DEVELOPMENT OF DIGITAL MODELS FOR A VECTOR CONTROLLED PERMANENT MAGNET SYNCHRONOUS MOTOR DRIVE

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ABSTRACT

It is generally true that analogue implementations of speed and current controllers in ac or dc servo drives tend to preceed microprocessor based implementations. In particular, most of the permanent magnet synchronous motor drives developed up to now have used analogue type controllers although digital implementations are beginning to be reported. In most microprocessor based implementations that have been developed only the speed or position loops have been implemented digitally. This is largely due to the high switching frequencies and bandwidths associated with the current loop thus complicating its digital implementation. Hence a digital model for a PMSM drive is developed assuming that only the speed controller is implemented digitally with the current controller implemented in hardware and thus freed from the problems associated with finite sampling times. However in anticipation of future drives with digital current controllers as well, a model is also developed assuming both the speed and current controllers are implemented digitally.

I. INTRODUCTION

Microprocessors have imparted a new philosophy to the control of electric machines. The concepts of hierachical and decentralized control have evolved as well as operating a number of machines in unison. In the latter, different motor drives perform different functions of the same overall task, with high speed and reliable communication between the drives being a necessary feature to obtain overall control and reliability. Here dedicated microprocessors, operating in the slave mode, control individual machines while obtaining commands from a master microprocessor or computer in charge of the overall task. Adaptive control also becomes an inherent feature of microprocessor based drives because of the possibility of changing the controller parameters which reside in specific locations in the microprocessor memory.

Normally analogue techniques are used first in the development of a motor drive with digital implementation following at a later stage. This has certainly been true of dc and induction motor drives. Permanent magnet (PM) motor drives that are currently on the market also use analogue implementations of the controllers although microprocessor based implementations are beginning to be reported [1].

A key difference between digital and analogue controllers is the observation of the output of the plant by the digital controller only at the instants of sampling. The control laws are based on these observations at discrete instants only and hence operates essentially on open loop between sampling instants. Whereas the Laplace Transform has proven to be a valuable tool in the study of continuous time systems, it is necessary to use a transform that includes the sampling frequency of the digital controller as a parameter. The Z transform does this and essentially converts a difference equation into an algebraic equation for easier manipulation and solution.

Most of the earlier work on the digital control of electrical machines dealt with the dc motor [2-7]. The development of a digital model for a dc motor and a comparative evaluation of different digital speed controller configurations was done in [2]. A grapho-analytical technique for controller design in the Z domain has also been developed [2]. Controller design under different performance criteria for the dc motor has been done [3]. The possibility of applying adaptive [6] and sliding mode control [7] to digitally controlled dc motor drives have been examined. Particular attention was paid to the electronics of digitally controlled drives in [8,9]. DC motor drives can either operate in the continuous or discontinuous current conduction modes. Digital models for a dc motor current loop when operating in these modes have been developed [10].

Although numerous papers have been written on digital or microprocessor based ac drives, most have concentrated on the implementation. Very few have systematically developed digital models and investigated stability problems due to the sampling frequency and the possibility of controller design in the Z domain. A vector controlled digital speed control system for an induction motor was developed in [11]. A microprocessor was used to implement the speed controller, with the vector controller and current loop being implemented in hardware outside the microprocessor. A similar procedure was adopted in [1] for a digitally controlled PMSM drive. Other microprocessor based implementations also implement the vector controller and current loops in hardware outside the microprocessor so that they are freed from the problems of finite sampling times associated with the microprocessor. This is largely due to the high switching frequencies and bandwidths associated with the current loop thus complicating its digital implementation.

Crucial to the successful operation of a digital controller is the sampling time needed in order to ensure stability of the drive. This necessitates the development of a digital model suitable for the assessment of stability. Hence a digital model for a PMSM drive is developed assuming that only the speed controller is implemented digitally with the current controller implemented in hardware and thus freed from the problems associated with finite sampling times. However in anticipation of future drives with digital current controllers as well, a model is also developed

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assuming both the speed and current controllers are implemented digitally. Here in general the current loop would probably be sampled at a higher rate than the speed loop because of its higher bandwidth. Hence the digital model developed is capable of handling samplers operating at different sampling rates.

The paper is organized as follows: Section II develops a linear model of a vector controlled PMSM from the nonlinear model and presents the overall transfer function. Section III develops digital models for a vector controlled PMSM drive system with single rate and multirate sampling. Section IV uses this model to investigate the effects of the sampling time on the stability of the drive system. Section 5 has the conclusions.

II. A LINEAR MODEL FOR A PMSM DRIVE

The complete nonlinear model of a PMSM without damper windings is as follows [12]:

$$v_q = Ri_q + pL_q i_q + \omega_s(L_d i_d + \lambda_{af}) \quad (1)$$

$$v_d = Ri_d + p\lambda_d - \omega_s L_q i_q \quad (2)$$

v_d and v_q are the d,q axis voltages, i_d and i_q are the d,q axis stator currents, L_d and L_q are the d,q axis inductances, R and ω_s are the stator resistance and inverter frequency respectively. λ_{af} is the flux linkage due to the rotor magnets linking the stator. The electric torque

$$T_e = 3P(\lambda_{af} i_q + (L_d - L_q) i_d i_q)/2 \quad (3)$$

and the equation for the motor dynamics is

$$T_e = T_L + B\omega_r + Jp\omega_r \quad (4)$$

P is the number of pole pairs, T_L is the load torque, B is the damping coefficient, ω_r is the rotor speed and J the moment of inertia. The inverter frequency is related to the rotor speed as follows:

$$\omega_s = P\omega_r \quad (5)$$

The machine model is nonlinear as it contains product terms such as speed with i_d and i_q. Note that ω_r, i_d and i_q are state variables.

During vector control, i_d is normally forced to be zero, therefore equation (2) which describes the dynamics of i_d is unnecessary in any controller design application. Further, (1) and (3) degenerate to the following equations:

$$v_q = Ri_q + pL_q i_q + \omega_s \lambda_{af} \quad (6)$$

$$T_e = 3P\lambda_{af} i_q/2 = K_t i_q \quad (7)$$

Hence the torque depends only on the q axis current and the nonlinearity produced by the product of currents in (3) is removed by forcing i_d to be zero. In addition the nonlinearity produced by the multiplication of the speed with the direct axis current in (1) is also removed by forcing i_d to be zero. From (6),(7) and (4), the transfer function in figure 1 can be derived which shows the machine operating essentially on open loop. In a practical speed controlled drive system, both the speed and current loops would be closed in an attempt to ensure zero steady state error in the commanded speed and current and to provide desirable dynamic performance. The method of closing the speed and current loops is shown in figure 2.

The speed reference is compared with the actual speed to generate an error. The error is operated upon by a speed controller to generate a reference quadrature axis current. This is compared to the actual current flowing to generate a current error. The output of the current contoller which operates on the current error is then input to the motor transfer function developed in figure 1.

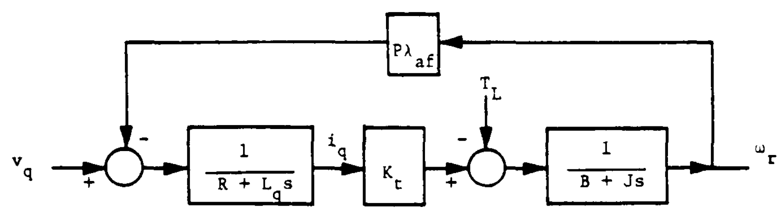


Figure 1. Block diagram of a PMSM

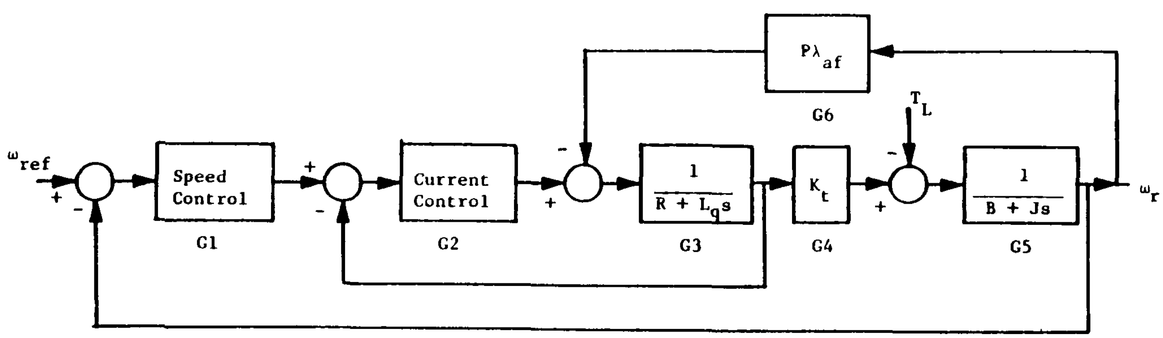


Figure 2. Block diagram of a PMSM Drive

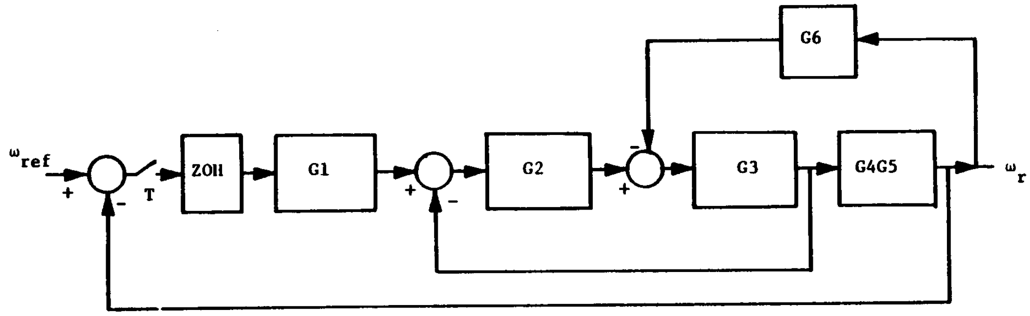


Figure 3. Single rate multiloop control system

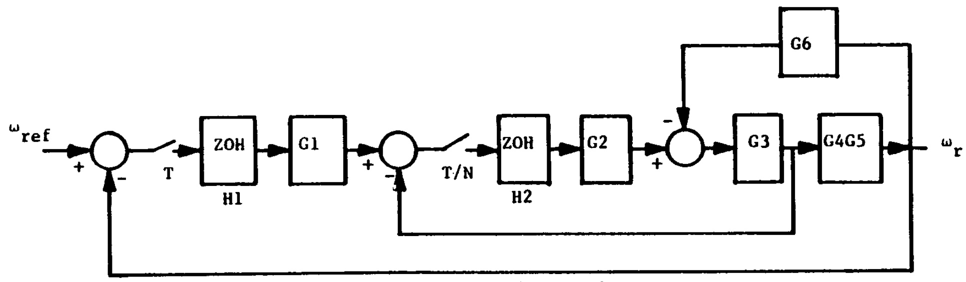


Figure 4. Multirate multiloop control system

III. DEVELOPMENT OF DIGITAL MODELS FOR A PMSM DRIVE

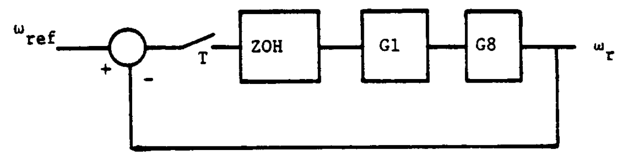
In order to develop a digital model for a vector controlled PMSM drive, it is necessary to discretize the system shown in figure 2.

Two cases are considered. In the first case, it is assumed that only the speed loop is implemented in the microprocessor with the current loop operating in continuous time. This is shown schematically in figure 3. In this case figure 3 can be described as a single rate, multiloop digital control system. The second case considered is where the current loop is also implemented digitally but where in general it may be sampled at a different rate from the speed loop. This can be described as a multirate, multiloop digital control system and is shown in figure 4.

It is assumed that error sampling is used on both the speed and current loops. In the multirate, multiloop system of figure 4, if the speed loop is sampled at period T, then it is assumed that the current loop is sampled at T/N where N is an integer. In addition it is assumed that the current and speed loop samplers are synchronized. In addition, a zero order hold (ZOH) is assumed after each sampler. This means that if the speed or current controllers are to be implemented digitally then the ZOH equivalent design is implemented in the microprocessor. From the models developed a technique for the assessment of the stability of the system particularly with respect to the sampling period T will be shown.

Single Rate, Multiloop System

Standard reduction techniques [13,14] can be used to obtain the transfer function of the output speed over the input speed in the Z domain. Although it is a reasonably simple matter to obtain the transfer function of the single rate, multiloop system shown in figure 3, a step by step procedure is given anyway so as to facilitate the analysis of the multirate, multiloop system shown in figure 4. The steps are as follows [13].



$$G8 = \frac{G2G4G5G7}{G4G5 + G2G7}$$

$$G7 = \frac{G3G4G5}{1 + G3G4G5G6}$$

Figure 5. Single rate singleloop control system

[1] Draw the original signal flow graph (SFG) with the samplers
[2] Choose a name for each sampler (eg.X) and denote the output of that sampler by X^*.
[3] Express the inputs of each sampler and the system output in terms of the sampler outputs and system input.
[4] Take the starred transform of the resulting equations and solve for the sampler outputs by some convenient method eg. signal flow graphs or any other technique suitable for the solution of simultaneous equations like Cramer's rule.

Before applying the above rules, figure 3 can be simplified as shown in figure 5. This reduces the algebra during the application of the above rules. The application of the first rule results in the SFG shown in figure 6. If the input to the sampler is E1 then the output by rule 2 is $E1^*$. Then applying rule 3 results in:

$$E1 = \omega_{ref} - G1*G8*ZOH*E1^* \quad (8)$$

$$\omega_r = G8*G1*ZOH*E1^* \quad (9)$$

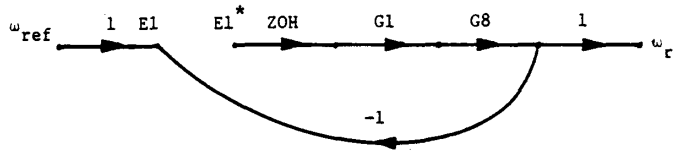


Figure 6. Signal flow graph of single rate system

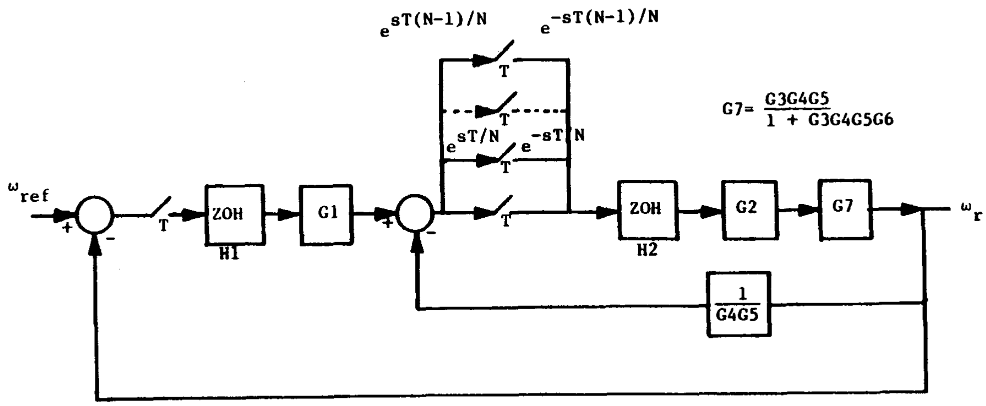


Figure 7. Equivalent single rate system

Taking the starred transform results in

$$E1^* = \omega_{ref}^* - (G1*G8*ZOH)^* E1^* \qquad (10)$$

and

$$\omega_r^* = (G8*G1*ZOH)^* E1^* \qquad (11)$$

Therefore

$$E1^* = (\omega_{ref}^*)/(1 + (G1*G8*ZOH)^*) \qquad (12)$$

$$\omega_r^* = (G8*G1*ZOH)^* \omega_{ref}^*/(1 + (G1*G8*ZOH)^*) \qquad (13)$$

$$\omega_r(z) = (G8*G1*ZOH)(z)\omega_{ref}(z)/(1+(G1*G8*ZOH)(z)) \qquad (14)$$

If it is necessary to ascertain the stability of the drive system as a function of the sampling interval T, it is only necessary to use the characteristic equation (CE) of the system which is given by

$$1 + (G1*G8*ZOH)(z) \qquad (15)$$

In general, the Z transform of a transfer function can have T, T^2 or exp(-aT) as factors. This leads to a transcendental equation in T for which a closed form solution is generally unobtainable. Hence if it is desired to apply the Jury stability test to assess the stability of the drive system, numerical iteration would have to be used to determine whether each of the criteria of the Jury test is satisfied. Since some form of numerical technique would be used anyway, an alternative is to solve directly for the roots of the characteristic equation and to ascertain whether they lie within the unit circle for stability of a discrete time system. This latter technique was chosen since the number of criteria to be checked using the Jury test is directly proportional to the order of the CE.

Multiloop-Multirate System

The modeling of a multirate, multiloop system is far more complicated than that of a single rate, multiloop system. The technique used here was developed in [15] and explained in [16] and [17] and is known as the sampler decomposition method. The sampler operating at T/N is replaced by N samplers operating at period T, with time advance and delay components as shown in figure 7. Now single rate reduction techniques can be applied to determine the input-output transfer function or the CE. It should be remembered that the time advanced elements on one side of the sampler do not multiply with the time delayed elements on the other side of the sampler to produce unity. Instead each of the time advanced elements must be multiplied by G1 before taking the Z transform while each of the time delay elements must be multiplied by the transfer functions on the other side of the multirate sampler as well.

Two points need to be remembered when attempting to determine the transfer function or CE of figure 7. The first is that each of the time delay and advance elements contain certain fractions of the sampling period T. This implies the use of the modified Z transform. Whereas the ordinary Z transform can have the sampling period T as a parameter, the modified Z transform in general has T and m as factors. The Z transform of a time delayed function (where the delay is a fraction of T) is given by:

$$Z(G\exp(-ksT/N)) = Zm(G) \text{ "evaluated at } m=1-k/N\text{"} \qquad (16)$$

k/N is the fraction of T while Z and Zm indicate the Z transform and modified Z transform respectively. The Z transform of a time advanced function is given by

$$Z(G\exp(ksT/N)) = z*Zm(G) \text{ "evaluated at } m=k/N\text{"} \qquad (17)$$

where the lower case z refers to the parameter z in the Z transform. Tables of z transforms are available for example in [15] and are used in a similar fashion to Laplace Transforms.

The second point that must be remembered is that the ZOH operating after the high speed sampler has a different transfer function from that operating at a lower frequency. If the transfer function of the ZOH associated with G1 is (1-exp(-sT))/s, then the transfer function of the high rate ZOH is (1-exp(-sT/N))/s [15].

The signal flow graph of the system in figure 7 is given in figure 8. The sampler names are chosen conveniently and labelled on figure 8. The above constitutes rules 1 & 2 in the derivation. Let the low speed ZOH be called H1 and the high speed ZOH be H2. Then applying rule 3, the following equations are obtained

$$E1 = \omega_{ref} - \sum_{k=0}^{N-1} (H2*G2*G7*exp(-ksT/N))X_k^* \quad (18)$$

$$X_i = (G1*H1*exp(sTi/N))E1^* - \sum_{k=0}^{N-1} ((H2*G2*G7*exp(-ksT/N)exp(isT/N))X_k^*)/(G4*G5) \quad (19)$$

$$\omega_r = \sum_{k=0}^{N-1} (H2*G2*G7*exp(-ksT/N))X_k^* \quad (20)$$

Let G1xH1 = GH, H2xG2xG7 = G9 and (H2xG2xG7)/(G4xG5) = G10 then

$$E1^* = \omega_{ref}^* - \sum_{k=0}^{N-1} (G9*exp(-ksT/N))^* X_k^* \quad (21)$$

$$X_i^* = (GH*exp(sTi/N))^* E1^* - \sum_{k=0}^{N-1} (G10*exp(-ksT/N)exp(isT/N))^* X_k^* \quad (22)$$

$$\omega_r^* = \sum_{k=0}^{N-1} (G9*exp(-ksT/N))^* X_k^* \quad (23)$$

As an example, consider N=3. The following set of equations result:

$$E1(z) = \omega_{ref}(z) - G9(z)X_0(z) - G9exp(-sT/3)(z)X_1(z) - G9exp(-2sT/3)(z)X_2(z) \quad (24)$$

$$X_0(z) = GH(z)E1(z) - G10(z)X_0(z) - G10exp(-sT/3)(z) X_1(z) - G10exp(-2sT/3)(z)X_2(z) \quad (25)$$

$$X_1(z) = GHexp(sT/3)(z)E1(z) - G10exp(sT/3)(z)X_0(z) - G10X_1(z) - G10exp(-s+T/3)X_2(z) \quad (26)$$

$$X_2(z) = GHexp(2sT/3)(z)E1(z) - G10exp(2sT/3)(z)X_0(z) - G10exp(sT/3)X_1(z) - G10X_2(z) \quad (27)$$

$$\omega_r(z) = G9(z)X_0(z) + G9exp(-sT/3)(z)X_1(z) + G9exp(-2sT/3)(z)X_2(z) \quad (28)$$

One technique suggested by [13] to solve the above set of equations is to use the signal flow graph for the above system. This is shown in figure 9. Mason's rule can then be used to determine the transfer function. However for stability, only the denominator of the transfer function is needed which from Mason's gain formula, is produced by the well known formula of 1-(sum of all individual loops)+(sum of the product of non touching loops taken 2 at a time) etc. This technique can conveniently be applied only for small N. For large N, equations (24)-(28) may be solved by Cramer's rule.

IV. RESULTS

One of the reasons for developing the digital models in this paper was for the assessment of the stability of the digital control systems. The Z transforms of the transfer functions in equation (15) were calculated. This is the CE for the system with the continuous time current loop. Note that in this case only ordinary Z transforms are needed since there are no time delays in the model. The resulting Z transform had T, T^2 and exp(aT) as factors. A program was written to solve for the roots of the CE as a function of T. In other words T was set and the resulting equation in z was solved. The resulting roots indicated stability or instability depending on whether they were inside the unit circle or not. It was easier to determine the stability using this technique than to use the Jury test because of the nonlinearity in T.

The results indicated that a sampling time less than or equal to 0.1s was sufficient to guarantee stability. Of course in a practical implementation a sampling rate many times higher than that which guaranteed stability would be used.

Some researchers completely neglect the current loop when developing the digital model of a drive system like that presented in figure 3. Essentially this means that the output of G1 which gives the current reference is input directly into G4 without first going through the dynamics associated with the current controller (G2) or the stator time constant (G3). Obviously this would produce an error in the determination of the required T for stability. The faster the dynamics of the current controller and the lower the stator time constant, the less erroneous would be the result. This approximation was made and the CE of the simplified transfer function calculated and solved for the limiting value of T for stability. The resulting T was 0.135s. This answer makes sense since by neglecting the current loop dynamics the sampling period to ensure stability would be determined by the speed loop dynamics only and would always be larger than if nonzero time constants of the

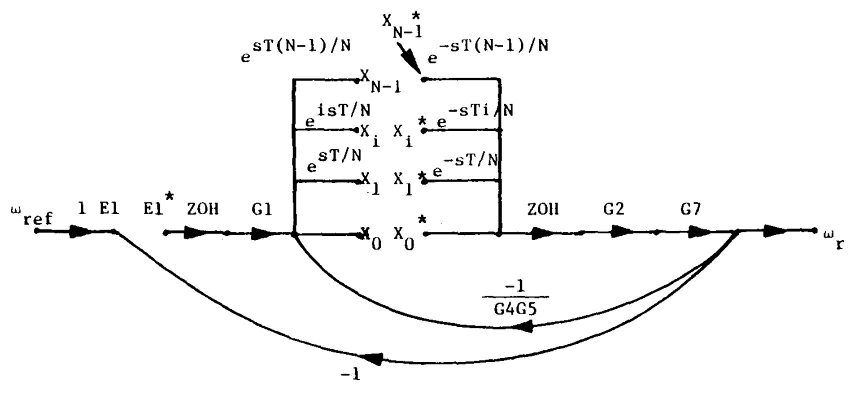


Figure 8. Signal flow graph of multirate multiloop system

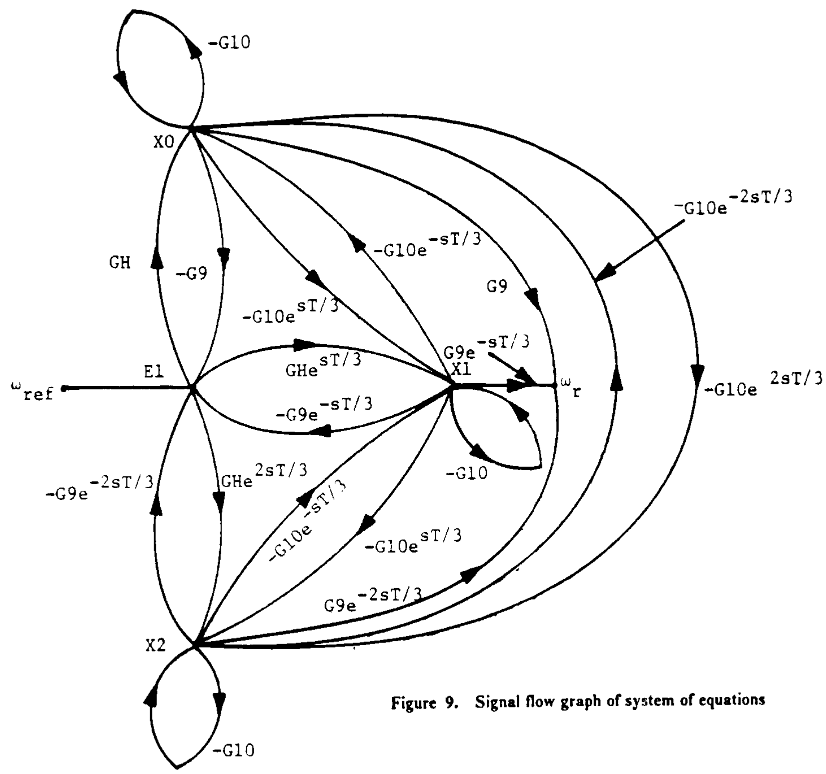


Figure 9. Signal flow graph of system of equations

current controller and stator were included. The error in the T for stability is 35%. In motor drive systems with larger stator time constants this error could be even larger.

From the SFG of figure 9 the CE was determined and the Z transforms calculated. Terms with T and exp(aT) resulted which meant that the CE was again nonlinear in T and numerical techniques were needed to solve. For N = 3, which meant that the current loop was being sampled at 3 times the rate of the speed loop, a T of 0.0001 was needed to ensure stability. This gives an indiction of the reason that the current loop is still implemented in continuous time. The sampling rate is quite high. In order to solve the system for large N requires a large amount of computation since for example for N=100, a 100x100 matrix of rational functions of 3rd degree would result when applying the rules presented earlier for such a system. It will then become necessary to use packages developed for the analysis of systems of large order. The equations developed in this paper are valid however for any N.

V. CONCLUSIONS

Two models for the digitally controlled PMSM drives have been developed. The first does not neglect the current loop but assumes that it is implemented in continuous time. This corresponds to the level of the technology today. For the system considered a T of 0.1s was needed to ensure stability. When the current loop is neglected altogether, than an error of 35% was made in the determination of T. Hence the current loop should not be neglected in such a calculation.

The second model developed assumed that both the speed and current loops were implemented digitally. Since in practice these loops have different responses, it is reasonable to assume that the loops would be sampled at different frequencies. It was shown that the modelling of such a mulitrate, multiloop system is far more complicated than a single rate multiloop system. This is because if the current loop is sampled at N times the speed loop then the current sampler is replaced by N samplers operating at T but with time advanced and delayed elements thus implying the use of the modified Z transform to a high order system. Although the resulting system structure is easy to visualize, large computer resourses would be needed to solve the resulting high order system consisting of rational functions. For small N the SFG technique may be used.

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List of Symbols

B	damping constant, N.m/rad/sec
i_a, i_b, i_c	a,b and c phase currents, A
i_d, i_q	d and q axis stator currents, A
J	moment of inertia, kg-m^2
L_d, L_q	stator d,q inductances
N	ratio between current and speed loop sampling
p	derivative operator
P	number of pole pairs
R	stator resistance, Ohms
T	sampling period
T_e	electric torque, N-m
T_L	load torque, N-m
v_d, v_q	d and q axis stator voltages
ω_r	rotor speed, rad/sec
ω_s	synchronous speed, rad/sec
λ_{af}	mutual flux linkage between rotor and stator due to magnet, Wb-turn
*	superscript indicating output of sampler