Enhanced multi-dimensional power network planning based on ant colony optimization

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SUMMARY

One ultimate goal of power network planning is to construct power systems that are environmentally friendly. However, it is still a challenging issue to efficiently consider environmental factors in power network planning problems due to the complexity and the diversity of environment. This paper presents a nonlinear approach to formulate the multi-dimensional spatial power network planning problem based on raster map in geographic information systems. With the objective of minimizing investment and operation costs, the proposed model integrates five dimensions: the substation location, the substation sizing, the line type selection, the optimal electric line routing, and the power network evaluation in terms of the ac power flow calculation. Furthermore, the branch capacity constraints and the node voltage limitations at peak load hours are examined to ensure the operational security of power systems. To overcome difficulties in solving the mixed-integer nonlinear optimization problem, an enhanced ant colony optimization with the four-search-space structure and the improved state transition rule is introduced. The case results validate the proposed model and the algorithms. Copyright © 2014 John Wiley & Sons, Ltd.

KEY WORDS: geographic information systems; ant colony optimization; power transmission planning; the substation location and sizing; line routing

1. INTRODUCTION

Power network planning is a classical optimization problem in power systems, and many studies on this topic have been reported in literatures. In [1], a bi-level programming model for market-based transmission network expansion planning was presented with the consideration of transmission profit, operation security and reliability, as well as social welfare. A hybrid algorithm integrated with niche genetic algorithm and primal-dual interior method was used to obtain the final optimal solution. [2] presented a mixed-integer linear programming (MILP) formulation for the long-term transmission expansion planning problem in a competitive pool-based electricity market, which considered investment and operating costs, transmission losses, generator offers, and demand bids. A new technique known as harmony search was proposed for the transmission network expansion planning [3]. A risk-based approach for the transmission network expansion problem under nonrandom uncertain deliberate outages was presented in [4], while the risk was implemented through the minimax weighted regret paradigm. A coordination of transmission and generation capacity planning was proposed in [5–7]. Based on a linear disjunctive model, power transmission network design problems were solved via a Benders decomposition approach [8,9].

With the ever-increasing importance of environmental protection, investment costs of power network planning projects are rising rapidly. To efficiently take complicated environmental elements into consideration, the electric power line routing problem was studied based on the vector map and the raster map of

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geographic information systems in many literatures. It is difficult to develop a standard line routing approach on vector map because irregular vector graphics are used to represent different regions and infrastructures [10–16]. Thus, the raster map is an efficient way for formulating the completed geographical environments, in which the region covered by each cell is assumed to have similar environmental and altitude characteristics if cells on the raster map are small enough [17–19].

Most of the previous studies dealt with the electric power line routing and the power network planning in a separate manner, which may result in unnecessary losses. [20] presented an innovative formulation and an effective methodology for the spatial power network planning in complicated environments, which considered variant environmental factors in the electric line routing formulation and integrated it with the traditional power network planning formulation. However, the following issues are not covered in [20]: (i) the substation location and sizing, (ii) the line type selection, (iii) the cost of network losses, and (iv) the difficulty for solving large-scale problems due to the curse of dimensionality.

The substation location and sizing problem cannot be ignored in the power network construction. [21] presented a genetic algorithm for the optimal location and sizing of HV/MV substations with the consideration of electricity load uncertainty. [22] presented an evolution strategy-based method for designing large rural low-voltage distribution networks. In [23], a constructive heuristic algorithm was used to solve the distribution system planning problem while considering the location and the size of substations and circuits. [24] proposed an improved genetic algorithm for the optimal design of large-scale distribution systems in order to provide optimal sizing and location of high and medium voltage substations, as well as medium voltage feeder routing. The optimal distribution substation expansion planning was solved via a hybrid heuristic and learning automata-based algorithm [25]. [26] presented a hybrid algorithm for the distribution system planning, in which the quasi-Newton procedure actuated in the search for optimal substation location co-ordinates and the genetic algorithm procedure actuated in the design of optimal network topologies. A genetic algorithm-based approach was proposed in [27] for the long-term transmission substation expansion planning.

Obviously, the substation location and sizing are closely associated with the line type selection, the optimal electric line routing, and the power network evaluation in terms of the ac power flow calculation. Each of them can be regarded as one dimension of the power network planning problem. The authors’ previous work [20] also addresses the similar topic on the spatial power network planning problem while considering complicated environments via the identical map rasterization technology. However, fundamental differences between the two papers are twofold. First, [20] considered optimal electric line routing and the power network evaluation, which overlooked other complicated factors and derived an MILP problem. In this paper, the proposed multi-dimensional power network planning is to find optimal size and location of substations as well as type and route of lines on a raster map, while considering electrical and geographical constraints, which derives a mixed-integer nonlinear programming (MINLP) problem. Second, [20] used state-of-the-art commercial MILP solvers, such as CPLEX, to solve the MILP spatial power network planning problem. In this paper, the ant colony optimization (ACO) with the four-search-space structure and the improved state transition rule is adopted to effectively solve the complicated MINLP problem.

The rest of the paper is as follows. The proposed multi-dimensional spatial power network planning model is described in section 2. The ACO with a four-search-space structure and the improved state transition rule is discussed in section 3. Comprehensive case studies are presented in sections 4, and section 5 contains our conclusions.

2. MULTI-DIMENSIONAL POWER NETWORK PLANNING PROBLEM FORMULATION

2.1. The accumulated cost matrix and the altitude matrix

As mentioned in [19,20], an image map can be rasterized into $N_R \times N_C$ square cells. The accumulated cost matrix proposed in [20] considers variant environment for each type of line. In this paper, the similar method is adopted to construct accumulated cost matrices for different types of lines and substations.
where \( N_V \) is the total number of candidate line types. \( v \) is the line type index, \( v = [0, 1, \ldots, N_V] \). Here, \( v = 0 \) means this line is not built. \( E_v \) is the accumulated cost matrix for type \( v \) line. If \( v = 0 \), the corresponding \( E_v = 0 \). \( e_{v,i,j} \) is per unit value for the accumulated cost of type \( v \) line in cell \( C_{i,j} \). \( N_W \) is the total number of candidate substation types. \( w \) is the substation type index, \( w = [0, 1, \ldots, N_W] \). Here, \( w = 0 \) means this substation is not built. \( E_w \) is the accumulated cost matrix of type \( w \) substation. If \( w = 0 \), the corresponding \( E_w = 0 \). \( e_{w,i,j} \) is the accumulated cost of type \( w \) substation in cell \( C_{i,j} \).

The altitude matrix of cells proposed in [20] is given as (3).

\[
H = \begin{bmatrix}
h_{1,1} & h_{1,2} & \cdots & h_{1,N_C} \\
h_{2,1} & h_{2,2} & \cdots & h_{2,N_C} \\
\vdots & \vdots & \ddots & \vdots \\
h_{N_C,1} & h_{N_C,2} & \cdots & h_{N_C,N_C}
\end{bmatrix}
\]  

(3)

where \( h_{i,j} \) is per unit value of the altitude of \( C_{i,j} \).

In [20], the index \( d \) was used to denote eight directions, by which an electric line moves from a cell to eight neighboring cells, where \( d = 1, 2, \ldots, 8 \). The line length between centers of two cells is calculated by horizontal and vertical differences between the centers of the two cells [20].

2.2. Objective function

The objective function of the proposed model considers fixed and variable costs of the power network planning. Fixed costs represent the investments for the installation of new lines and substations. Variable costs are the costs of technical energy losses. Thus, the objective function is modeled as (4)–(9).

\[
\text{Min}F = \min \left\{ \sum_{l=1}^{N_L} \sum_{(i,j) \in \Omega_l} 0.5e_{v,i,j}D_{i,j,l}x_{l,v} + \sum_{l=N_L+1}^{N_L+N_S} \sum_{(i,j) \in \Psi_l} e_{w,i,j}x_{l,w} + \sum_{t=1}^{N_T} \left( 1 + \phi \right)^{-t} \sum_{l=N_L+1}^{N_L+N_S} \sum_{(i,j) \in \Omega_l} 8760 \bar{c}P_{w,0}x_{l,w} + T_{\text{max}} \bar{c}P_{\text{loss}} \right\}
\]

(4)

\[
D_{i,j,1} = \sqrt{2 + (h_{i-1,j-1} - h_{i,j})^2} \quad D_{i,j,2} = \sqrt{1 + (h_{i-1,j} - h_{i,j})^2} \\
D_{i,j,3} = \sqrt{2 + (h_{i-1,j+1} - h_{i,j})^2} \quad D_{i,j,4} = \sqrt{1 + (h_{i,j+1} - h_{i,j})^2} \\
D_{i,j,5} = \sqrt{2 + (h_{i+1,j+1} - h_{i,j})^2} \quad D_{i,j,6} = \sqrt{1 + (h_{i+1,j} - h_{i,j})^2} \\
D_{i,j,7} = \sqrt{2 + (h_{i+1,j-1} - h_{i,j})^2} \quad D_{i,j,8} = \sqrt{1 + (h_{i,j-1} - h_{i,j})^2}
\]

(5)

\[
\Omega_l = \{(i,j,d) | \text{if line } l \text{ selects direction } d \text{ in cell } C_{i,j}\}
\]

(6)

\[
\Psi_l = \{(i,j) | \text{if substation } l \text{ selects cell } C_{i,j}\}
\]

(7)
\[
\sum_{v=0}^{N_v} x_{l,v} = 1 \quad (8)
\]
\[
\sum_{w=0}^{N_w} x_{l,w} = 1 \quad (9)
\]

where \( l \) is the branch index, and \( N_L \) is the total number of branches including existing branches, candidate lines, and candidate substations. \( N_B \) is the total number of candidate lines, which are indexed from 1 to \( N_B \). \( N_S \) is the total number of candidate substations, which are indexed from \( N_B + 1 \) to \( N_B + N_S \). The remaining branches are existing lines. Binary variable \( x_{l,v} \) represents the line construction decision, where 1 means line \( l \) is built using type \( v \) line, otherwise 0. Binary variable \( x_{l,w} \) represents the substation construction decision, where 1 means substation \( l \) is built using type \( w \) substation, otherwise 0. \( e \) is the annual average electricity energy price. \( T_{\text{max}} \) is the maximum load duration. \( P_{\text{loss}} \) is the variable loss of power network. \( t \) is the year index, and \( N_T \) is the payback period. \( \varphi \) is the annual interest rate.

### 2.3. Constraints

1. **Coupling constraints between candidate lines and substations**

\[
\sum_{l=1}^{N_B} \sum_{v=1}^{N_v} \tilde{A}_{l,v} x_{l,v} \leq M_0 \sum_{w=1}^{N_S} x_{l,w} \quad (10)
\]

where \( \tilde{A}_{l,v} \) represents an element of the substation-branch incidence matrix, and \( M_0 \) is a big enough positive number.

2. **Minimum number of candidate lines connected to a bus**

\[
\sum_{l=1}^{N_B} \sum_{v=1}^{N_v} |A_{l,v}| x_{l,v} \geq M_n \quad (11)
\]

where \( A_{l,v} \) represents an element of the bus-branch incidence matrix, and \( M_n \) is the minimum required number of candidate lines which are connected to bus \( n \).

3. **Nodal power balance equations**

\[
\frac{\tilde{S}_n}{\tilde{U}_n} = \sum_{n=1}^{N_B} Y_{mn} \bar{U}_{n1} \quad (12)
\]

where \( \tilde{S}_n \) is the conjugate injected apparent power of bus \( n \). \( \tilde{U}_n \) is the conjugate voltage of bus \( n \). \( N_B \) is the total number of buses. \( Y_{mn} \) is admittance between buses \( n \) and \( n_1 \). \( \bar{U}_{n1} \) is voltage of bus \( n_1 \).

4. **Line power flow equations**

\[
\tilde{S}_l = \bar{U}_{F(l)} \left[ \frac{\hat{U}_{F(l)} - \hat{U}_{T(l)}}{Y_l} \right] \quad (13)
\]

\[
\begin{cases}
    Y_l = \sum_{v=1}^{N_v} x_{l,v} Y_{0,v} & l \leq N_B \\
    Y_l = \sum_{w=1}^{N_S} x_{l,w} Y_{0,w} & N_B < l \leq N_B + N_S 
\end{cases} \quad (14)
\]

where \( \tilde{S}_l \) is apparent power transferred on branch \( l \). \( \hat{U}_{F(l)} \) and \( \hat{U}_{T(l)} \) are conjugate voltages of sending and receiving buses of line \( l \), respectively. \( \hat{Y}_l \) is conjugate admittance of branch \( l \). \( Y_{0,v} \) is the per unit admittance of type \( v \) line, and \( Y_{0,w} \) is the admittance of type \( w \) substation. \( s_l \) is the length of line \( l \), which is given in (15).
\[
sl = \sum_{(i,j,d) \in \Omega} 0.5D_{i,j,d}
\]

(5) Line power flow capacity limits

\[
\begin{cases}
S_{l,\text{min}} \leq Sl \leq S_{l,\text{max}} & I > N_B + N_S \\
-(x_{l,0} - 1)S_{l,\text{min}} \leq Sl \leq -(x_{l,0} - 1)S_{l,\text{max}} & I \leq N_B + N_S
\end{cases}
\]

(16)

(6) Voltage limits

\[
U_{n,\text{min}} \leq U_n \leq U_{n,\text{max}}
\]

where \(S_{l,\text{min}}\) and \(S_{l,\text{max}}\) are the lower and upper capacity limits of branch \(l\). \(U_{n,\text{min}}\) and \(U_{n,\text{max}}\) are the lower and upper voltage limits of bus \(n\).

3. SCHEME OF ACO

3.1. ACO paradigm

The ACO is a heuristic intelligence method inspired from the natural behavior of ants for solving optimization problems. Its powerful capability for solving high-dimension optimization problems has been proven in literatures [28–30]. The main rules of the ACO are as follows:

(1) State transition rule: The state transition rule (18) is a random-proportional rule to determine the probability with which an ant in city \(i\) chooses to move to city \(j\). Obviously, the exploration of new paths is biased towards short and high trail edges.

\[
\begin{align*}
p_k(i,j) & = \begin{cases} 
\frac{[\tau(i,j)]^{\beta} [\eta(i,j)]^{\beta}}{\sum_{m \in J_k} [\tau(i,m)]^{\beta} [\eta(i,m)]^{\beta}} & j \notin J_k \\
0 & \text{otherwise}
\end{cases} \\
\end{align*}
\]

where \(k\) is the ant index, \(k = 1, 2, \ldots, N_K\), and \(N_K\) is the total number of ants. \(i\) and \(j\) are city indices, \(J_k\) is the set of ants that are not neighbors to ant \(k\) when ant \(k\) is at city \(i\). \(\tau(i,j)\) is the pheromone deposited on edge \((i,j)\). \(\eta(i,j)\) is the inverse of the length of edge \((i,j)\). \(\beta\) is a parameter to trade off impacts of the pheromone and the length.

(2) Local updating rule: When an edge is chosen by an ant, its pheromone value is changed by applying the local trail updating formula:

\[
\tau(i,j) = (1 - \alpha)\tau(i,j) + \alpha \tau_0
\]

where, \(\tau_0\) is the initial pheromone value, and \(\alpha\) is a heuristically defined parameter.

(3) Global updating rule: Once all ants have completed their tours, only the best ant deposits pheromone on visited edges. The global trail updating formula is

\[
\tau(i,j) = (1 - \rho)\tau(i,j) + \rho \delta^{-1}
\]

where \(\delta\) is the distance of the globally best tour from the beginning of the trail, and \(\rho\) is the pheromone decay parameter.
3.2. ACO for spatial power network planning

In [31], search spaces of ant placement and reconfiguration are formed to solve the integrated feeder reconfiguration and capacitor placement problem. In this paper, four search spaces, corresponding to substation location, line type selection, and line routing, are created in ACO for solving the proposed multi-dimensional power network planning problem. The full search space of the proposed ACO is shown in Figure 1. Dark circles and thick lines constitute the traveling path of an ant.

In Figure 1, the city index in the search space of substation sizing is identical to the substation type index. That is, city 0 means substation is not built, otherwise, means the corresponding substation type is built. Therefore, there are \( N_W + 1 \) cities for a substation. The number of cities in the search space of substation location is \( N_R \times N_C \) since a substation could be located in any cell. Consequently, the index of city \( \tilde{i} \) in cell \( C_{i,j} \) is \((i-1)N_R + j\). Similarly, the city index in the search space of line type selection is identical to the line type index. Therefore, there are \( N_V + 1 \) cities for a line. A line cannot be built if no station connected to it is built. Once locations of substations are determined, the positions of sending and receiving buses of lines are also determined. Starting from sending buses and heading towards receiving buses, ants proceed along cells to explore routes for lines to be built. Consequently, the number of cities is \( N_R \times N_C \) for a line in the search space of line routing, and the index of city \( \tilde{i} \) in cell \( C_{i,j} \) is \((i-1)N_R + j\). In addition, considering constraints that couple adjacent cells, the feasible neighbor of ants in cell \( C_{i,j} \) is \( \{C_{i-1,j-1}, C_{i-1,j}, C_{i,j-1}, C_{i+1,j-1}, C_{i,j+1}, C_{i+1,j+1}\} \).

An ant prefers shorter edges with higher levels of pheromone trails, which consequently increases the pheromone trails on shorter paths. Therefore, ACO is similar to a reinforcement learning scheme, in which better solutions get higher reinforcement. In order to effectively solve the proposed multi-dimensional power network planning problem, the heuristic value \( \lambda(\tilde{i},\tilde{j}) \) related to the specific search space is introduced in the state transition rule as shown in (21).

\[
p_{k}(\tilde{i},\tilde{j}) = \begin{cases} \frac{[\tau(\tilde{i},\tilde{j})][\eta(\tilde{i},\tilde{j})]^\beta[\lambda(\tilde{i},\tilde{j})]^\gamma}{\sum_{m \in J_{\tilde{i}}} [\tau(\tilde{i},m)][\eta(\tilde{i},m)]^\beta[\lambda(\tilde{i},m)]^\gamma} & \text{if } \tilde{j} \not\in J_{\tilde{i}+1} \\ 0 & \text{otherwise} \end{cases} \tag{21} 
\]

In the search spaces of substation sizing and line type selection, ants move according to the pheromone trail. In other words, \( \eta(\tilde{i},\tilde{j}) = 1 \) and \( \lambda(\tilde{i},\tilde{j}) = 1 \). In the search space of substation location, the movements of ants among cities depend on both the pheromone trail and the distance. That is, \( \lambda(\tilde{i},\tilde{j}) = 1 \) and \( \eta(\tilde{i},\tilde{j}) \) can be formulated as (22).

\[
\eta(\tilde{i},\tilde{j}) = \frac{1}{b_{w,i,j}} \quad \text{if } \tilde{j} = (i-1)N_R + j \tag{22} 
\]

In the search space of line routing, the traveling path of an ant is determined by the pheromone trail, the distance, and the heuristic value \( \lambda(\tilde{i},\tilde{j}), \eta(\tilde{i},\tilde{j}) \), and \( \lambda(\tilde{i},\tilde{j}) \) can be formulated as (23) and (24), respectively.

\[
\eta(\tilde{i},\tilde{j}) = \frac{1}{b_{w,i,j}} \quad \text{if } \tilde{j} = (i-1)N_R + j \tag{23} 
\]
\[
\lambda(i,j) = \frac{1}{|\vec{\vartheta}|}
\] (24)

Where \( \vartheta \) is the angle between the vector from sending bus to receiving bus of a line and the vector from city \( e_i \) to city \( e_j \). Ants can avoid unnecessary detour using \( \lambda(i,j) \).

In ACO, the fitness function (25) estimates the performance of each ant, where \( \lambda_1 \) and \( \lambda_2 \) are punishment factors. \( CV_1 \) and \( CV_2 \) are the summation of squared bus voltage violations and the summation of squared line capacity violations, respectively.

\[
F_1 = F + \lambda_1 CV_1 + \lambda_2 CV_2
\] (25)

Here, the power flow calculation is applied to evaluate the fitness of each ant, which means that outputs of generating units (excluding those connected to the slack bus) are set to be constant. It could be easily extended to consider variable generating unit outputs together with their capacity limits, by replacing the power flow calculation with the optimal power flow model.

### 3.3. Computational procedures

The main computational procedure of the proposed ACO is briefly stated below.

Step 1) **Initiation.** The number of ants and the maximum iteration number are experimentally determined, and initial pheromone is set to \( \tau_0 \).

Step 2) **Termination of the algorithm.** End the process if the maximum iteration number is reached; otherwise, go to the next step.

Step 3) **The tour of ants.** At this step, an ant enters search spaces as shown in Figure 2 to explore the optimal solution to the proposed multi-dimensional power network planning problem. The ant employs a roulette selection algorithm based on (21) to choose direction. First, the ant

![Figure 2. The image and raster maps of the planning region.](attachment:image.png)
explores search spaces of substation sizing and substation location to decide substation size and location. Next, the ant travels in search space of line type selection to find desirable types for every line considering the status of substations connected to lines. If a line will be built, the ant travels in the raster map to explore optimal line routes from sending bus to receiving bus decided by substation locations. A full tour of an ant is a feasible solution to the multi-dimensional power network planning problem.

Step 4) **Local updating.** When an ant has completed its tour, the local updating rule (19) is applied to change pheromone trail on edges.

Step 5) **Estimation of the fitness.** When all ants have completed their tours, the power flow is calculated to evaluate the fitness of each ant by (25). The level of fitness represents the performance of ant.

Step 6) **Global updating.** The best ant deposits pheromone on its visited edges by applying the global updating rule (20), and the other edges remain unchanged. Go to step 2).

4. CASE STUDIES

The proposed ACO for the multi-dimensional power network planning model was implemented and tested by several case studies utilizing Matlab 6.5 on a Pentium-4 2.40-GHz personal computer. The maximum generation is set as 500. Other parameters of the proposed algorithm are selected as $N_k = 60$, $\beta = 0.1$, $\gamma = 2$, $\alpha = 0.1$, and $\rho = 0.05$.

4.1. The effect of multi-dimensional power network planning

Three examples are investigated to demonstrate the effectiveness of the proposed multi-dimensional power network planning approach. The first example illustrates the tradeoff between the investment cost and the operation cost. The second example shows the necessity of simultaneously taking into account the substation location, the substation sizing, the line type selection, and the optimal electric line routing. The performance of the proposed ACO approach and the MILP[20] are compared in the third example.

We use $40 \times 40$ cells to rasterize the image map of planning region in Figure 2(a). The accumulated cost and the altitude of each cell are shown in Figures 2(b) and 2(c), respectively. Table I lists the colors as well as their corresponding accumulated costs and altitudes. For simplicity, we assume that different types of stations and lines have the same raster map. Meanwhile, based on Table I, accumulated costs of different types of stations and lines are scaled by ratios shown as “$Tm$” column of Tables II and III, respectively.

A test system shown in Figure 3 has 2 voltage levels, 16 candidate lines, 1 source bus, 7 load buses, and 2 candidate substations. There are 2 parallel transformers in each candidate substation. We assume only one line is connected to each load bus. Candidate line routes and substations can be located in any cell.

(1) example 1: the first example contains the following three scenarios. In this section, all scenarios simultaneously take into account the substation location, the substation sizing, the line type selection, and the optimal electric line routing.

<table>
<thead>
<tr>
<th>No</th>
<th>Color</th>
<th>Cost ($10^4$)</th>
<th>Altitude (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>10.00</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>21.25</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>32.50</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
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</tr>
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<td></td>
<td>55.00</td>
<td>2.0</td>
</tr>
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</tr>
<tr>
<td>7</td>
<td></td>
<td>77.50</td>
<td>3.0</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>88.75</td>
<td>3.5</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>500</td>
<td>4.0</td>
</tr>
</tbody>
</table>
Scenario 1: Considering investment cost only.
Scenario 2: Considering operation cost only.
Scenario 3: Considering both investment cost and operation cost, which is the base scenario.

Figure 4(a) uses red squares to indicate locations of all existing buses. Optimal substation locations and line routes are shown in Figures 4(b)–(d). Here, red squares with red circular frames represent source buses, red squares with red square frames indicate constructed substation, and different colors show routes of constructed lines. Optimal station sizing and line type are listed in Tables IV and V, respectively. Figure 5 shows the three-dimensional optimal electric line routes and substation locations of Scenario 3. The computational results of the three scenarios are listed in Table VI. As shown in Figures 4(b)–(d) and Tables IV and V, the station sizing and location, the line type, and the line route of the three scenarios are all different. It can be observed from Table VI that Scenario 1 has the minimum investment cost, Scenario 2 has the lowest operation cost, and Scenario 3 can obtain the minimum total cost result.

(2) example 2: the second example contains the following three scenarios. In this section, all scenarios consider both investment cost and operation cost.
Scenario A: Given the least cost routes of lines, considering the substation location, the substation sizing, and the line type selection.

Scenario B: Given the substation locations derived in Scenario A, considering the line routing, the line type selection, and the substation sizing.

Scenario C: The base scenario that is the same as Scenario 3 in example 1.

Figure 6 and Table VII show results of Scenarios A and B. Total costs of Scenarios A, B, and C indicate that simultaneous consideration of the substation location, the substation sizing, the line type selection, and the optimal electric line routing is more effective than considering them separately.
Table IV. Line expansion results of Scenarios 1–3.

<table>
<thead>
<tr>
<th>Bus to bus</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type</td>
<td>Length (km)</td>
<td>Type</td>
</tr>
<tr>
<td>1–2</td>
<td>1</td>
<td>22.295</td>
<td>4</td>
</tr>
<tr>
<td>1–3</td>
<td>2</td>
<td>18.681</td>
<td>4</td>
</tr>
<tr>
<td>3–4</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>3–5</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>3–6</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>3–7</td>
<td>1</td>
<td>7.715</td>
<td>0</td>
</tr>
<tr>
<td>3–8</td>
<td>1</td>
<td>6.103</td>
<td>0</td>
</tr>
<tr>
<td>3–9</td>
<td>1</td>
<td>6.753</td>
<td>0</td>
</tr>
<tr>
<td>3–10</td>
<td>1</td>
<td>9.815</td>
<td>0</td>
</tr>
<tr>
<td>2–4</td>
<td>1</td>
<td>10.619</td>
<td>0</td>
</tr>
<tr>
<td>2–5</td>
<td>1</td>
<td>12.403</td>
<td>0</td>
</tr>
<tr>
<td>2–6</td>
<td>1</td>
<td>5.302</td>
<td>0</td>
</tr>
<tr>
<td>2–7</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2–8</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2–9</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2–10</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

Table V. Substation expansion results of Scenarios 1–3.

<table>
<thead>
<tr>
<th>Bus</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type</td>
<td>Location</td>
<td>Type</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>(27,28)</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>(18,31)</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 5. The three-dimensional spatial power network planning of Scenario 3.

Table VI. Spatial network planning costs of the example 1 (104$).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Operation cost</th>
<th>Lines investment cost</th>
<th>Substation investment cost</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7105</td>
<td>3580</td>
<td>1048</td>
<td>11,733</td>
</tr>
<tr>
<td>2</td>
<td>2934</td>
<td>35,770</td>
<td>38,783</td>
<td>77,487</td>
</tr>
<tr>
<td>3</td>
<td>5348</td>
<td>4587</td>
<td>1638</td>
<td>11,573</td>
</tr>
</tbody>
</table>
example 3: the proposed ACO approach and the MILP are compared using different resolutions of the raster map, i.e. 30 × 30, 40 × 40, 50 × 50, and 60 × 60. For the sake of comparison, we assume that only the routes of lines are optimized (while locations and types of substations, and types of lines are given) and the network formulation is linearized (bus voltages and operation costs are ignored). The maximum iteration of ACO is set to 500. Results are shown in Table VIII. Note that: (i) For the raster map with a low resolution, the MILP can converge to the targeted relative gap and obtain a better result than ACO. However, the results of ACO are fairly satisfied. (ii) The CPU time of the MILP increases much faster than that of the ACO with the increased resolution of the raster map. (iii) The MILP cannot converge to a tolerable relative gap for the raster map with 50 × 50 resolution before running out of memory. The MILP even cannot obtain a feasible solution for the raster map with 60 × 60 resolution.

### Table VII. Spatial network planning costs of the example 2 (104$).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Operation cost</th>
<th>Lines investment cost</th>
<th>Substation investment cost</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3531</td>
<td>25,345</td>
<td>874</td>
<td>29,750</td>
</tr>
<tr>
<td>B</td>
<td>6711</td>
<td>5053</td>
<td>1048</td>
<td>12,812</td>
</tr>
</tbody>
</table>

### Table VIII. Numerical results of example 3.

<table>
<thead>
<tr>
<th>Items</th>
<th>MILP[20]</th>
<th>ACO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time (s)</td>
<td>Gap (%)</td>
</tr>
<tr>
<td>30 × 30</td>
<td>159</td>
<td>0.5</td>
</tr>
<tr>
<td>40 × 40</td>
<td>1805</td>
<td>0.5</td>
</tr>
<tr>
<td>50 × 50</td>
<td>3774</td>
<td>45.9</td>
</tr>
<tr>
<td>60 × 60</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
This example indicates that the proposed ACO is more efficient than the MILP approach for applications to the high-resolution raster map.

4.2. The influence of peak load level

For comparison, the following four cases are considered based on Scenario 3, for exploring the impacts of different peak load levels on the proposed multi-dimensional power network planning problem.

Case 0: Original load level
Case 1: The load level decreases to 20%
Case 2: The load level decreases to 60%
Case 3: The load level increases to 140%

Figure 7 and Tables IX and X show results of Cases 1–3. Comparing results of Case 1 with those of Case 0 as shown in Figure 4(d) and Table VI, only one substation and a few lines with smaller capabilities are constructed because the peak load level in Case 1 is low. Although substation locations and line routes are the same in Cases 0 and 2, their types are different. As shown in Figure 4(d) and Figure 7 (c), only substation locations remain the same in Cases 0 and 3, while line routes and types of both substations and lines are different. Comparing results of the four cases, we notice that the peak load level has an effect on the substation size and location, as well as types and routes of lines.

Figure 7. The optimal results of Cases 1–3.
4.3. Large-scale test system

In this case study, the proposed approach is applied to a large-scale power network and a high-resolution raster map. The image map of the planning region in Figure 2(a) is rasterized by 110 × 110 cells. The accumulated costs and the altitudes are the same as those listed in Tables I–III, except that numbers in the “Tm” column of Table II are decreased by a factor of 2.75. In Figure 2(a), a new network is to be expanded based on the original IEEE 118-bus system as shown in Figure 8(a). The detailed data can be found in motor.ece.iit.edu/data/SCUC_118test.xls. The entire test system has 2 voltage levels, 54 units, 186 existing branches, 118 existing buses, 327 low-voltage candidate lines, 18 high-voltage candidate lines, 46 new load buses, and 6 candidate substations which can be connected to buses 20, 21, and 22. There are 2 parallel transformers in each candidate substation. The peak load level is 0.6 + j0.3 MVA for all new load buses. Fixed-location buses, candidate lines, and candidate substations of the new network are shown in Figure 8(b), in which the fixed-location buses, including buses 20, 21, and 22 and all new load buses, are indicated using red squares with red labels (the font size of labels of buses 20, 21, and 22 is relatively larger), while the freely located candidate substations are represented as black squares with black underline labels. Here, high-voltage and low-voltage candidate line routes are represented as purple and blue straight lines, respectively. The following three cases are studied to verify the effectiveness of the proposed multi-dimensional power network planning approach in large-scale systems.

Case A: Consider both investment cost and operation cost with the original load level.
Case B: Consider investment cost only with the original load level.
Case C: Consider both investment and operation cost with the load level increased to 1.2 + j0.6 MVA.

The results of the multi-dimensional power network planning for Cases A–C are shown in Figure 9 and Table XI. Different line colors in Figures 9(a), (c), and (e) represent different line routes of Cases A, B, and C, respectively. That is, different line colors are used in order to distinguish individual line routes.
routes connected to the same bus. For example, both line 1 and line 2 are connected to bus 1. Thus, two different colors are used for line 1 and line 2 to indicate that they are two individual invested lines.

Figure 8. IEEE 118-bus system and the new network to be expanded.
rather than one single line. In Figures 9(b), (d), and (f), different line colors represent different types of lines of Cases A, B, and C, respectively. Blue is type 1, pink is type 2, green is type 3, and brown is type 4.

Table XI. The summary of optimal results of Cases A–C.

<table>
<thead>
<tr>
<th>Cases</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operation cost ($10^4$)</td>
<td>76,961</td>
<td>77,086</td>
<td>77,810</td>
</tr>
<tr>
<td>Lines investment cost ($10^4$)</td>
<td>704</td>
<td>644</td>
<td>981</td>
</tr>
<tr>
<td>Substation investment cost ($10^4$)</td>
<td>188</td>
<td>203</td>
<td>359</td>
</tr>
<tr>
<td>Total cost ($10^4$)</td>
<td>77,853</td>
<td>77,933</td>
<td>79,150</td>
</tr>
<tr>
<td>Number of type 1 lines</td>
<td>48</td>
<td>48</td>
<td>44</td>
</tr>
<tr>
<td>Number of type 2 lines</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Number of type 3 lines</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Number of type 4 lines</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Length of type 1 lines (km)</td>
<td>260.28</td>
<td>217.68</td>
<td>236.49</td>
</tr>
<tr>
<td>Length of type 2 lines (km)</td>
<td>19.21</td>
<td>19.52</td>
<td>26.49</td>
</tr>
<tr>
<td>Length of type 3 lines (km)</td>
<td>0</td>
<td>0</td>
<td>39.14</td>
</tr>
<tr>
<td>Length of type 4 lines (km)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Number of type 1 substations</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Number of type 2 substations</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Number of type 3 substations</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

In Case A, there is a tradeoff between line investment cost, substation investment cost, and operation cost. For example, the load bus N1 is connected to substation S2 in Case A. If we switch it to N2, the line investment cost is decreased to $694*10^4$, the substation investment cost remains the same, the operation cost is increased to $76,981*10^4$, and the total cost is increased to $77,863*10^4$. The location of substation S3 is (29, 77). If it is moved to (27, 79), the line investment cost is increased to $707*10^4$, the substation investment cost is increased to $346*10^4$, the operation cost is decreased to $76,959*10^4$, and the total cost is increased to $78,012*10^4$.

Load buses are connected to the network via lower investment cost lines, which will lower the investment cost while increasing the operation cost. For example, the load bus N12 is connected to the neighboring bus N10 in Case B, while it is connected to substation S4 in Case A. Thus, the total cost of Case B is higher than that of Case A.

Most load buses are connected to substations in Case C, which shows that the effect of operation cost in total cost is increasing while the load level increases.

The network structure, type, and position of substations, as well as type and route of lines, are different in Cases A–C, which indicates that the interaction among the five dimensions of power network planning is complicated and it is hard to derive the optimal result without integrating all the five dimensions into one optimization model.

5. CONCLUSIONS

This paper proposes a multi-dimensional power network planning problem, which considers the substation sizing, the substation location, the line type selection, the optimal electric line routing, and the power network evaluation using the raster map. An ACO with a four-search-space structure and the enhanced state transition rule is introduced to solve the complicated mixed-integer nonlinear optimization problem. This approach does not need pre-determined line routes and candidate substation locations, which are common premises in previous studies. Following conclusions are observed through numerical case studies:

1. Considering both operation cost and investment cost is more effective than considering them individually.
2. Simultaneous consideration of the substation location, the substation sizing, the line type selection, and the optimal electric line routing is more effective than considering them separately.
3. The proposed multi-dimensional power network planning can derive optimal substation sizes and locations, as well as line types and routes with respect to peak load levels.
4. The reinforcement learning scheme and the parallel search make the ACO method suitable for applications to large-scale systems.

6. LIST OF SYMBOLS AND ABBREVIATIONS

\[ v \] Index for line type
\[ w \] Index for substation type
\[ d \] Index for direction
\[ l \] Index for branch
\[ t \] Index for year
\[ k \] Index for ant
\[ i \] and Indices for city
\[ j \] Number of candidate line types
\[ w \] Number of candidate substation types
\[ l \] Number of branches
\[ B \] Number of candidate lines
\[ S \] Number of candidate substations
\[ T \] Payback period
Number of buses $N_N$

Number of ants $N_K$

Accumulated cost matrix for type $v$ line $E_v$

Per unit value for the accumulated cost of type $v$ line in cell $C_{i,j}$ $e_{v,i,j}$

Accumulated cost matrix of type $w$ substation $E_w$

Accumulated cost of type $w$ substation in cell $C_{i,j}$ $e_{w,i,j}$

Per unit value of the altitude of $C_{i,j}$ $h_{i,j}$

Line construction decision $x_{l,v}$

Substation construction decision $x_{l,w}$

Annual average electricity energy price $\bar{e}$

Maximum load duration $T_{max}$

Fixed loss of type $w$ substation $P_{w,0}$

Variable loss of power network $P_{loss}$

Annual interest rate $\phi$

An element of the substation-branch incidence matrix $A_{l,w}$

An element of the bus-branch incidence matrix $A_{l,v}$

Minimum required number of lines which are connected to bus $n$ $S_n$

Conjugate injected apparent power of bus $n$ $U_n$

Conjugate voltage of bus $n$ $\bar{U}_n$

Conjugate apparent power transferred on branch $l$ $S_l$

Conjugate voltages of sending and receiving buses of line $l$ $\bar{U}_l$

Conjugate admittance of branch $l$ $Y_l$

Per unit admittance of type $v$ line $Y_{0,v}$

Admittance of type $w$ substation $Y_{0,w}$

Length of line $l$ $s_l$

Lower and upper capacity limits of branch $l$ $S_{l,\min}$ $S_{l,\max}$

Lower and upper voltage limits of bus $n$ $U_{n,\min}$ $U_{n,\max}$

Set of ants that are not neighbors to ant $k$ when ant $k$ is at city $\bar{i}$ $J_{k\sim}$

Pheromone deposited on edge $(\bar{i},\bar{j})$ $\tau(\bar{i},\bar{j})$

Inverse of the length of edge $(\bar{i},\bar{j})$ $\eta(\bar{i},\bar{j})$

Parameter to trade off impacts of the pheromone and the length $\beta$

Initial pheromone value $\tau_0$

Heuristically defined parameter $\alpha$

Distance of the globally best tour from the beginning of the trail $\delta$

Pheromone decay parameter $\rho$

Angle between the vector from sending bus to receiving bus of a line and the vector from city $\bar{i}$ to city $\bar{j}$ $\vartheta$

Punishment factors $\lambda_1$ $\lambda_2$

and Summation of squared bus voltage and line capacity violations $CV_1$

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China, under grant 51347003, and the Fundamental Research Funds for the Central Universities, under grant 12MS19. This work was also supported in part by the U.S. National Science Foundation grants ECCS-1102064 and ECCS-1254310.

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