Tachyon Paradoxes*

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It is sometimes inferred from tachyon paradoxes that there is no consistent motion of a system permitting faster than light signals. Using an argument of Wheeler and Feynman, we show how this dilemma can be avoided although in the process there is some modification of the ideas of cause and effect. The resulting point of view is one that has been adopted by science fiction writers concerned with time travel.

Two kinds of uneasiness are generated by tachyon paradoxes. The first is the breakdown in the usual notion of causality. This (in some form) is the problem usually studied most seriously. However, in order to demonstrate the existence of this problem, a common device is the exhibition of a closed causal cycle, i.e., a sequence of events in which the "last" prevents the "first" from occuring. This leads to the second dilemma: What actually happens in these cycles? Does the paradoxical nature of the cycle mean that a mechanics with tachyons is necessarily inconsistent? An affirmative answer to this question should lead to *a priori* rejection of tachyons,[1] while if all that is necessary is re-examination of causality then the idea of the tachyon should be welcome.

I will now discuss Newton's paradox[2] and resolve it—at least as far as the second dilemma is concerned—in exactly the same way that Wheeler and Feynman[3] handled a similar problem 25 yr ago. This will permit me, later in this article, to pose a question about causality and "free will" in a relatively concrete fashion. It will be seen that the results of this discussion will be in general accord with a consensus of science fiction writers who have dealt with this theme.

Figure 1 is the space–time diagram for Newton's paradox. Rockets I and II are variously in motion or at rest (for finite or infinite slope, respectively) and the dotted lines indicate simultaneity. Tachyon signals are sent from A to B and from C to D. A switch S with position X and Y is in rocket I. The paradox is built as follows: Just before time A, S is in position X. Whenever S is in X, the emitter in rocket I emits a type Y signal. B therefore receives a Y signal and induces its apparatus to emit a Y signal at C. At D a Y signal is received. The effect of a Y signal on the apparatus in rocket I is to change S from X to Y. If S is in Y the emitter in rocket I sends out a type X signal at A. This works its way around and on reaching S sends the switch from the Y position to the X position. What happens?

The discussion which follows is essentially the same as that given by Wheeler and Feynman. They were concerned with a similar problem that arose from time-symmetric electrodynamics.

For simplicity, let the X position of S be "on" and the Y position "off." For S "on" a signal is sent from A to B. This causes a signal to reach D which turns the emitter in rocket I off. To state the paradox, we introduce an earlier simultaneous time $E-E'$ at which initial conditions are given. The initial conditions are: At E, S is on. Question: Is there a signal or not?

In Fig. 2 is a plot of the position of S at the time A as a function of the intensity, J, of the signal from C to D. J is the independent variable and the plot takes into account the fact that there is some threshold intensity needed to flip the switch. On the other hand, the position of S determines J since it determines whether or not emission takes place at A. This dependence is indicated in Fig. 3, where "position of S" is the

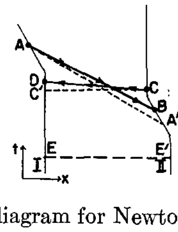

FIG. 1. Space–time diagram for Newton's paradox.

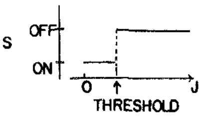

FIG. 2. J is the intensity of the signal from C to D and is considered the independent variable in this plot. Dependent on this is the position of the switch S at the time (event, really) A.

independent variable. The fact that the graphs of Figs. 2 and 3 have no overlap (were they to be superimposed) indicates that there is no one consistent motion. This is another way of stating the paradox.

The way out is to recognize or postulate continuity in nature. If the threshold region of Fig. 2 is magnified sufficiently, it will show a continuous curve from the lower line to the upper one. For these borderline intensities, the switch will be in some intermediate position at the time A. As a corollary, it follows that S *can* be in intermediate positions. The dependence of J on this position will, in general, be very complicated but as above we know (or postulate) that there is some continuous curve connecting the points in Fig. 3. In Fig. 4, the two continuous curves are superimposed. The meeting point (there must be at least one) represents a consistent motion of the system.

In the remainder of this article, I will assume the validity of the foregoing argument. However, in an appendix deficiencies in the argument are discussed.

An observer always at rest and for whom E and E' are simultaneous would see the following: First, a tachyon signal of a very particular strength interacts with the apparatus in rocket II at B (our observer actually sees this as an emission). Next, a tachyon whose intensity falls within the spread in the threshold for flipping S is emitted at C and received at D. This pushes S (from its "on" position) just enough so that when time A is reached the apparatus is in a position to

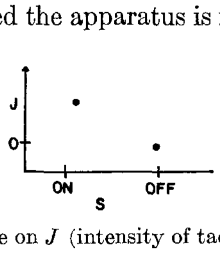

FIG. 3. Dependence on J (intensity of tachyon signal from C to D) on position of the switch S.

receive (observers in rocket I see this as emitting) a signal of just the right intensity to have originated from B.

Thus, a complete cycle of events occurs, each dependent upon the others but not necessarily at increasing times. Such a cycle is another example of generalized causality as considered by Csonka[4] and Newton.[5] Any of the events at A, B, C, or D or anywhere in between may be considered a cause or an effect. In fact, if the world is as here suggested, it would seem that these terms are without fundamental meaning. This would leave open the problem of explaining their excellent approximate validity, perhaps in physical, perhaps in psychological terms.

In any case, whatever the implications of this thought experiment for causality, the motion itself is not inconsistent. I emphasize that whether or not there is a switch with only two positions is

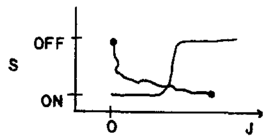

FIG. 4. In the curve passing through (0, on), J is the independent variable. For the other curve, position of S is independent.

not a technological question as technology can only make the curve rise more steeply. In the context of contemporary physics, it is reasonable to assume that if one starts from continuous microscopic coordinates and involves any finite number of particles interacting through continuous, bounded forces, the final system will possess the properties required for this discussion.

To feel the full crunch of the paradox, it is necessary to set up initial conditions at $E-E'$. Although there is no guarantee that a tachyon from a later time will not reach back and influence the initial state (as the signal from A affects D) one may assume that in the actual motion this does not take place and the evolution is as described.

For the time-symmetric electrodynamics[3] the problem of establishing initial conditions is more severe (and hence the paradox less compelling) since the interaction always reaches both forward and back.

One should not be too quick to accept the argument offered in this note since he may find the implications for what may loosely be called "free will" distasteful. Suppose the switch S is a man. The incoming tachyon signal at D lights a bulb in view of this man. He has been instructed to send a tachyon signal at time A (he is "on") unless he sees the bulb light. At time D the bulb may light very dimly and he may not be quite sure what to do. He thinks it over, vacillating, until at a time A he decides to send the signal and presses the button. However, he is slightly late and does not manage to get off a full-strength signal so that the bulb at D is confusingly dim. Did this man really "decide" on his course of action?

The point in this discussion is that history is a set of world lines essentially frozen into space time. While subjectively we may feel strongly that our actions are determined only by our backward light cone, this may not always be the case—as for the human switch above. Presumably this would have implications for the philosophy of science since the assumption that we have control over events[2] enters in the formulation of scientific law.

Science fiction writers have confronted some of these problems in stories involving time travel. Heinlein,[6] in the story "'All You Zombies'" enriches the basic time travel paradox with a few additional flourishes but handles history as we do, as an "already" accomplished affair. By going back and forth in time and by means of an unusual medical phenomenon he manages to have an individual be both his own parents. All the time lines mesh, however, and it all works out. For example, the father goes back in time to impregnate the mother only after she has had a sex changing operation. In addition, he takes the female infant (himself) back about 20 yr to allow time for growing up. (But I still do not know what determined his/her/its genes.)

There are other stories of this kind,[7–9] and as might be expected, some very strange situations develop. In "Behold the Man" by M. Moorcock,[7] Jesus turns out to be a twentieth century time traveler who is so interested in the crucifixion that he goes back to watch—and gets crucified. Generally speaking, in all these stories[10] the idea is the same: A self-consistent sequence of events occurs but it is no longer possible to distinguish cause from effect. Some efforts are also made to understand the psychological state of the protagonist and his cumulative conscious processes are studied and often made the basis for the sequencing of the story. In any case, I feel these stories are useful for developing an intuition which is not bound to the idea of cause and effect.

APPENDIX

In reality the parameter spaces of the switch and the signal are not one dimensional. Let M be the space of values for the switch position and N the space in which the parameters describing the signal take their values. Then Fig. 2 is a schematic graph of $f:N\rightarrow M$ and Fig. 3 describes $g:M\rightarrow N$. Statements about continuity are of course meaningless unless both N and M are topological spaces. What we need is that the map $u=g\circ f:N\rightarrow N$ have a fixed point. A reasonable description of N is that it be part of a function space with, say, an L_2 norm. If we knew that N was convex and compact, we could invoke the Schauder–Tychonoff fixed point theorem.[11] However, this kind of statement can only be made through a model of the physical apparatus. Another way to handle the problem might be as Wheeler and Feynman did which is to assume that somewhere along the line a piece of the apparatus depends on only one parameter (the velocity of the shutter at 5:59 pm in their example) so that the fixed point property becomes obvious. Since we are dealing with problems of principle and since by straddling the threshold—as a self-consistent solution is expected to do—we often encounter pathological situations, it is not clear that the Wheeler-Feynman assumption ought to be made.

In any case, the purpose of this note is to show that the paradox does not prove that tachyon mechanics is inconsistent. The burden of proof rests on one who wishes to make an *a priori* rejection of tachyons. He would have to exhibit a function u on a space N which does not have a fixed point.

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[1] Or at least to the idea that one can send tachyon signals. See F. A. E. Pirani, Phys. Rev. **D1**, 3224 (1970)

[2] R. G. Newton, Science **167**, 1579 (1970).

[3] J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. **21**, 425 (1949).

[4] P. Csonka, Phys. Rev. **180**, 1266 (1969), particularly Sec. 5. Also Phys. Today **23**, 15 (May 1970).

[5] R. G. Newton, Phys. Rev. **162**, 1274 (1967).

[6] R. A. Heinlein, " 'All You Zombies' " in *The Best from Fantasy and Science Fiction: 9th Series*, edited by R. P. Mills (Ace, New York, 1958).

[7] M. Moorcock, "Behold the Man" reprinted in *Nebula Award Stories Three*, edited by R. Zelazny (Doubleday, New York, 1968).

[8] R. A. Heinlein, "By His Bootstraps" reprinted in *Spectrum*, edited by K. Amis and R. Conquest (Berkeley, New York, 1961), first appeared in *Astounding Science Fiction* (Street and Smith, New York, 1941).

[9] B. W. Aldiss, "Man in His Time" in *Nebula Award Stories—Number Two*, edited by B. Aldiss and H. Harrison (Doubleday, New York, 1967).

[10] But there are dissenting opinions. Two stories which skirt the paradoxes are: R. Silverberg, *The Time Hoppers* (Doubleday, New York, 1967); and R. S. Scott, "Who Needs Insurance," in *Nebula Award Stories—Number Two*, edited by B. Aldiss and H. Harrison (Doubleday, New York, 1967).

[11] R. E. Edwards, *Functional Analysis* (Holt, New York, 1965), p. 161.

Normal-Mode Frequencies of Finite One-Dimensional Lattices with Single Mass Defect: Exact Solutions

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The normal-mode longitudinal-vibration frequencies are determined for a finite one-dimensional lattice of mass particles joined by identical massless ideal springs, when at most one of the particle masses differs from all the rest. Results are derived for all boundary conditions of common physical interest by a method that is both elementary and straightforward. The results are expressed through solutions of trigonometric equations readily solvable with arbitrary precision by numerical methods. Considerable information about the solutions is presented by means of simple graphical representations.

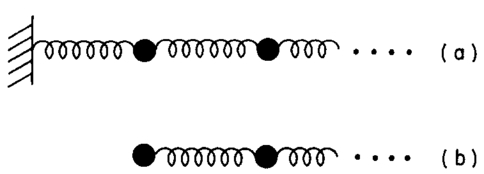

FIG. 1. Boundary conditions (shown at left-hand ends only): (a) "fixed end," (b) "free end."

I. INTRODUCTION

Consider a finite one-dimensional mass-point lattice in which nearest neighbors are joined by massless ideal springs (a "linear chain"); the particle at either end may be otherwise unconstrained ("free end") or may be connected by a spring to a fixed outside point in the line of the lattice ("fixed end"). (The two cases are illustrated in Fig. 1.) The springs, including any used to effect a fixed end, are identical; each is subject to both compression and extension, with the same Hooke's constant for either. The major purpose of this paper is to determine, for the various end (or "boundary") conditions indicated, the normal-mode longitudinal-vibration frequencies of the chain when all the particle masses, *with one possible exception*, are equal.

My search of the relevant literature has revealed no solution of the single-defect problem for the boundary conditions indicated in the preceding paragraph. In a 1967 paper P. Dean determines the longitudinal normal-mode frequencies for the chain with a single mass defect when the system is subject to the essentially artificial "cyclic," or "periodic," boundary condition only.[1] This condition—whereby in effect one supposes that each end particle serves as a nearest neighbor to the other—is treated in Sec. X by the method introduced below; the resulting normal-mode frequency spectrum coincides with Dean's result.