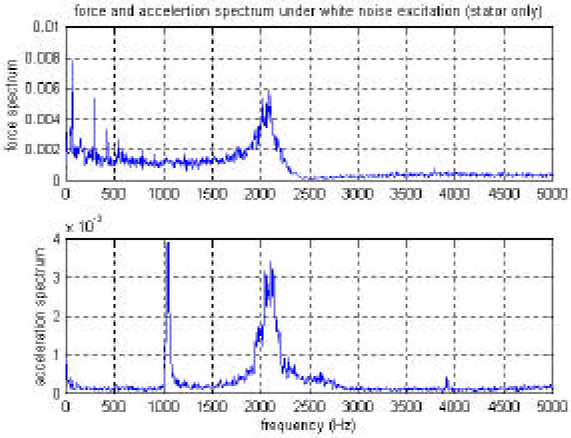
Fig.6 SRM with and without end-bells

Furthermore, the effect of ribs and end-bells on the resonant frequency should be included in order to improve the accuracy of calculation.

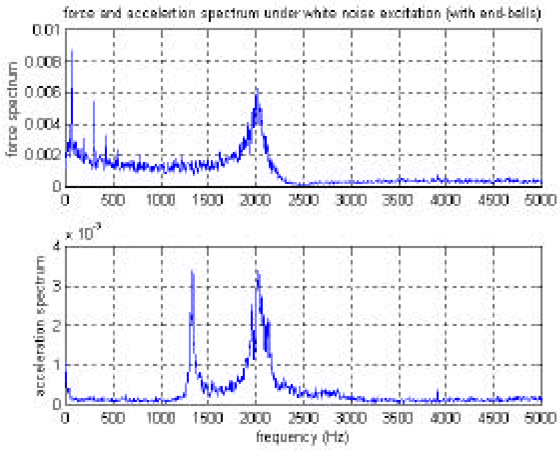
(5) *Measurement Results of the White Noise Excitation*

Figs 7(a), (b) and (c) show the measured force and acceleration spectra under white noise excitation. (a) is the result with stator only, (b) is the result for the stator with end-bells (no rotor), and (c) is the result for the entire motor (with end-bells and rotor). It can be seen from the white noise excitation results that the 2^{nd} mode resonant frequency is 1045Hz for the stator only and 1320Hz for the stator with end-bells and 1330Hz for the entire motor (stator with end-bells and rotor). The other peak in the acceleration spectrum (2100Hz for the first case and 2020Hz for the second and third cases) is caused by the experimental system (there is a steel push rod connecting the shaker and the stator of SRM), which can be confirmed by looking at the peak of the force spectrum.

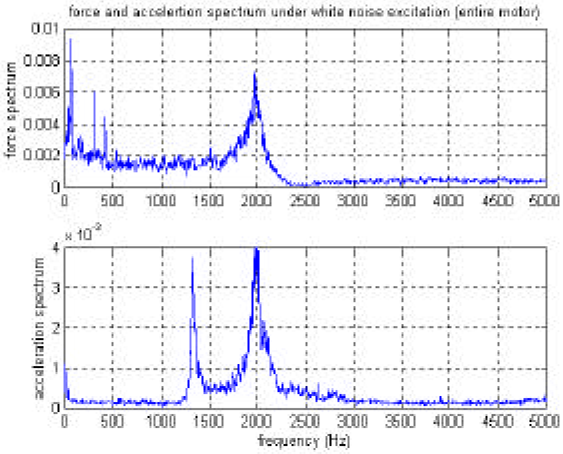
The measured 2^{nd} mode resonant frequencies for the stator with (1045Hz) and without (1320Hz) end-bells, and for the entire motor (1330Hz) are fairly close to the results of the other methods mentioned above, which further confirms that both methods are reliable.



(a) with stator only (no end-bells, no rotor)



(b) with end-bells (stator with end-bells, no rotor)



(c) entire motor (stator with end-bells and rotor)

Fig.7 Force and acceleration spectrums under white noise excitation

Compare Fig.7 (b) and (c) (both have end-bells but without or with rotor), the 2^{nd} mode resonant frequencies are very close. The reason for the slight difference is complicated, it is a combination of adding the rotor mass, as well as the damping and stiffness changes caused by adding rotor and bearing, etc. This result shows that the resonant frequencies of the stator with end-bells (but without rotor) can be used to represent the frequencies of the entire motor, which will simplify the experimental procedures. Furthermore, modal results from finite element method show very good correlation without losing much accuracy [2,3].

(6) *Measurement Results of the No-load Normal Running Test*

Fig.8 shows the measured acceleration spectra for four channels (accelerometers on different locations of the SRM stator). It is easy to verify that the acceleration response around the 2^{nd} mode resonant frequency (it is 1305Hz) in this case) is much higher than the other frequencies.

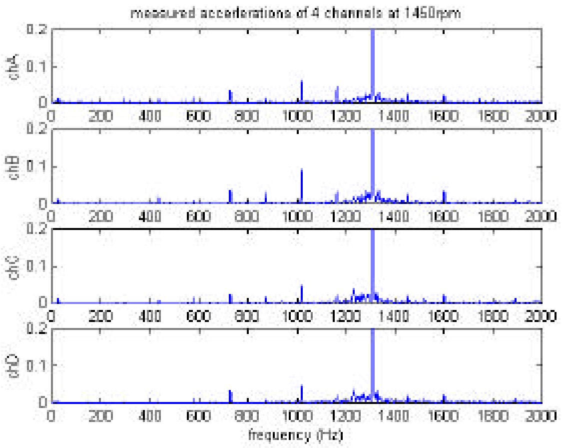


Fig.8 Measured accelerations under no-load normal running condition

The measurement results of several other speeds also show that the acceleration response amplitude around the 2nd mode resonant frequency is much higher relative to the others. This can also confirms that the 2nd mode resonant frequency is the most important to consider for a reduced noise design of SRMs with 8/6 poles.

V. CONCLUSIONS

The study of vibrations and the development of experimental techniques for their reduction are necessary to allow SRMs to reach its full potential in industrial applications. A real time method through phase current excitation is applied for identification of the vibration parameters of the SRM. An indirect frequency transfer function method for modal tests is introduced in the paper. These methods can be used to perform an experimental study of SRM vibrations without force transducer. The test results compare favorably to the traditional modal test method. The methods presented are used for modal testing of a 5hp 8/6 SRM stator with and without end-bells. The effects of the end-bells on the resonant frequencies of the SRM are presented, which shows that the end-bells cause a significant increase of resonant frequencies of the SRM, especially for the 2nd order mode shape. The rotor has little effect on the stator vibrations.

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