Opposite Thermodynamic Arrows of Time

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Abstract. A model with two weakly coupled systems having opposite thermodynamic arrows of time is exhibited. The essential conceptual step is the phrasing of the problem as a two-time boundary value problem. Such a system could arise in a big bang-big crunch cosmology. Causal paradoxes, in the event of communication between the regions, are discussed and resolved. The main issue confronted in the present article is the nature and possibility of such communication. A degree of signaling may be possible, but is likely to be overwhelmed by noise.

INTRODUCTION

Eddington [1, 2] famously made the 2nd law of thermodynamics supreme among physical laws and absolutely inviolable. For the departure from the 2nd law that I discuss here, there is the complication that the ubiquity of the law can make language an obstacle to the contemplation of its breakdown. The particular snare is the near tautological time-sequencing of (macroscopic) cause and effect.

Begin with a non-directional time parameter, “\( t \),” for microscopic motion, as in \( F = m d^2x/dt^2 \). Macroscopically there emerges directionality, the thermodynamic arrow. As I have argued,\(^1\) the selection of the direction of this arrow is equivalent to the selection of which are to be macroscopic initial conditions and which are to be final. Ignoring this point can lead to circularity, and for contemplating simultaneous opposite arrows the choice of macroscopic boundary conditions is of paramount importance.

AN EXAMPLE OF OPPOSITE ARROWS

In [4] I showed that with unprejudiced boundary conditions there is no bar to the simultaneous existence of opposing arrows in a system of weakly interacting subsystems. Only minimal details are given here; see [4, 6]. The “cat map,” \( \phi \), sends the unit square, \( I^2 (I = [0, 1]) \) into itself. A second map, \( \theta_\alpha, \alpha \in \mathbb{R} \), is also used. Define

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\phi : \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x + y \\ x + 2y \end{pmatrix}, \quad \theta_\alpha : \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x + \alpha y \\ y \end{pmatrix},
\]

both transformations modulo 1. The dynamical system is a gas of \( N \) “atoms” in system “A,” \( N \) in system “B.” They are weakly coupled in that each atom in A interacts with

\(^1\) See [3] and references therein; also [4, 5].
a particular atom in B, so the total motion is the dynamics of N points on \( \mathbb{R}^4 \), \((x_A,y_A)\) for A, \((x_B,y_B)\) for B. A single time step has 3 parts: \( \theta_{\alpha/2} \) acting on \((x_A,y_B)\) and \((x_B,y_A)\); \( \phi \) acting separately on A and B, and finally a repeat of the first part. The cat map, the middle part, provides the chaos. Parts 1 and 3 couple A and B. At time-0 A is specified to be in a small region of \( I^2 \), B is unconstrained. A coarse graining is defined and, with it, an entropy. Let \( p_\beta = \) fraction of atoms in grain \( \beta \); then \( S = - \sum p_\beta \log p_\beta \) (coarse grains of equal measure). For \( \alpha = 0 \) this yields independent motion in which \( S(A) \) increases for \( t \) going from 0 to \( T \). By contrast \( S(B) \) decreases from 0 to \( T \); no surprise, since it has “final” conditions from A’s perspective. This gives A an arrow in the direction of increasing microscopic \( t \), and B’s arrow opposite. What may be a surprise is that with \( \alpha > 0 \) this situation persists. Using \( N \sim 500 \), \( T \sim 10 \), \( 0.1 \times 0.1 \)-square coarse grains, for \( \alpha \sim 0.2 \) each system retains its arrow. Similarly, it is not difficult to show that “macroscopic causality” follows the entropy-increase arrows. For illustrations see [4, 6].

**THE CA(RPE)T PARADOX**

Let there be opposite arrow regions with sentient beings A and B, or to adopt the cryptographer’s convention, Alice and Bob. At 2 p.m. her time, Alice sees rain entering Bob’s open window, ruining his carpet, or annoying his cat, to use the physicist’s convention on paradoxes. Half an hour later she sees rain “beginning” in Bob’s part of the universe. At 3 p.m. her time she signals Bob: “It’s going to rain (bad for your cat, carpet), close your window.” Bob then closes his window half an hour before it will rain, by his clock. But with Bob’s window closed, at Alice’s 2 p.m. she does not see it raining in. So she won’t signal, but then . . . ?

The resolution [7] involves a formulation of the paradox as a boundary value problem, setting “initial” conditions for each participant. Alice, at 1:45 p.m. has good intentions: if Bob has a problem, she’ll help. For definiteness let the rain begin at 10:30 a.m. Bob’s time, so Alice sees the rain coming in at Bob’s 11 a.m. He gets her “close the window” signal at his 10 a.m. Bob’s “initial” conditions are given at his 9:45 a.m. They are: window wide open and the cat in a small room, unable to avoid the rain should it blow in. One now appeals to continuity in Nature, as in [8, 9, 10]. This yields a self-consistent solution to the boundary value problem,\(^2\) as follows. Alice, at her 2 p.m. (11 a.m. Bob’s time) sees his window open just a crack, with a few drops hitting the cat. She’s unsure whether this warrants signaling another galaxy, but finally, at her 3 p.m., sends: “Bob, it will rain, you might close the window.” His reaction to this weak admonition is not to slam the window shut, but to mostly close it, preferring a bit of fresh air to a totally happy cat. And—this is what Alice sees at her 2 p.m.\(^3\)

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\(^2\) There could be several solutions, as a result of the macroscopic nature of the boundary values. However, in the microscopic formulation considered in [7], there may also be more than one solution.

\(^3\) This corresponds to the resolution adopted for time-travel paradoxes by authors such as Heinlein and Moorcock (see [7]). Another writer, Egan [11], takes a similarly deterministic view of the universe and uses an opposite-arrow region to read the “future”!
SIGNS

Consider how a signal could be sent. In [4] I argued for electromagnetic signals between opposite arrow regions. I no longer make this claim, nor, by the way, do I necessarily believe it to be false. Instead I imagine that for A to signal B a solid object is sent, a rock or clay tablet with an inscription; similarly for signals from B to A.

Consider Alice’s information on Bob’s window. At 2 p.m. she has a rock with an inscription about rain, windows and cats. Later it is no longer with her, and (assuming 5 min. travel time—but time intervals could be rescaled) by (say) 2:05 her time, 11 a.m. his time, the rock is with Bob. Where is it at 1:50 p.m. her time? Quite possibly it was in her possession. As far as she is concerned the rock has “always” been with her, and at 2 p.m. it left her. Bob will have the same impression (it was “always” with him). His idea of its history is that he sends it to her at 11 a.m., his time.

Furthermore, for both of them the rock departs, is outgoing, i.e., becomes more distant as subjective time increases. To see this consider the space-time diagram (Fig. 1) of the transmission. If Alice and Bob are at rest relative to one another, their world lines can be pictured as parallels to the t-axis. The world line of the rock necessarily has slope more than c. Recall that Alice, Bob, and the rock, are all ordinary matter. No electrons are going backward in time looking like positrons. The opposite-arrow story only involves the statistics of the matter.

Assume then that Alice has had this rock, and perhaps others, for a long time. Why should she think it’s a message from the future? There could be many clues. It could (but need not) be in a language not traceable to any civilization on Alice’s planet. It could be inscribed on a rock older than the universe. By this I mean that the rock’s age could be estimated by measuring isotope abundances of long lived nuclides, as in Carbon or Rubidium/Strontium dating. For the present application one would need extreme longevity, as for example in Ge\textsuperscript{76} with a lifetime of about 10\textsuperscript{21} yr [12], as well as an a priori idea of the rock’s “initial” composition. Another clue, presumably included

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4 As discussed below, it is not clear whether they impart velocity or whether they would see a violation of the second law in which the system picked up velocity at the expense of local heat sources.

5 This discussion is based on a suggestion of Frank Avignone and Richard Creswick.
intentionally by Bob, could be verifiable predictions about Alice’s future, predictions about unstable, chaotic systems, such as the weather, for which a multi-decade prediction about (e.g.) the onset time of rainfall is impossible (both because of computing power and data collection). Given these clues, given the theoretical possibility of interacting opposite-arrow systems, and given the absence of other explanation, Alice will deduce that the messages come from such a system.

I next consider the vexing issue of who wrote the message. We’ve assumed it was Bob, keeping a diary on the discomfiture of his cat. But now I want to look at the rock from Alice’s perspective. Suppose she had been privileged to watch that rock from its formation in the cooling of her planet. As the rock hardens would it develop just those indentations corresponding to a written language, giving information about future weather (similarly for baked clay)? In the present context, where the 2nd law is not assumed, such suggestions must be considered. Nevertheless, I do not think this happens. My principle for estimating the likelihood of scenarios is that, given constraints, select the least unlikely scenario consistent with the constraints. This is conventional probability, except that I also contemplate constraints consisting of partial information at widely different times. With this principle, equilibration may not follow the conventional path (cf. [3]) but the macroscopic time dependence is nevertheless the most likely under the circumstances.

To see whether spontaneous writing can occur, consider normal writing. Certainly when Bob creates patterns on the surface of a clay tablet the result is rare among those pieces of clay that would otherwise emerge from a kiln. So it’s Bob’s ability to select particular states, to provide information, that enables the writing. Where does Bob get this power? From lunch, from the negentropy stream that powers all life. This recalls Schrödinger’s theme [13] and also the arguments of Gold [14, 15] relating the thermodynamic and cosmological arrows. The lowering of the rock’s entropy is performed at the expense of increased entropy for photons coming from the sun: they arrive at 6000 K and leave at 300 K. Thus Bob’s message, although lowering entropy in the rock itself, is part of a larger process of overall increase of entropy. Moreover, the low entropy at earlier epochs, which drives the solar activity, food production and writing, is part of the overall cosmological history of the universe. Gold ascribes this to expansion, but one can be more specific. In [3] I reason from the early-epoch low entropy, and justify that low entropy in terms of expansion. Specifically, at “recombination” the universe was uniform (with deviations $< 10^{-5}$). Uniformity at equilibrium is characteristic of systems dominated by short range forces. On the other hand, the subsequent expansion leads to a universe dominated by gravitational forces. Under gravity uniformity is an extremely unlikely state. So it is expansion that takes a (relatively) high entropy state at one epoch and makes the same state (relatively) low entropy later on.

By contrast, a scenario in which a meaningfully inscribed clay tablet emerges from a kiln with no macroscopic “cause” would, in generous terms, be called a “rare fluctuation.” It would be like a rock rising from water at the expense of energy in the water.

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6 See for example [3], Fig. 4.3.2, page 141. For boundary conditions for a big bang-big crunch universe, the separated times could be “recombination,” roughly 300,000 yr after the big bang, and a corresponding time before the big crunch.
Ripples form in anticipation and the water cools—a scenario that is reasonable in one direction of time, but not the other. (This could also occur for the departure of the rock from Alice toward Bob.) Could this happen? The way to answer this is to look at the boundary conditions. For boundary conditions we take those alluded to in Footnote 6, but supplement them with the demand that at the space-time event midway along the rock’s trajectory in Fig. 1 there is a rock moving at the appropriate velocity with an inscription concerning cats, weather, etc. For the usual evolutionary scenarios for the universe this is not a great restriction on states: at this moment there is a tablet of that sort moving away from earth with cartoons about the human race—NASA’s “golden plaque” on the 1972 PIONEER 10 mission. What I now ask though is that given that Bob has created his inscribed rock with the indicated information (as we created the golden plaque), what is the least unlikely scenario that includes its being received in an opposite-arrow region of space-time?

As indicated, one scenario is to have the scratches spontaneously appear. Rocks have scratches; all that’s required is that they convey information in some language. Alice might be the curator of a museum of inscribed rocks, devoted to studying civilizations of puzzling provenance. One could argue that the entropy of one configuration of scratches is the same as another. Marks on a baked clay tablet would be more difficult to explain as spontaneous. Potters don’t find cuneiform inscriptions on vases that entered the kiln smooth.

Another way for writing to appear is by human agency. From an entropic standpoint, the spontaneous appearance of chipped inscriptions in a rock (involving perhaps $10^{23}$ atoms) is a greater fluctuation than having a few neurons inside someone’s head “accidentally” direct him to chip away at a stone. This fluctuation is smaller if the writer is part of a civilization in which rock chipping is already a form of recorded language. Since this has happened it is not too severe a demand on the satisfying of the boundary conditions.

Or one could go further and imagine circumstances in which those neurons respond to events in a larger society, one where predictions about the long range future may be common. Again, we have instances in our own history. On Earth however such inscriptions are so common (with competing claims of authenticity) that the signal to noise ratio may be too small for reliable communication. Perhaps the criterion will involve finding the cult that manages to send its message to the stars.\(^7\)

Assuming signaling does occur, one can ask who started it, Alice or Bob? There may be no answer—there are boundary conditions and there is a solution. This lack of traditional causality is reminiscent of Heinlein’s, “All you zombies—” [17]. The life of a single individual is a loop in which he/she is both his/her parents. What fixes this creature’s genome?

Finally, confronted by the problems of signaling, there is another possibility. The

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\(^7\) I here allude to a common feature of religious texts, predictions about the future. I comment that the notion of a deity introduces conflicts with the thermodynamic arrow of time. An example is in the Kuzari [16]. The apparent contradiction between free will and an omniscient deity is there addressed by the assertion that the knowledge of things to come no more determines their outcome than knowledge of things past. In a similar vein, omniscience and entropy are conflicted, since coarse graining—loss of information—is required to define entropy.
requirement placed on the dynamical system—remote boundary conditions plus an inscribed rock—may be too restrictive for any solution to exist. In that case there could very well be opposite arrow systems extant, and one could contemplate communicating with them, but communication in fact does not take place. In mathematical terms, the remote boundary conditions (physically motivated, as in Footnote 6) could have a significant measure for solutions in which simultaneous opposite arrows exist, but if you add the condition that they signal one another, the measure shrinks to zero.\footnote{As discussed elsewhere \cite{3}, the use of probability (measure) for deciding what the universe is doing is justified only by imitation of probability’s use on smaller systems.}

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REFERENCES

2. Mackey, M. C., “Microscopic Dynamics and the Second Law of Thermodynamics,” in \cite{18}, pp. 49–65, Mackey quotes Eddington: “... If your theory is found to be against the second law ... there is nothing for it but to collapse in deepest humiliation.