# Wavelet Modeling of Motor Drives Applied to the Calculation of Motor Terminal Overvoltages

Jianguo Liu, Pragasen Pillay, *Senior Member, IEEE*, and Hugh Douglas, *Student Member, IEEE*

***Abstract*—The wavelet is a powerful tool that has emerged for the calculation of power system transients. In this paper, a wavelet motor drive model is developed and applied to the overvoltage problem which occurs with high $dv/dt$ inverters. This technique is fundamentally different from EMTP and Pspice, which are time-domain methods. Wavelet models of typical passive parameters are derived and then applied to the problem of motor terminal overvoltages, which occurs with long cable lengths. It is proved that the simulation results with wavelet modeling is faster in the solution of this complex, power system simulation problem. Additional insight is possible by examining the wavelet coefficients.**

***Index Terms*—Induction motors, overvoltage, power quality, wavelet modeling.**

## I. INTRODUCTION

INDUCTION machines fed by variable-speed drives through long cables are used in many industrial applications where the motor and inverter must be at separate locations. Examples include subsea oil exploitation, water pumping in mining, and shipboard drives. Fig. 1 shows a motor–inverter system composed of an inverter, cable, motor, and load.

Most industrial motor drives use pulsewidth-modulated voltage-source inverters switched by power semiconductor switching devices such as gate-turn-off thyristors (GTOs), bipolar junction transistors (BJTs), and insulated gate bipolar transistors (IGBTs). A high switching frequency of the power electronic device is required to improve current waveform quality and reduce audible noise. In recent years, switching frequencies of 2–20 kHz are common with IGBT technology while allowing for power levels of over 200 kW.

The high switching frequency and high rate of voltage rise ($dv/dt$) of 0–600 V in less than 0.1 $\mu$s has adverse effects on the motor turn insulation and can contribute to bearing currents. AC motor transient overvoltages resulting from the motor-cable response to inverter pulse voltages have steadily increased in magnitude as semiconductor rise times and fall times have decreased from the GTO, to the BJT, and presently to the IGBT. Cable lengths had to exceed 1000–2000 ft for GTO drives and 500–1000 ft for BJT drives before producing overvoltages at the motor terminals. Presently, IGBT drives may create overvoltages that exceed the motor ratings for cable lengths as low as 50–200 ft [1].

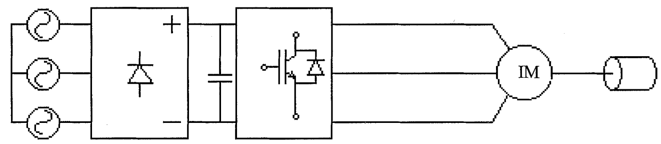



Fig. 1. Typical structure of motor–inverter system.

Long cable lengths also contribute to a damped high-frequency ringing at the motor terminals due to the distributed nature of the cable leakage inductance and coupling capacitance (*L*–*C*). This high-frequency oscillation results in overvoltages that further stress the motor insulation. The voltage reflections are a function of the inverter output pulse rise time and the length of the motor cables, which behave as a transmission line for the inverter output pulses [2]–[4]. It has been found that the pulses travel at approximately half the speed of light (150–200 m/$\mu$s or 500 ft/$\mu$s), and if the pulses take longer than half the rise time to travel from the inverter to the motor, then a full reflection will occur at the motor terminals and the pulse amplitude will approximately double [5].

A physical explanation of the long-cable overvoltage problem has been provided in [6]. The modeling and simulation of this problem and other transients has been based on EMTP, Matlab, or Pspice or other time-domain methods. The computation time using these packages can be large so an alternative method, the wavelet transform, is proposed here to examine its effectiveness in solving the motor terminal overvoltage problem.

The wavelet modeling of power systems and its application to the solution of differential equations is beginning to attract attention. It is proposed in [7] that it is possible to use wavelets to solve a class of power engineering circuit problems relating to the calculation of transients in the system. The wavelet solution of a differential equation is an example of the use of multiresolution analysis. [8] is based on the discrete-time domain approximation of linear power system components while the discrete wavelet transform is applied to achieve the wavelet-domain approximation. The wavelet-domain impedance and equivalent circuits are, thus, formed to solve the linear power system circuit.

This paper first presents the wavelet modeling method [8]. Wavelet models of basic circuit components are given. Finally, the wavelet modeling is applied to the overvoltage problem that exists with the long-cable problem in high $dv/dt$ motor drives.

Manuscript received March 29, 2001; revised November 8, 2002. Abstract published on the Internet November 26, 2003.

J. Liu is with Bechtel Corporation, Frederick, MD 21703 USA (e-mail: jliu1@bechtel.com).

P. Pillay is with the Department of the Electrical and Computer Engineering, Clarkson University, Potsdam, NY13699 USA (e-mail: pillayp@clarkson.edu).

H. Douglas is with Clarkson University, Potsdam, NY13699 USA, on leave from the Department of Electrical Engineering, University of Cape Town, Rondebosch 7701, South Africa (e-mail: hdougla@eng.uct.ac.za).

Digital Object Identifier 10.1109/TIE.2003.822085

## II. Wavelet Transform and Modeling

The wavelet transform has been used in the field of signal processing as well as for the study of power transients, because of its ability to deal with nonstationary signals.

In the solution of the long-cable problem, the model used is the $\pi$ equivalent circuit, which consists of resistors, inductors, and capacitors. For wavelets to be used in this problem, it is necessary to transform all these models to the wavelet domain. After the wavelet-domain solution is obtained, the inverse transform is used to get the time-domain solution.

The wavelet transform is being used in the solution of differential equations and opens a new field to its applications to power system transients. The basic transients in a typical power system can be formulated as ordinary differential equations (ODEs). Based on this, [8] provides a new wavelet model to solve power system transients.

The discrete wavelet transform (DWT) is the discrete form of the wavelet transform. A signal vector $X$ has its image $Y$ through a linear operator $T$

$$Y = T \cdot X. \tag{2.1}$$

Let $WX$ be the wavelet coefficients of the vector $X$, and $WY$ be the wavelet coefficients of the vector $Y$, then

$$WX = DWT \cdot X \tag{2.2}$$
$$WY = DWT \cdot Y \tag{2.3}$$
$$DWT^{-1} = DWT^{T}. \tag{2.4}$$

Then, this operator is represented in the wavelet domain as

$$WT = DWT \cdot T \cdot DWT^{T} \tag{2.5}$$

and

$$WY = WT \cdot WX. \tag{2.6}$$

Equations (2.5) and (2.6) represent the wavelet-domain representation of the discrete time domain (2.1). For fast numerical computation, matrix $WT$ is calculated from the DWT. The wavelet transform of each column of matrix $T$ is calculated and stored as the column of $WT$, and then the wavelet transform of each row of the resulting matrix is computed to obtain the $WT$ matrix.

The basic elements of a typical electrical power system can be represented as resistors, inductors, and capacitors.

### A. Resistor Transient Model

For a resistor, the $V$–$I$ characteristic is represented as

$$v(t) = R \cdot i(t). \tag{2.7}$$

Its transient model in the wavelet domain is

$$WV = R \cdot U \cdot WI \tag{2.8}$$

in which $WV$ and $WI$ are the DWT coefficients of the voltage $V$ and current $I$.

### B. Inductor Transient Model

For an inductor, the $V$–$I$ characteristic is represented as

$$v = L \cdot \frac{di}{dt}. \tag{2.9}$$

Its transient model in the wavelet domain is

$$WV = L \cdot WD_T \cdot WI - L \cdot i(0) \cdot WVL0 \tag{2.10}$$

where $WD_T$ is the transient differential operator, and $LWD_T$ is the wavelet-domain inductor impedance. $WVL0$ is the wavelet coefficients of the vector $VL0$ related to the initial current

$$WD_T = DWT \cdot D_T \cdot DWT^{T} \tag{2.11}$$
$$VL0 = \frac{1}{\Delta T} \cdot [1\ 0\ 0 \cdots 0]^{T} \tag{2.12}$$
$$D_T = \frac{1}{\Delta T}\begin{bmatrix} 1 & & & & \\ -1 & 1 & & & \\ & -1 & & 1 & \\ & & \ddots & & \ddots & \\ & & & -1 & & 1 \end{bmatrix}. \tag{2.13}$$

### C. Capacitor Transient Model

The time-domain equation of a capacitor is

$$v(t) = v(0) + \int_0^t \frac{i(t)}{C} dt. \tag{2.14}$$

Its wavelet-domain equivalent is

$$WV = v(0) \cdot WVC0 + \frac{1}{C} WIN_T \cdot WI \tag{2.15}$$

where $WIN_T$ is the transient integral operator, and $WIN_T/C$ is the wavelet capacitor impedance. $WVC0$ is the wavelet coefficients of the vector VC0 which is related to the initial voltage

$$WIN_T = DWT \cdot IN_T \cdot DWT^{T} \tag{2.16}$$
$$VC0 = [1\ \cdots\ 1\ \cdots\ 1]^{T} \tag{2.17}$$
$$IN_T = \Delta T \begin{bmatrix} 1 & & & \\ 1 & 1 & & \\ \vdots & \ddots & \ddots & \\ 1 & 1 \cdots & 1 & 1 \end{bmatrix}. \tag{2.18}$$

## III. Application to Simple $RLC$ Circuit Transients

A typical power system can be modeled as different combinations of *R*, *L*, and *C*. An *RLC* series circuit, with dc power source is given in Fig. 2. The dc source will be switched at time $t = 0$.

The wavelet model of the *RLC* series circuit with zero initial conditions is given in Fig. 3. Its $V$–$I$ equation is the following:

$$WVD = R \cdot U \cdot WI + L \cdot WD_t \cdot WI + \frac{1}{C} \cdot WIN_t \cdot WI. \tag{3.1}$$

It shows that the original time-domain differential equations are transformed into a linear equation in the wavelet domain. Conventional linear circuit theory can be applied to solve the equations easily. The problem is that in the wavelet modeling

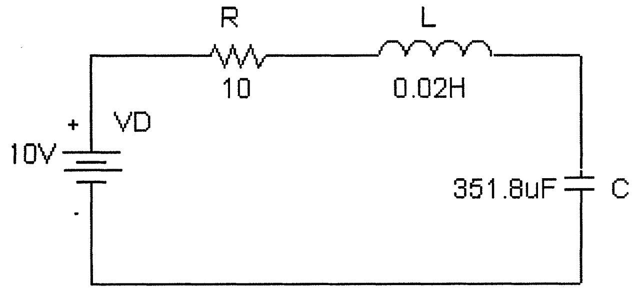



Fig. 2. *RLC* series circuit with a dc source.

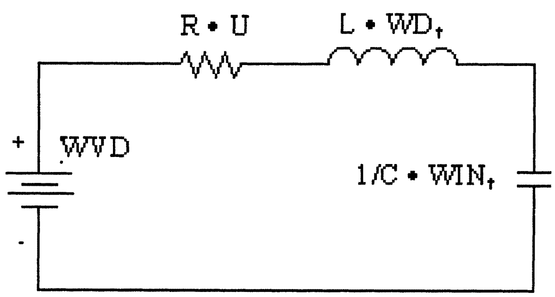



Fig. 3. Wavelet model of *RLC* series circuit with zero initial conditions.

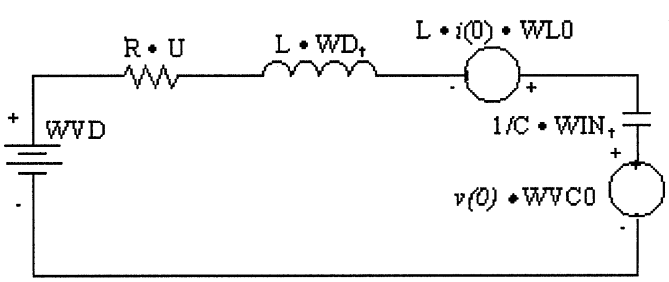



Fig. 4. Wavelet model of *RLC* series circuit with initial conditions.

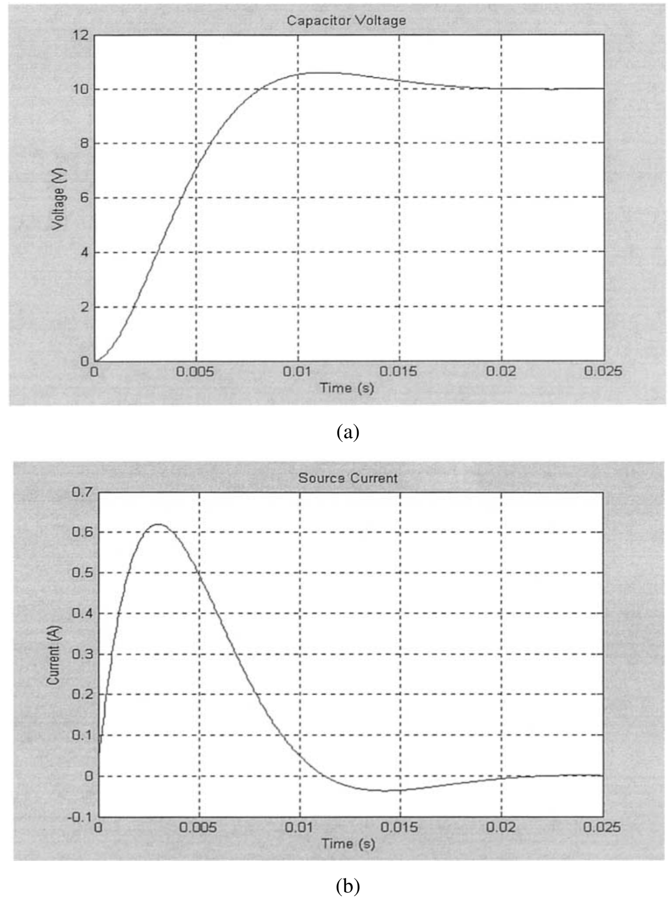



Fig. 5. (a) Inductor current and capacitor voltage using wavelet modeling. (b) Inductor current and capacitor voltage using wavelet modeling.

method there are computations of a large-dimension matrix if done simultaneously, thus resulting in the considerable increase of computation time. It is found that with a constant dc voltage value, such as $VD$ in the above circuit, the wavelet transform coefficients $WVD$ shows significant values only in a very small percentage of the total coefficients. This will greatly reduce the computation time in the matrix operation since only a small number of columns in the matrix need to be computed.

The wavelet model of the *RLC* series circuit with initial conditions is presented in Fig. 4. Its $V$–$I$ equation is the following:

$$WVD + L \cdot i(0) \cdot WVL0 - v(0) \cdot WVC0 = R \cdot U \cdot WI + L \cdot WD_t \cdot WI + \frac{1}{C} \cdot WIN_t \cdot WI. \quad (3.2)$$

In this case, on the left-hand side of the equation, there are additional terms resulting from the initial conditions of the inductor and capacitor. $WVL0$ is the wavelet coefficients of the vector $VL0$. It is clear that this case requires additional computations than the zero-initial-condition case.

The solution of the *RLC* series circuit is carried out using the methods above. The inductor current and capacitor voltage are shown in Fig. 5. Compared with the Pspice simulation results in Fig. 6, it is obvious that the accuracy is comparable.

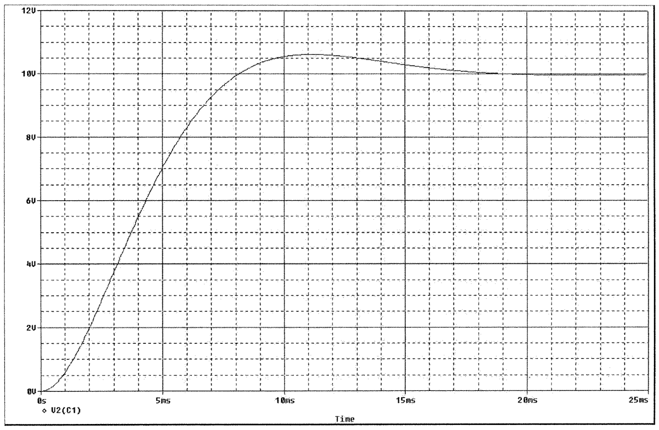
## IV. Long-Cable Overvoltage

### A. Long-Cable Model

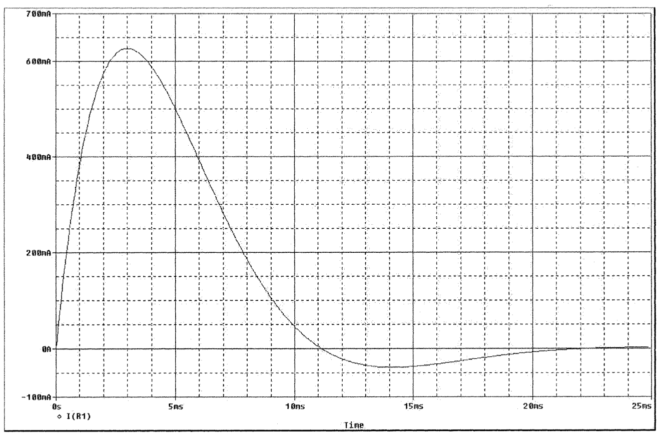
Fig. 7 shows a one-phase equivalent model of the long cable to simulate the overvoltage at the motor terminals. The cable used in this model is AWG #14, and the switching frequency of the PWM inverter is 2 kHz. The cable model used in the simulation is a three-sector lumped model. The output of the PWM inverter is shown in Fig. 8. The motor terminal voltage response to a single PWM pulse is shown in Fig. 9. Compared with the experimental results of the motor terminal voltage for a single PWM pulse in Fig. 10, the wavelet method shows its accuracy.

### B. Effect of Cable Length on the Overvoltage

Fig. 11 shows the overvoltage at the motor terminal for a single PWM pulse with the cable length changed to 250 ft, with



(a)



(b)

Fig. 6. (a) Capacitor voltage using Pspice simulation. (b) Inductor current and capacitor voltage from Pspice simulation.

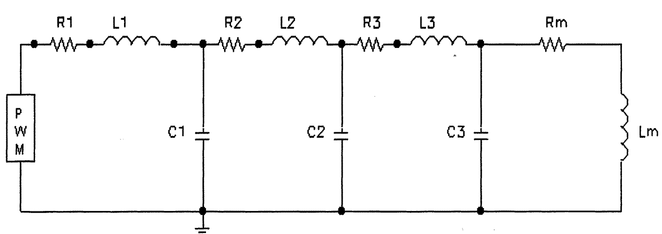



Fig. 7. Long-cable simulation model.

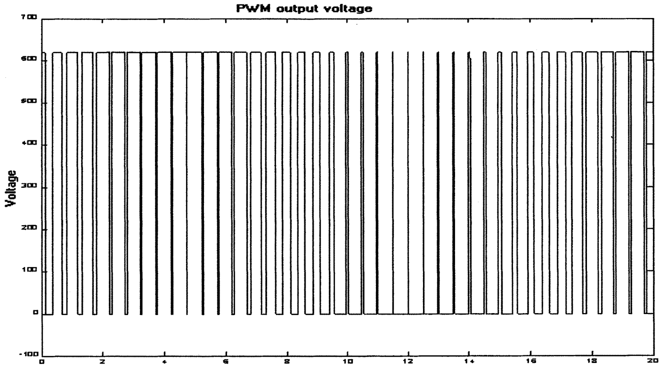



Fig. 8. Voltage from the PWM output.

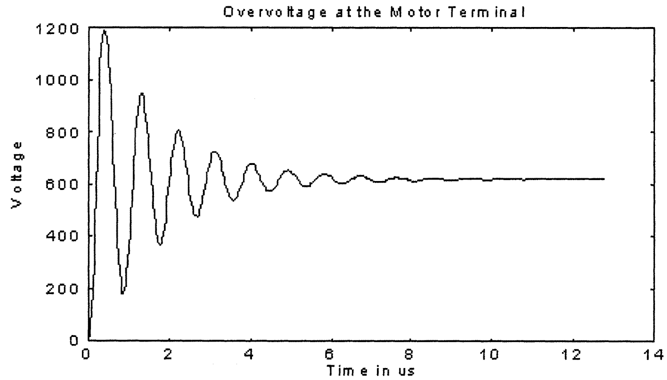



Fig. 9. Overvoltage at the motor terminal with long cable of 100 ft.

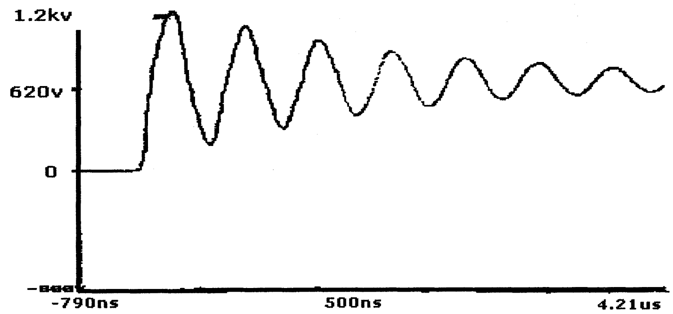



Fig. 10. Voltage at the motor terminal for 100-ft cable (measurement).

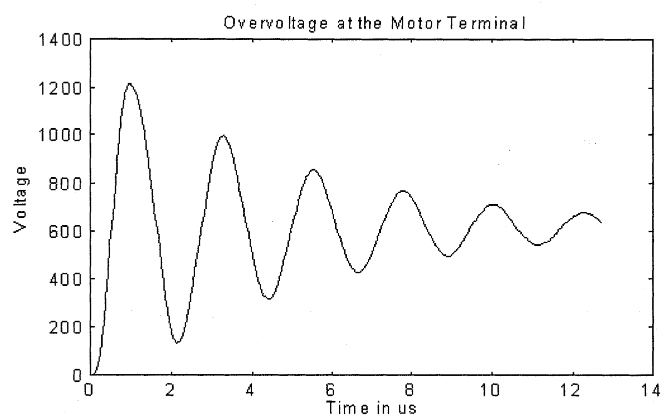



Fig. 11. Overvoltage at the motor terminal with long cable of 250 ft.

all other cable parameters kept unchanged. At first glance, there is no significant difference between this voltage waveform and the one for the 100-ft cable. The ringing frequency in Fig. 9 is more serious than that in Fig. 11. To explore the difference, the motor terminal voltage ringing caused by a single PWM pulse with 500 ft is shown in Fig. 12, where the frequency of the oscillation is lower. The oscillation frequency is inversely proportional to cable length. The cable length is a dominant factor in determining the terminal voltage damping. Generally, the longer the cable, the longer the time taken for transients to decay.

### C. *Effects of Rise Time*

In the case of Fig. 9, the power semiconductors in the PWM inverter are assumed to be ideal and the effects of the rise time

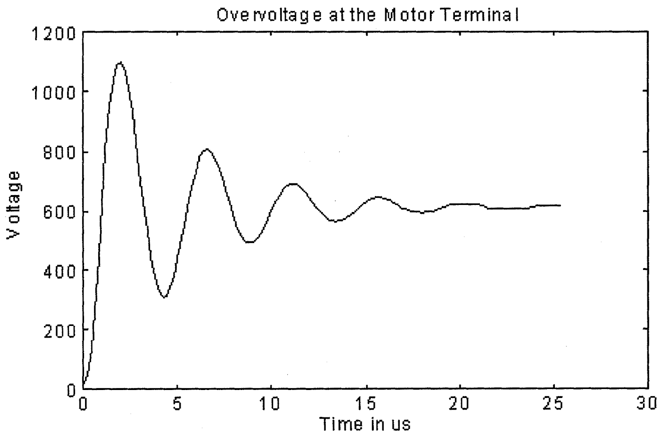



Fig. 12. Overvoltage at the motor terminal with long cable of 500 ft.

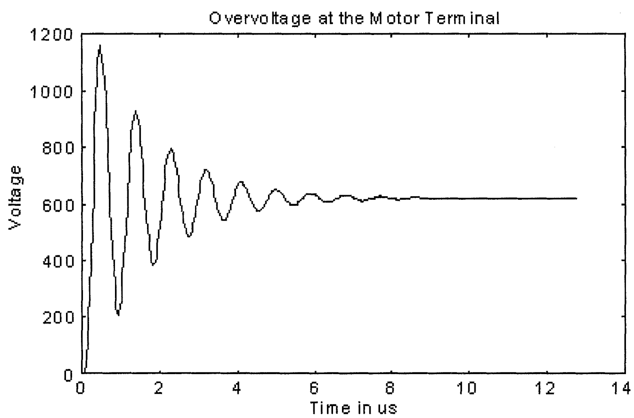



Fig. 13. Overvoltage at the motor terminal of a 100-ft cable with rise time = 0.2 $\mu$s.

of the switches are not included in the simulation. In fact, the rise time of the power electronics has a considerable effect on the overvoltage of the motor terminals.

Fig. 9 shows the motor terminal voltage for switches with a zero rise time. In Fig. 13, the motor terminal voltage is shown with a rise time of 0.2 $\mu$s. The voltage waveform with a rise time of 2 $\mu$s is shown in Fig. 14. By comparing Fig. 14 with Fig. 9, it is obvious that the peak voltage decreases with an increase in the rise time. That means that the peak voltage increases with an increase in $dv/dt$. Then, the effects of the rise time of the power electronic devices should be carefully considered when used in industrial applications.

*D. Simulation Results*

All the wavelet simulations presented in the sections above are based on the condition of a single PWM pulse as the input to the long-cable circuit. As stated in the wavelet model of the circuit elements, the selection of the step size determines the dimension of the matrices related to the model. For the 100-ft cable, the cable oscillation frequency will be 1.32 MHz. According to the Nyquist Rule, the sampling frequency must be larger than 2.64 MHz for an effective representation. The step size must be smaller than 0.37 $\mu$s. For a simulation with 100 $\mu$s, the transform matrices will be in the order of 270. The actual

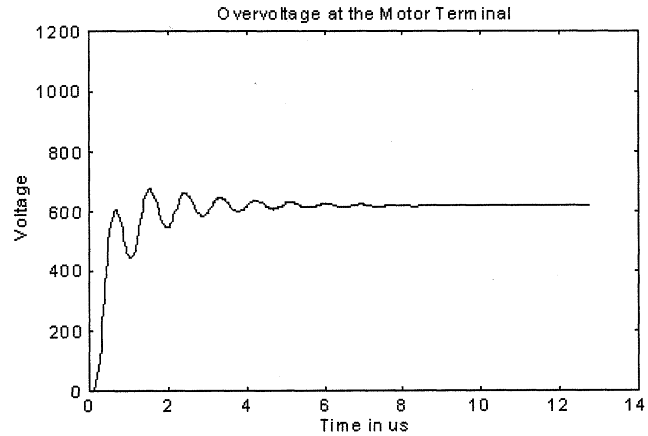



Fig. 14. Overvoltage at the motor terminal of a 100 feet cable with rise time = 2 $\mu$s.

simulation includes many operations of matrix inversion and multiplication. It takes more than 1 h in a Pentium 200-MHz digital computer to obtain the result.

In (2.10) and (2.15), the initial inductor current and the initial capacitor voltage are used in the wavelet domain characteristic equation of inductor and capacitor. The simulation is divided into short periods. This reduces the order of the matrices in the simulation, thus reducing simulation time. The final value of each inductor current and capacitor voltage for each period is used as the initial condition in the next period of simulation.

For a 270-square matrix, its inversion takes on the order of $270 \times 270 = 72\,900$ operations. If there are only eight sampling points in each period, the required time will be approximately 35 operations of the matrix inversion of order 8, which requires $35 \times 8 \times 8 = 560$ operations. The 35 operations is derived by 270/8 plus a few operations after the matrix inversion. The first method requires at least 130 times more operations than that required of the second method.

To illustrate this principal the circuit in Fig. 2 was simulated using Matlab 4.2 and Orcad version 9.2 on a Pentium 466 MHz. The simulation time was 25 ms and 512 points were calculated. When using Matlab and the large matrix method proposed in [8], a total simulation time of 70.5 s was observed. Using the smaller proposed $8 \times 8$ matrix method resulted in a total computation time of 0.05 s. The Orcad simulation time for the same number of data points was 0.32 s, i.e., 6.4 times the execution time using wavelets. The resulting waveforms for the $8 \times 8$ matrix method as well as the Orcad simulation can be seen in Figs. 5(a) and (b) and 6(a) and (b).

Fig. 15 shows the motor terminal voltage for a 100-ft-long cable during a 20-ms period. The step size $\Delta$T is 0.05 $\mu$s and there is a total of 40 000 data points. The direct use of the wavelet model in (2.8), (2.10), and (2.15) will make the implementation prohibitive. With the selection of 8 points each period, the dimension of the matrices is reduced to 8. After including the method from Fig. 7, the total computation time is reduced.

It is proposed in [8] that wavelet modeling will need extensive computation and lose its effectiveness when used to solve systems with high dimension or high resolution. However, it has

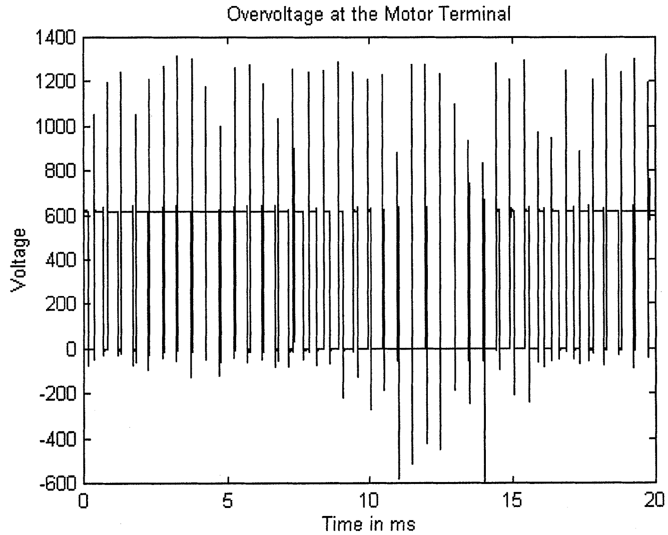



Fig. 15. Overvoltage at the motor terminal in 20 ms.

been proved in this paper that with the selection of a short basic computation period, the dimension of the matrices in the simulation can be greatly reduced and hence the computation time.

## V. Conclusion

Wavelet modeling has been used to solve the long-cable overvoltage problem. It is proved that the wavelet modeling is effective in the solution of the power system problem with the method proposed.

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**Jianguo Liu** received the Bachelor's degree from Shanghai Jiaotong University, Shanghai, China, in 1986, the Master's degree from Shanghai University, Shanghai, China, in 1989, and the Ph.D. degree in electrical engineering from Clarkson University, Potsdam, NY, in 2000.

He was a Power Engineer with the Chengdu Heat and Power Plant until 1996. Since 2000, he has been a Senior Electrical Engineer with Bechtel Corporation, Frederick, MD. His research interests are power system modeling and power transient analysis and power system generation control and protection.

**Pragasen Pillay** (S'84–M'87–SM'92) received the Bachelor's degree from the University of Durban-Westville, Durban, South Africa, in 1981, the Master's degree from the University of Natal, Durban, South Africa, in 1983, and the Ph.D. degree from Virginia Polytechnic Institute and State University, Blacksburg, in 1987.

From January 1988 to August 1990, he was with the University of Newcastle-upon-Tyne, U.K. From August 1990 to August 1995, he was with the University of New Orleans. Currently, he is with Clarkson University, Potsdam, NY, where he is a Professor in the Department of Electrical and Computer Engineering and holds the Jean Newell Distinguished Professorship in Engineering. His research and teaching interests are in modeling, design, and control of electric motors and drives for industrial and alternate energy applications.

Dr. Pillay is a member of the IEEE Power Engineering, IEEE Industry Applications (IAS), IEEE Industrial Electronics, and IEEE Power Electronics Societies. He is a member of the Electric Machines Committee and Past Chairman of the Industrial Drives Committee of the IAS and Past Chairman of the Induction Machinery Subcommittee of the IEEE Power Engineering Society. He has organized and taught short courses in electric drives at IAS Annual Meetings. He is a Fellow of the Institution of Electrical Engineers, U.K., and a Chartered Electrical Engineer in the U.K. He is also a member of the Academy of Science of South Africa. He was also a recipient of a Fulbright Scholarship during his Ph.D. studies.

**Hugh Douglas** (S'03) received the Bachelor's and Master's degrees from the University of Cape Town, Cape Town, South Africa, in 1997 and 2000, respectively. He is currently working toward the Ph.D. degree in electrical engineering from Clarkson University, Potsdam, NY.

He is a faculty member at the University of Cape Town, where he specializes in the fields of power electronics, machines, and drives. His research interests include signal processing applied to power applications.