The time reversal operator \( T \) for unstable particles is defined such that the fact of decay of the particle is not mistaken for a dynamical violation of \( T \) in the decay. This definition is not valid where the medium in which the decay takes place may have its own intrinsic \( T \) violation. Based on this limitation, some effort is made to restrict the conclusions drawn from apparent \( T \) violation in \( K \) meson decay.

In a previous paper [1] (hereafter denoted as I) we defined unstable particles in terms of representations of a system designated the Poincare semigroup \( P' \) which is a semigroup formed from the Poincare group \( P \) but including only those elements of \( P \) whose space-time translation is into the absolute future, or zero. The physical idea is that the formation of an unstable particle is an irreversible process, that for each specimen of a particular unstable particle there is a time before which it simply did not exist, and that therefore there is no reason to assume that indefinite translation back in time is an operation that can be applied to the state of an unstable particle. We then developed the idea that if there is such a thing as an elementary unstable particle, its state can be defined, isolated and propagated forward in time homogeneously. This led to the representation theory of \( P' \) and to the identification of a reasonable candidate for the class of representations corresponding to physical unstable particles. These representations can be formally extended to nonunitary representations of \( P \) and we now mention that, unknown to us, an identification of unstable particles as nonunitary representations of the Poincaré group had been suggested by Zwanziger [2]. While the physical motivations in [1] and [2] differ widely, they do agree on the quantum numbers and some other properties of the unstable particle.

One problem left outstanding in I is the definition of the time reversal operator \( T \) for unstable particles. The addition of \( T \) to the transformations of \( P' \) introduces
operators previously ruled out because of irreversibility. Nevertheless, there are physical situations where the action of $T$ is of paramount interest—$K$ decay, for example—so that an analysis of just what is meant by this operation is necessary. Now while applying $T$ to unstable particles has never before been hindered by the fact that one leaves the Poincaré semigroup, it is nevertheless the case that any definition of $T$ must be subject to the same physical considerations—and problems—which led to the definition of the Poincaré semigroup in the first place.

The purpose of this paper, which may be considered a sequel to I, is to present a time reversal operator for unstable particles and to analyze what is meant by invariance under this operation.

The standard definition [4] of $T$ is illustrated in Fig. 1. The system is described by points $\omega$ in a space $\Omega$. For example, $\Omega$ may be phase space [$\omega = (x, p)$] or a Hilbert space. $T$ is an operator mapping $\Omega$ into itself. For a particular system, $\omega \in \Omega$ describes its state at some time, and $T$ assigns to $\omega$ another point $T(\omega)$ at the same time. Statements about invariance under $T$ involve dynamics, described by an evolution function $U_\tau : \Omega \rightarrow \Omega$, which assigns to each state $\omega$, the state that a system starting in $\omega$ reaches a time $t$ later.

Suppose at time $t = -\tau$ a system is in $\omega$. At $t = 0$, the state is $U_\tau(\omega)$. Now apply $T$ and let the system develop until $t = \tau$. The state is $U_\tau[TU_\tau(\omega)]$. If this state is the image under $T$ of the original state $\omega$, then the system is said to be invariant under $T$. Thus,

$$T(\omega) = U_\tau[TU_\tau(\omega)],$$

then $U_\tau$ is $T$ invariant. (A)

For example, if $\omega = (x, p)$, the system travels some path until it reaches $(x', p')$ at $t = 0$. $T$, to conform to the usual definition, makes of this $(x', -p')$ and this is allowed to propagate an additional time $\tau$. A $T$-invariant system is expected to retrace its original path and return to $x$ but with reversed momentum.

For nonrelativistic quantum mechanics it is a bit more complicated to show how (A) corresponds to Wigner time reversal and we refer the reader to [4] for elaboration.
For unstable particles it is desirable to go beyond (A). If we take as the definition of $T$ reversal of momentum and of spin, and include within $U$, the fact of decay then any unstable particle violates $T$ according to (A). This violation is not real, however, in the sense of not being dynamical and it is clear that to separate real violations from kinematical ones, either the definition of $T$ or the condition (A) must change.

To illustrate our points in what follows, we introduce the example of a particle falling through air and governed by the equation

$$\ddot{y} + ky = g,$$  

(1)

where $y$ is measured downward, $k > 0$ and dots are time derivatives. The solution of this equation for which $y(0) = y'(0) = 0$ is

$$y(t) = \frac{(g/k)t}{1 + (g/k^2)e^{-kt} - 1}. \quad (2)$$

If $T$ involves only the transformation $\dot{y} \rightarrow -\dot{y}$, then by (A) this system is not time reversal invariant (if it were, then we should have $y(-\tau) = y(\tau)$).

What one would like to know about this system is if apart from the dissipative mechanism described by $k$, there is $T$ invariance. The extension of the concept of $T$ invariance that immediately suggests itself is to define $T$ so as to have the additional action $k \rightarrow -k$. The rationale would be that in the time reversed world to which we wish $T$ to bring us, friction acts oppositely and particles acquire velocity at the expense of the thermal excitation of the medium. This seems to be the definition adopted for unstable particles. Specifically, $T$, besides its usual action, reverses the sign of the decay parameter or, for $K$ decay, of the imaginary part of the mass matrix [5, 6].

It is just this definition which, if nature has been sufficiently wily, misses an important possibility. It may noted that the definitions given are not derived in any fundamental way but are rather suggested as natural extensions of the usual $T$. We now give what we consider to be the correct definition of $T$ and then by means of the system of Eq. (1) illustrate its difference from other definitions and its physical significance.

The example of the particle in air suggests that in order to eliminate the kinematical $T$ violation, the source of the viscosity must be acted upon by $T$ also. (This was the purpose of $k \rightarrow -k$, but the cure is incomplete.) When the viscous medium is included within the dynamical system there is no problem in formulating $T$ invariance. What was previously considered the "vacuum" or background is included in the action of $T$—in the usual way—once the microscopic coordinates of the background are identified, and some action of $T$ on them given. Similarly for unstable particles $T$ would act on all decay products switching their momenta and
spins and possibly acting on what we call the vacuum too in case it has some time structure (we shall return to this point).

But now the trouble with taking \( T : k \rightarrow -k \) is obvious. Suppose there is a violation of \( T \) in the equations of motion (or \( U \)) of the air. There would be no reason to believe that with velocities reversed the medium will produce an antifriction effect. That requires very fine tuning indeed and given even a small violation (in either air or particle) one would expect ordinary friction to obtain within a short time. On the other hand, it seems desirable to have the transformation \( k \rightarrow -k \) in cases where the medium is time symmetric but the particle may not be. For this reason we change the definition \( A \).

A state \( \omega \) is given at \( t = -\tau \) which includes reference to the medium. An operation \( T \) on states is given. The state \( \omega \) evolves to \( t = 0 \) by means of \( U_\tau \) to yield \( U_\tau(\omega) \). Thus far we have dealt with an initial value problem. The second part of the definition involves a boundary value problem in which the final state is given. Let \( \nu \) be a state at \( t = 0 \) which evolves (by means of \( U \)) to \( T \omega \) at \( t = \tau \). Then if

\[
U_\tau(\omega) = T \nu \tag{B}
\]

for some\(^2\) such \( \nu \), the system is time reversal invariant.

With this definition we expect to see antifriction from \( t = 0 \) to \( t = \tau \). If the medium is \( T \) invariant then the friction coefficient simply ought to reverse sign (even if the particle motion violates \( T \)), while if the medium is \( T \) violating one still expects antifriction but there may be some change in the absolute value of the coefficient.

The above assertions are difficult to prove, for even if one were given some microscopic (or hydrodynamic) equations of motion of the medium it would not be trivial to extract the existence and value of a friction coefficient. We shall nevertheless attempt to justify the assertions by making use of an opposite running time parameter (this is where many discussions of time reversal start). Let

\[
s = -t.
\]

At \( s = -\tau \) the initial (from the standpoint of \( s \)) state is \( T \omega \). The motion of the medium is assumed \( T \) invariant. Implicit in our earlier statements was the assumption that the state (at \( t = -\tau = -s \)) described a quiescent medium. For a hydrodynamic description of the medium this might mean a fluid at rest; a microscopic description as air molecules would involve some equilibrium state. In any case, the condition of the medium described by the state \( T \omega \) should be likewise quiescent. Either one continues to have a fluid at rest, or else another microscopic state, but one which is also in equilibrium (since the equations of motion are \( T \) invariant, the equilibrium distribution may be expected to be symmetric under \( T \)). Thus at

\(^2\) In general \( \nu \) may not be unique. For example, in the differential equation \( \ddot{y} + k \text{ sign}(\dot{y}) \sqrt{\vert \dot{y} \vert} = 0 \), \( y(t_{\text{final}}) = y(t_{\text{final}}) = 0 \) do not define unique initial conditions.
s = −τ the quiescent medium contains a particle which begins some series of motions. While the detailed motion may be different (for reasons intrinsic to the particle) from that which moved forward in t from \( t = -\tau \), nevertheless, a treatment of interaction of medium and particle should be identical whether one is moving forward in s or t (from the value −τ). This interaction is found to be described by a coefficient of friction which is therefore the same in both cases. (If a coefficient of friction is a poor description and some other description preferable, then this preference will also be the same). The use of a coefficient of friction (or other macroscopic description) is a consequence of both dynamical and statistical assumptions, both of which hold for evolution in the positive s or t directions from −τ.

If the motion of the medium violates \( T \), the underlying statistical assumptions about the quiescent state (which may itself no longer be \( T \) invariant) suggest frictional effects for motion away from the quiescent state (whether forward in \( s \) or in \( t \)), provided the \( T \) violation is not too drastic, but there is now no reason to obtain the same value for the coefficient of friction.

The advantage of this last definition is that if one has theoretical certitude of the \( T \) symmetry of the medium and if he is sure the medium’s interaction with his particle is completely accounted for by a coefficient of friction \( k \), then he can confine his \( T \) operation to the particle alone but with the proviso that \( T \) sends \( k \) to \(-k\).

Such an operation might arise in the following circumstances. Suppose one had two particles falling through the air [as in (1)] but in addition subject to some mutual interaction. The \( T \) invariance of the air is assured and it is that of the particles’ mutual interaction we wish to study. Experimentally we determine \( U \), (or differential equations), which include \( k \). The coefficient \( k \) is considered a property of the particle (in this medium) and subject to the transformation \( T \); in particular, it suffers a change of sign under \( T \). With this \( T \), the definition (A) is restored.

Conversely, barring this certitude concerning the medium, theoretical studies cannot draw conclusions about violation by the particle alone of the medium’s transformation properties have been absorbed into the operation \( k \rightarrow -k \).

Turning now to a proposed definition of \( T \) for states of unstable particles in quantum mechanics we find the same dilemma. The decay of the particle takes place in a “medium” consisting the decay products and the vacuum. Unless we are certain of the \( T \) invariance of this medium, no \textit{a priori} definition of \( T \) on the state of the unstable particle alone can be made nor is any statement about \( T \) violation in the dynamics of the decay possible.

Before examining whether the distinctions we are drawing have any practical or even philosophical interest, we summarize our conclusion for the time reversal operator for unstable particles. Consider the entire physical system—unstable
particle, decay products and vacuum. Suppose the vacuum and decay products (the "medium") are themselves simple enough for an unambiguous definition of $T$ to obtain:

(a) If there is sufficient reason to believe $T$ is not violated in the medium, then $T$ applied to the state of the unstable particle is defined to reverse the sign of its lifetime (in addition, to whatever $T$ does to momenta, etc.).

(b) If the $T$ properties of the medium are not known, then by (B) unless there is some way of separating the unstable particle from this $T$ violating medium no definition of $T$ on the state unstable particle alone can be made. There is no reason to believe that (B) can ever be tested in an experimental situation, since one does not set up final conditions in experiments. (This is in agreement with our point of view in I.) In effect, we have been forced to drop so many operationally useful assumptions that we are finally driven to say that if you want to test time reversal invariance you must set up a time reversed world.

We have gone to some effort in this article to draw fine distinctions. They may be of interest if one is worried about the definition of time reversal for unstable particles, but do they have any relevance for $K$ meson decay? There are two levels at which to pose this question. First, does it matter whether we choose to attribute the observed $T$ violation [7, 8] to the $K$ meson or to the medium? The point is that the violation is observed. Second, granted license to place the guilt on the vacuum, how can this be done to produce the observed violation and no other?

To the first question I think there is a definite answer. It makes a big difference to the theory of weak interactions whether or not there is to be a $T$ violating term present. A $T$ violating vacuum (presuming $\pi - \pi$, etc., interactions are $T$ invariant) would leave us free to employ $T$ invariant weak dynamics.

It is more difficult to make a case for a $T$ violating vacuum that is capable of providing the observed violation. We offer some comments on this point, but emphasize the speculative nature of our remarks.

The failure of $T$ invariance in the observed (generally macroscopic) world is often phrased in terms of an "arrow of time". There is thermodynamic arrow, a cosmological arrow, a $K$-decay arrow, an electrodynamic arrow, and possibly others. There are arguments to support the idea that the thermodynamic (entropy increases) and electrodynamic (retarded Green’s function rather than symmetric combination of retarded and advanced) arrows follow the cosmological one [9, 10, 11]. For example, Hogarth claims that Wheeler’s and Feynman’s arguments [12] for the absorption by the universe of advanced interactions only work for some cosmologies and for some of these only in one direction (Wheeler–Feynman theory could just as well have absorption of retarded waves). This selection of arrows may depend on an arbitrarily small difference seen between looking in the future or the past of an (say)expanding universe.
My point in mentioning this work is to suggest the general idea that although small regions of space-time have almost zero participation in the overall expansion of the universe, local dynamics can nevertheless be strongly affected. This is presumably because of a symmetry breaking effect—it does not take much to establish a direction in time, but once established it becomes an important feature. One need not accept the details of Hogarth's thesis to believe that the choice of direction of time is a symmetry breaking effect. It follows that the gross asymmetries of the universe need not be large on a terrestrial scale in order to produce a vacuum for elementary particle physics which is not invariant under $T$.

How this vacuum asymmetry is supposed to do its job in $K$ decay (and only there) I do not know. Those theories which attribute the $CP$ violation to cosmological origins [13, 15] are examples of what I have in mind (although these authors speak in terms of the preponderance of matter over antimatter rather than an arrow of time) but not necessarily the only examples. From our point of view the mere fact of vacuum asymmetry precludes a definition of time reversal for the $K$ alone, and thus prevents the separation of vacuum violations from dynamical violations.

It may be noted that the usual definitions of $T$ violation in $K$ decay involve relative reality of certain coupling constants; this criterion is in turn derived from various matrix elements. The vacuum in these matrix elements is assumed invariant under $T$ (when deriving the consequences of $T$) and it is here that one may propose to introduce the asymmetry of the vacuum discussed above.

**Acknowledgments**

I would like to thank Professor George Patsakos for many useful and stimulating discussions of time, its arrows, reversals and invariances. I am also grateful to Professor Jerry Franklin for helpful discussions on $K$ decay.

**References**


595/72/2-13