Resonant Frequencies and Mode Shapes of Switched Reluctance Motors

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***Abstract*—The wide application of switched reluctance motors (SRM) is limited by their higher vibration and acoustic noise. An accurate determination of resonant frequencies and mode shapes of the SRM stator is therefore essential for low acoustic noise design. Based on the structural finite element method and elasticity theory, 3D vibration modes and resonant frequencies of a 5 hp SRM with 8/6 poles are demonstrated and calculated in this paper. The stator stack with both a smooth frame and a ribbed frame are examined. The effects of an encased frame and a keyed frame on the free vibration modes and resonant frequencies of SRM stator are compared. Accelerometer test results on a SRM verify some of the numerical computations.**

***Index Terms*—Acoustic noise, mode shapes, resonant frequency, switched reluctance motors, vibrations.**

I. Introduction

SWITCHED reluctance motor (SRM) drives are being used in domestic and industrial applications from power steering to washing machines to traction. The increased application is due to advantages over competitive motors in manufacturing, reliability and robustness as well as lifetime. Their wide application is mainly obstructed by two disadvantages: acoustic noise and torque ripple.

The existing research on vibration and acoustic noise of the SRM can be categorized into time domain methods [3], [5] and frequency domain methods [1], [2], [4]. The time domain method can reveal the links between acoustic noise, stator vibration, shape and timing of the applied voltage and the current in the phase winding of the motor. To understand the effects of the power electronic converter on the vibration and acoustic noise of the SRM, the time domain is particularly useful. The frequency domain method is useful to show the spectrum and dominant components of the noise and vibrations, and it can be used to predict the resonant frequencies in terms of the geometry of the motor and the material properties.

Analytical and experimental results have widely confirmed a magnetic origin as the dominant noise source of the SRM [1]–[4]. When the phase windings of the SRM conducts current, a magnetic attraction is produced between the stator pole and the rotor pole. While the tangential component of the forces on the pole surfaces contributes to torque production, the radial magnetic attraction causes radial deformation of the SRM stator. The latter leads to stator vibration and acoustic noise. The phenomenon becomes particularly severe when the frequency of vibration approaches or coincides with that of the mechanical resonance of the stator [2]. The resonant frequencies of the low order mode shapes, especially the 0th, 2nd and 4th order, play a significant role in the vibration of the stator in a 4 phase SRM with 8/6 poles. To suppress the acoustic noise in the SRM, smoothing or current shaping and switch angle control as well as active control strategies have been presented [2], [3], [5]. From the viewpoint of both design and control, it is essential to be able to predict the natural resonant frequencies of the stator accurately, to understand the effects of different design parameters on these frequencies.

All of the mechanical components contribute to acoustic noise and vibration. The natural frequency and mode shapes of some components, such as the stator can be extracted either from a closed-form solution of analytical equations [7], [8] or from finite element analysis [1], [4], [6]. Because of the complexity of some of the components and nonlinear coupling between structural components, the calculated model needs to be verified empirically [1], [9], [11]. Although some previous work has been done in this area, many practical problems are still left unsolved or invite a deeper investigation. The ANSYS or COSMOS/M programs, are capable of predicting the mode shapes and the corresponding natural frequencies of the 3D physical model of the components in the SRM, and is used in this work.

With the aid of the structural finite element method and elasticity theory, 3D vibration modes and their corresponding resonant frequencies are calculated for a 5 hp 8/6 SRM with 4 phases. A modal analysis of the stator lamination stacks with smooth frame and with ribbed frame is performed. The effects of an encased and a keyed structure frame on the vibration modes and the resonant frequencies of the SRM stator are compared. Accelerometer tests on a 5hp SRM verify some of the numerical computations.

II. Numerical Models of Stator Vibration

A. Analysis Basis

The analysis is based on 3D elasticity theory. Hamilton's principle is introduced to determine the motion equation of the vibration system [13]:

$$\int_{t_2}^{t_2} [\delta(T-U)+\delta W_{nc}]\, dt = 0 \qquad (1)$$

where W_{nc} is the virtual work; The potential energy U and the kinetic energy T in the 3D solid of volume V enclosed by

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surface S at time t can be expressed as a function of the independent displacements and velocity respectively

$$U = \tfrac{1}{2} \int_V \{\varepsilon\}^T [\mathbf{D}] \{\varepsilon\}\, dV$$
$$T = \tfrac{1}{2} \int_V \rho(u + v + w)\, dV. \tag{2}$$

The potential energy refers to the elastic potential energy or strain energy. In the above equations, the displacement components of an element in the direction of the r, θ and z axes, can be expressed in cylindrical polar coordinates:

$$u = u(r, \theta, z, t), \quad v = v(r, \theta, z, t), \quad w = w(r, \theta, z, t) \tag{3}$$

whose derivatives with respect to time t are the velocity of the element; ρ is the specific mass density of the structure. In this case the strain components are given as follows [12]:

$$\begin{aligned}\{\varepsilon\}^T &= [\varepsilon_r \quad \varepsilon_\theta \quad \varepsilon_z \quad \gamma_{zr} \quad \gamma_{r\theta} \quad \gamma_{\theta z}] \\ &= \Bigg[\frac{\partial u}{\partial r} \quad \left(\frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta}\right) \quad \frac{\partial w}{\partial z} \\ &\qquad \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \quad \left(\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r}\right) \quad \left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}\right) \Bigg]\end{aligned} \tag{4}$$

and the stress-strain relationships are given by the elasticity matrix $\mathbf{D}$:

$$[\mathbf{D}] = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix}$$
$$[\mathbf{A}] = \begin{bmatrix} (1-v) & v & v \\ v & (1-v) & v \\ v & v & (1-v) \end{bmatrix},$$
$$[\mathbf{B}] = \tfrac{1}{2}(1-2v)[\mathbf{1}] \tag{5}$$

where

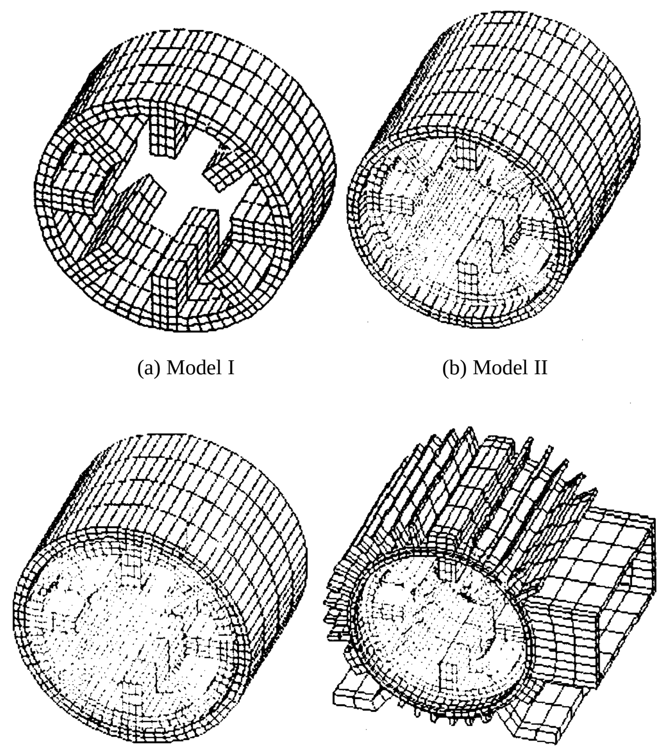
E	is Young's modulus;
v	is Poisson's ratio;
$[\mathbf{0}]$ and $[\mathbf{1}]$	are 3×3 null and unit matrices, respectively.

The virtual work δW_{nc} in (1), done by the nonconservative forces on the surface S, vanishes since the vibration analysis in this paper concentrates on determining the natural frequencies and mode shapes of free vibration of the SRM stator. The motion equation for an undamped system can be expressed in matrix notation as follows when the free vibration is periodic:

$$([\mathbf{K}] - \omega_i^2[\mathbf{M}])\{\phi\}_i = \{\mathbf{0}\} \tag{6}$$

where the structure stiffness matrix $[\mathbf{K}]$ including prestress effects, is positive definite or positive semi-definite, and the inertia matrix $[\mathbf{M}]$ is positive definite. They are formed from the elementary stiffness matrix and the elementary inertia matrix, respectively [6]. Eigenvector $\{\phi\}_i$ represents the mode shape at the ith natural angular frequency. Equation (6) has a nontrival solution when the determinant of the coefficients vanish, i.e.

$$\det([\mathbf{K}] - \omega_i^2[\mathbf{M}]) = 0. \tag{7}$$



(a) Model I (b) Model II

(c) Model III (d) Model IV

Fig. 1. Different three dimensional models.

The polynomial equation has n eigenvalues, i.e., roots ω_i^2 ($i = 1, 2, \ldots, n$) which correspond to n natural frequencies:

$$f_i = \frac{\omega_i}{2\pi}, \qquad i = 1, 2, \ldots, n. \tag{8}$$

Furthermore, the eigenvectors $\{\phi\}_i$ in (6) can be solved for up to n vectors where n is the degree of freedom of the system.

B. Computation Models

Four basic models were built to study the resonant frequencies and mode shapes of the SRM stator. These models are shown in Fig. 1 and represent the structure of the SRM stator for different industrial applications. The computational results are compared and analyzed to find the effects of various mechanical constructions on the stator vibration, whose dimensions are given in Appendix.

All the models in the paper are based on 3D finite elements, and hexahedral elements are chosen to guarantee free deformation in all degrees of freedom. Model I in Fig. 1(a) is the stator lamination stack. The lamination stack with a perfect frictional contact to the smooth frame is in model II. Model III has longitudinal keys between the lamination stack and the smooth frame. Finally, model IV includes the details of a ribbed frame structure with a terminal box. The mechanical properties of this stator stack material are: Young's modulus $E = 2.07 \times 10^{11}$ N/m^2, Poisson's ratio $v = 0.3$ and mass density $\rho = 7800$ kg/m^3; The iron-casting frame has $E = 1.6 \times 10^{11}$ N/m^2, $v = 0.275$ and $\rho = 7800$ kg/m^3.

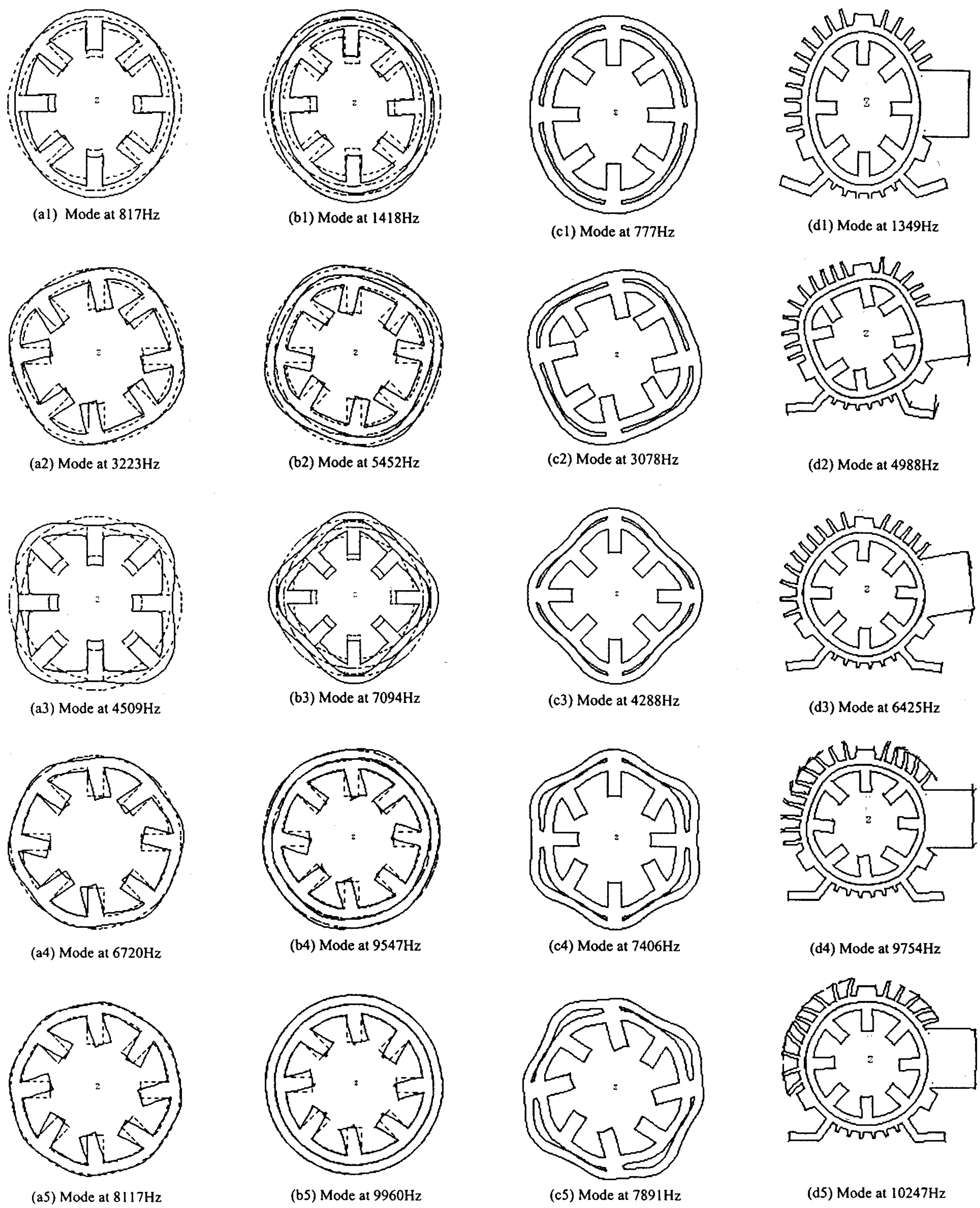


Fig. 2. The selected vibration mode shapes of 4 basic computation models in SRM.

III. Computation and Measurement Results

A. Numerical Results of Model I

In Fig. 1(a), the stator stack without frame is modeled with dimensions shown in Fig. 7 of the Appendix. The motor structure with a suspended stator lamination stack can also be built into this model. The computational results of the model are similar to previous papers [1], [5]. Furthermore, the effects of both length and radius of the stator yoke on vibration modes and frequencies are developed in this paper. The results show only a slight influence of the length of stack on the natural frequencies of the first several in-plane mode shapes, but there are larger effects on the modes and the stack frequencies in the 3rd dimension, such as bending and torsional deformation. Fortunately the latter cannot be excited in normal SRM operation since there is no corresponding force wave in this plane. The resonant frequency change of the in-plane mode shapes due to the change of stator length can be omitted if the ratio of the length of stator to average yoke radius is between 1 and 2. The resonant frequency of the stator stack will decrease roughly linearly with an increase of the yoke radius when the yoke thickness is kept unchanged. Furthermore, the existence of the stator poles will reduce the frequencies of the first few order modes, and increase the higher order natural frequencies so that the geometry of the poles has some effect on the resonant frequencies of the SRM.

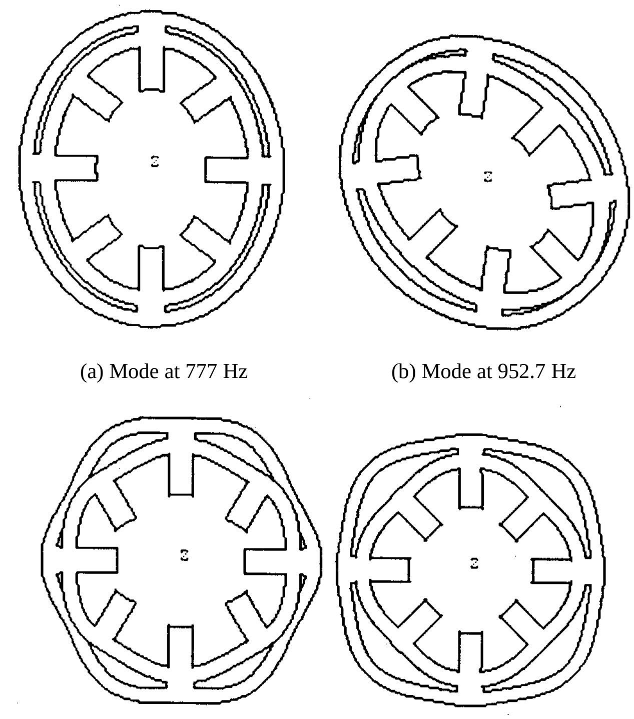
The dominant mode shapes and resonance frequencies are given in column (a) of Fig. 2.

B. The Effects of the Frame in Model II

When the lamination stack is installed into a cylindrical smooth frame, the finite element model can be built as shown in Fig. 1(b). The mode shapes in this model corresponding to model I can be seen in column (b) of Fig. 2. As long as the stiffness of the frame is sufficient, the resonant frequencies of the in-plane vibration modes will rise monotonically with an increase in the frame thickness. The calculations also show a slight decrease of the resonant frequency of the in-plane modes with an increase in the length difference between the frame and lamination stack. The decrease in the frequency is less than 2% for a conventional range of frame lengths. It has been found that the thicker the frame, the higher the resonant frequencies. Thus changing the frame thickness is an effective way of adjusting the frequencies of the mode shapes in motor design, but with an effect on cost. The 0th order mode shape, i.e., contraction and expansion deformation, may appear in a different position in the mode order sequence [1].

C. The Effects of the Frame in Model III

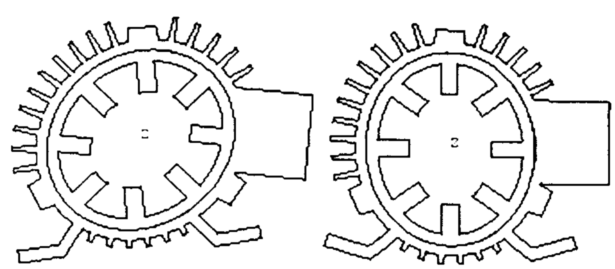
This stator model is composed of the lamination stack with a frame. The stack is the same as in model I, but it is installed on four longitudinal keys on the inner surface of a smooth frame, as shown in Fig. 1(c). The stack and the frame are made of materials with different mechanical characteristics, which are given in Section II. A few mode shapes corresponding to model I are shown in column (b) of Fig. 2. The calculation results demonstrate: 1) The frequencies of the in-plane mode shapes may decrease or increase with the existence of this frame, compared to Model I (lamination stack without frame). 2) The same resonant frequencies of some mode shapes in model I (say 823 Hz of the 2nd order mode) become separated from each other in model III, shown in Fig. 3(a) and (b). One is lower than 823 Hz of model I, while the other is higher than 823 Hz. These can cause some difficulties in canceling the acoustic noise since the resonant frequency of the second order mode shape is essential in the application of active noise control techniques [3]. 3) Additional vibration modes appear with this design, compared to model I. The mode shapes also become more complex. Besides column (b) in Fig. 2, several mode shapes are shown in Fig. 3(c) and (d). The frequency of the individual mode shape of a quasi-4th order in Fig. 3(d) may



(a) Mode at 777 Hz (b) Mode at 952.7 Hz

(c) Mode at 5551 Hz (d) Mode at 6132 Hz

Fig. 3. The mode shapes of model III (Frame thickness = 10.2 mm, key thickness = 4.2 mm, free vibration).



(a) Mode at 1262 Hz (b) Mode at 1349 Hz

Fig. 4. Mode shapes with 2nd order deformation.

be higher than the frequency of a quasi-6th order in Fig. 3(c). Therefore, model III causes a change in the sequence of the mode shapes.

D. The Effects of the Ribbed Frame in Model IV

This model is based on a real 5 hp SRM shown in Fig. 1(d). The ribs, keys and terminal box are all included in the model. Selected results are given in column (d) of Fig. 2. The resonant frequencies of the lower order modes are reduced with the addition of ribs to the smooth frame surface, shown in Fig. 1(d), but the modal frequencies of the higher order mode shapes are raised by the addition of ribs, terminal box, etc. This demonstrates that the effect of the ribs is like an extra mass to the lower order vibrations but an increase in stiffness to the higher order vibrations. This conclusion contradicts some analytical formulas [9], [11]. The mode shapes of the 2nd order vibration are shown in Fig. 3 for the keyed frame and in Fig. 4 for the ribbed frame. The difference with the 2nd order mode shapes is clearly evident. In addition, the existence of the ribs introduces many complex vibration mode shapes, but many of them come from the ribs and feet vibration modes. While attention has been focused on modal frequencies in the 20 k Hz range, many of these mode shapes cannot be excited under normal SRM operation.

The above computational results are based on free vibrations, i.e., no additional constraint on the model. If the motor is installed on an infinite base, its feet can be considered to being fixed (without deformation). These node constraints are exerted on the numerical model, then the mode shapes and the frequencies change. For example, the frequency corresponding to Fig. 4(b), but with feet fixed, becomes 1143 Hz, and the frequency corresponding to Fig. 4(a) with feet fixed increase to 1988 Hz, and so on. The mounting style of the motor has a considerable effect on the resonant frequencies, but is left for a future paper because of page limitations.

Compared to the model without considering the terminal box of the SRM stator, the results in model IV show that the mode shapes have additional changes, i.e., every mode shape of integer order is accompanied by some rib deformation. The frequencies of some in-plane modes of lower orders decrease slightly. Over 200 mode shapes were studied and compared with the results from the model without terminal box.

E. Measurement Results

To validate the computed results of the above numerical models, the vibration of the 2nd order mode is measured with accelerometers.

Since the windings are included in the measured motor stator, the effects of the windings have to be added to the finite element model. The 2nd mode frequency extracted by the FE computation is 1093 Hz.

The whole measurement system can be divided into two subsystems: excitation and acquisition, as shown in Fig. 5. The former is composed of a signal generator, power amplifier, permanent magnetic exciter and a push-rod, which produces an exciting force over a range of frequencies. The data acquisition consists of accelerometers and a multi-channel amplifying coupler as well as a digital oscilloscope. The accelerometers are screwed on the frame behind the poles of the SRM stator. For reliable results, 4 accelerometers are placed every 90 degrees.

The accelerometer outputs are acquired and the power spectrum density vs. frequency is obtained by FFT, as shown in Fig. 6. The measured resonant frequency of 1060 Hz verifies the result computed by the finite element method to within 3%.

IV. Conclusion

Four basic stator structures of the SRM are modeled to study the effects of different frame structures on mode shapes and resonant frequencies under free vibration. The influence of mounting constraints on the SRM vibration behavior is analyzed. The results from the finite element calculation are compared to measured values of a real SRM to validate the accuracy of the numerical calculations. Several conclusions are drawn on the effects of stator structures on the vibration behavior.

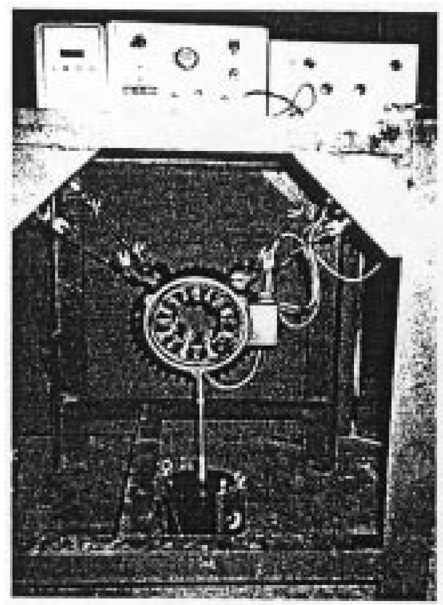


Fig. 5. Vibration test table (signal generator, amplifier, exciter, accelerometers and coupler as well as oscilloscope, etc.).

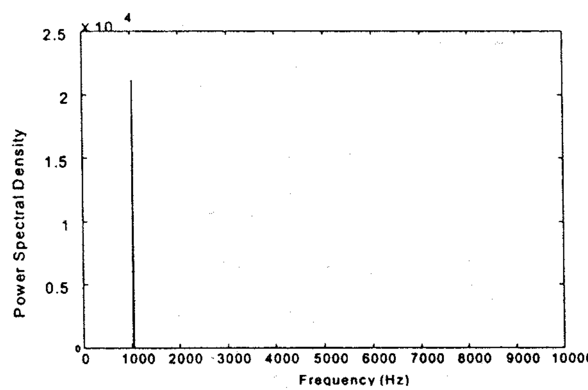


Fig. 6. The 2nd order measurement results ($f = 1060$ Hz).

1) The length of the stator stack and frame has a negligible effect on the frequencies of in-plane mode shapes;
2) A keyed contact between the stack and frame may split the resonant frequencies which are coincident in the lamination stack or the lamination stack with smooth frame. It would be difficult to reduce the vibration using active vibration control with this mechanical design.
3) The thickness of the yoke and frame of the stator has a significant effect on the resonant frequencies. The resonant frequency of the 2nd order mode shape can be changed by adjusting the frame thickness in models III and IV.
4) The ribs and terminal box reduce the frequencies of the first few mode shapes, but increases the frequencies of the higher order mode shapes. Their effects cannot be simply considered as an extra mass.
5) The mounting style of the motors has a significant effect on the resonant frequency. The influence should be taken

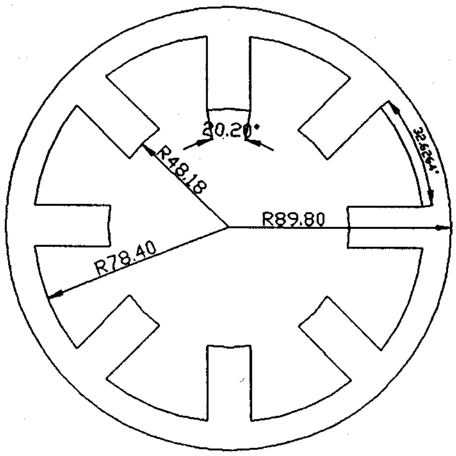


Fig. 7. Stator stack geometrical dimensions.

into. account while performing acoustic noise and vibration cancellation.

APPENDIX

The geometrical dimensions (mm) of the SRM stator stack are given in Fig. 7.

REFERENCES

[1] P. Pillay and W. Cai, "An investigation into vibration in switched reluctance motor," *IEEE Trans. on Industry Applications*, vol. 35, no. 3, pp. 589–596, May/June 1999.

[2] D. E. Cameron, J. H. Lang, and S. D. Umans, "The origin and reduction of acoustic noise in doubly salient variable-reluctance motors," *IEEE Trans. on Industry Applications*, vol. 28, no. 6, pp. 1250–1255, Nov./Dec. 1992.

[3] C. Y. Wu and C. Pollock, "Analysis and reduction of acoustic noise and vibration in the switched reluctance drive," *IEEE Trans. on Industry Applications*, vol. 31, no. 1, pp. 91–98, Jan./Feb. 1995.

[4] R. S. Colby, F. Mottier, and T. J. E. Miller, "Vibration modes and acoustic noise in a 4-phase switched reluctance motor," in *Conference Record of the 1995 IEEE Industrial Application Society, 30th IAS Annual Meeting*, vol. l, Orlando, USA, Oct. 8–12, 1995, pp. 441–447.

[5] P. Pillay, R. M. Samudio, M. Ahmed, and P. T. Patel, "A chopper-controlled SRM drive for reduced acoustic noise and improved ride-through capability using super-capacitors," *IEEE Trans. on Industry Applications*, vol. 31, no. 5, pp. 1029–1038, Sept./Oct. 1995.

[6] M. E. H. Benbouzid, G. Reyne, S. Dérou, and Foggia, "Finite element modeling of a synchronous machine: Electromagnetic force and mode shapes," *IEEE Trans. on Magnetics*, vol. 29, no. 2, pp. 2014–2018, Mar. 1993.

[7] S. P. Verma and R. S. Girgis, "Resonance frequencies of electrical machines stator having encased construction—Part II: Numerical results and experimental verification," *IEEE Trans. on Power Apparatus and Systems*, vol. PAS-92, no. 5, pp. 1586–1593, Sept./Oct. 1973.

[8] S. P. Verma and R. S. Girgis, "Method for accurate determination resonant frequencies and vibration behavior of stators of electrical machines," *Proceedings of IEE*, pt. B, vol. 128, no. 1, pp. 1–11, Jan. 1981.

[9] S. P. Verma and R. S. Girgis, "Experimental verification of resonant frequencies and vibration behavior of stators of electrical machines—Part II: Experimental investigations and results," *Proceedings of IEE*, pt. B, vol. 128, no. 1, pp. 22–32, Jan. 1981.

[10] R. K. Singal, K. Williams, and S. P. Verma, "The effect of windings, frame and impregnation upon the resonant frequencies and vibrational behavior of an electrical machine stator," *Experimental Mechanics*, vol. 30, no. 2, pp. 270–280, Sept. 1990.

[11] S. Watanabe, S. Kenjo, K. Ide, F. Sato, and M. Yamamoto, "Natural frequencies and vibration behavior of motor stator," *IEEE Trans. on Power Apparatus and Systems*, vol. PAS-102, no. 4, pp. 949–956, Apr. 1983.

[12] M. Petyt, *Introduction to Finite Element Vibration Analysis*: Cambridge University Press, 1990.

[13] S. S. Rao, *Mechanical Vibrations (3rd edition)*: Addison Wesley, 1995.

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