Ohm's and Kirchhoff's Laws: Circuits Containing Dependent Sources

Introduction

Each circuit in this set of problems:

1. consists of a small number of circuit elements.
2. consists of a dependent source, some resistors, voltage sources, current sources, and either a voltmeter or ammeter.
3. has constant-valued inputs.

The inputs to these circuits are the voltages of the voltage sources and/or the currents of the current sources. The output of each circuit is either a voltage measured by a voltmeter or a current measured by an ammeter. All of the inputs have constant values. Consequently, the outputs also have constant values.

Use Ohm's and Kirchhoff's laws to solve these problems.

Ohm's law is discussed in Section 2.6 of *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda. Kirchhoff's laws are discussed in Section 3.3. Independent voltage and current sources are described in Section 2.6. Dependent voltage and current sources are described in Section 2.8. Voltmeters and ammeters are described in Section 2.7.

Worked Examples

Example 1:
Consider the circuit shown in Figure 1. Find the value of the gain, *A*, of the CCCS (**C**urrent **C**ontrolled **C**urrent **S**ource).

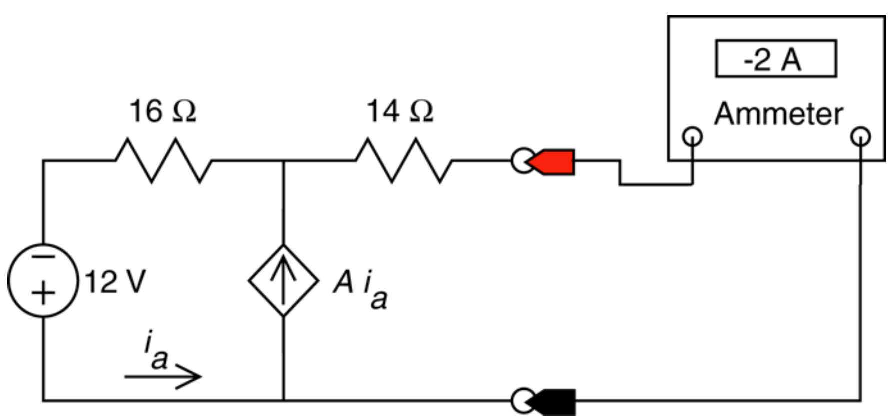

Figure 1 The circuit considered in Example 1.

Solution: Figure 2 shows the circuit from Figure 1 after replacing the ammeter by an equivalent short circuit and labeling the current measured by the ammeter. Also, the voltage across the short circuit is labeled.

Figure 3 shows the circuit after labeling the element currents and voltages as follows. Apply KCL at the bottom node of the voltage source and again at the top node of the voltage source to see that the current in the voltage source and the 16 Ω resistor are both equal to i_a, as labeled in Figure 3. Apply KCL at the right node of the 14 Ω resistor to see that the current in the 14 Ω resistor is –2 A, as labeled in Figure 3. Apply Ohm's law to see that the voltage across the 14 Ω resistor is –28 V, as labeled in Figure 3. (Since Ohm's law was used, the reference direction of the voltage across the 14 Ω resistor was chosen so that the reference directions of the voltage and current of the 14 Ω resistor adhere to the passive convention.) The voltage across the 16 Ω resistor is labeled as v_b. (In anticipation of using Ohm's law, the reference direction of the voltage across the 16 Ω resistor was chosen so that the reference directions of the voltage and current of the 16 Ω resistor adhere to the passive convention.)

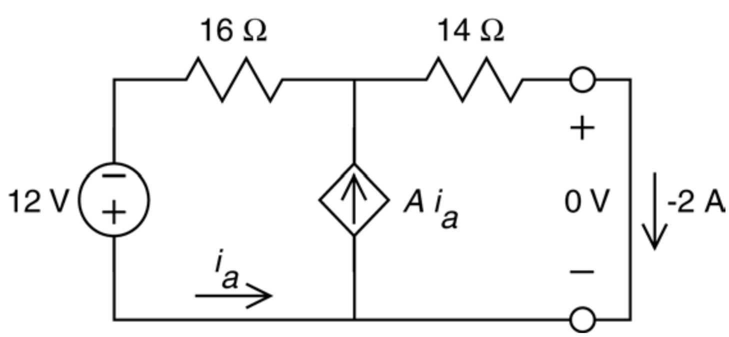

Figure 2 The circuit from Figure 1 after replacing the ammeter by a short circuit.

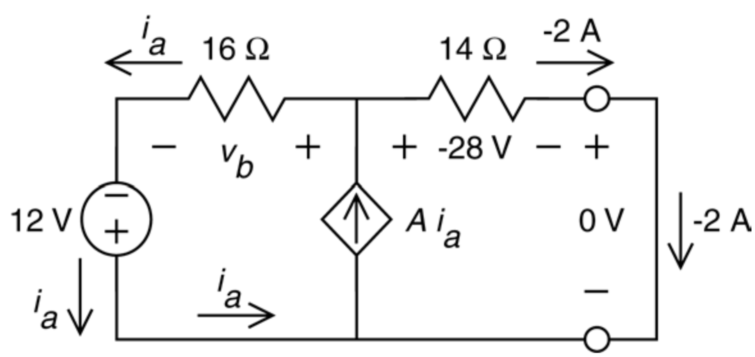

Figure 3 The circuit from Figure 2 after labeling the element currents and voltages.

Apply Ohm's law to the 16 Ω resistor to get

$$v_b = 16\, i_a$$

Apply KVL to the loop consisting of the voltage source, the two resistors and the short circuit that replaced to ammeter to get

$$-v_b + (-28) + 0 + 12 = 0 \quad \Rightarrow \quad v_b = -16 \text{ V} \quad \Rightarrow \quad i_a = -1 \text{ A}$$

Apply KCL at the top node of the dependent current source to get

$$A\, i_a = i_a + (-2) \quad \Rightarrow \quad A(-1) = -1 - 2 \quad \Rightarrow \quad A = 3 \text{ A/A}$$

Example 2:
Consider the circuit shown in Figure 4. Find the value voltage measured by the voltmeter.

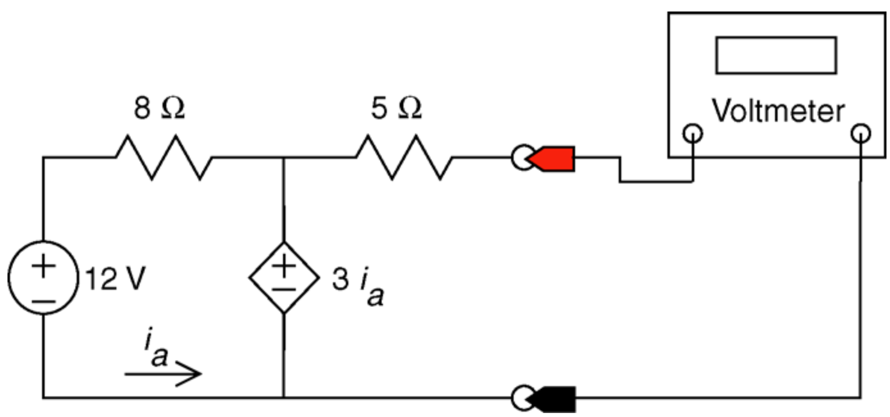

Figure 4 The circuit considered in Example 2.

Solution: See Example 3.3-4 in *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda.

Example 3:

Consider the circuit shown in Figure 5. Find the value of gain, A, of the CCVS. (**C**urrent **C**ontrolled **V**oltage **S**ource).

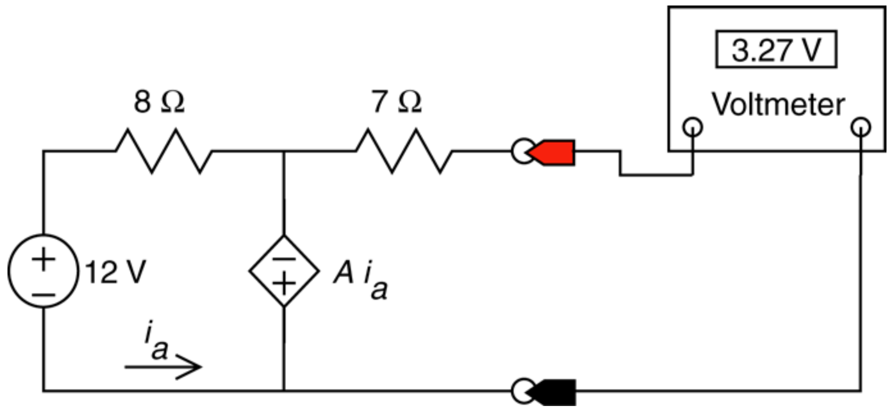

Figure 5 The circuit considered in Example 3.

Solution: Figure 6 shows the circuit from Figure 5 after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter.

Figure 7 shows the circuit after labeling the element currents and voltages as follows. Apply KCL at the bottom node of the voltage source and again at the top node of the voltage source to see that the current in the voltage source and the 8 Ω resistor are both equal to i_a, as labeled in Figure 7. Apply KCL at the right node of the 7 Ω resistor to see that the current in the 7 Ω resistor is 0 A, as labeled in Figure 7. Apply Ohm's law to see that the voltage across the 7 Ω resistor is 0 V, as labeled in Figure 7. The voltage across the 8 Ω resistor is labeled as v_b. (In anticipation of using Ohm's law, the reference direction of the voltage across the 8 Ω resistor was chosen so that the reference directions of the voltage and current of the 8 Ω resistor adhere to the passive convention.)

Apply Ohm's law to the 8 Ω resistor to get

$$v_b = 8\, i_a$$

Apply KVL to the loop consisting of the voltage source, the two resistors and the open circuit that replaced to voltmeter to get

$$-v_b + 0 + 3.27 - 12 = 0 \quad \Rightarrow \quad v_b = -8.73 \text{ V} \quad \Rightarrow \quad i_a = -1.09 \text{ A}$$

Apply KVL to the mesh consisting of the voltage source, the 8 Ω resistor and the dependent voltage source to get

$$-v_b - A\, i_a - 12 = 0 \quad \Rightarrow \quad -(-8.73) - 12 = A(-1.09) \quad \Rightarrow \quad A = 3 \text{ V/A}$$

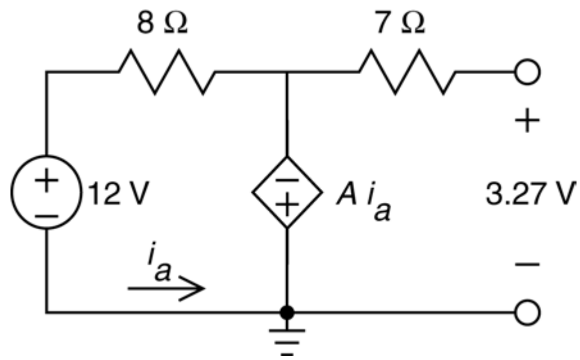

Figure 6 The circuit from Figure 5 after replacing the voltmeter by an open circuit.

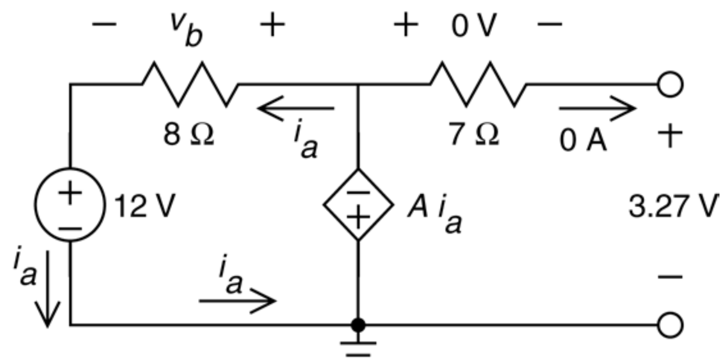

Figure 7 The circuit from Figure 6 after labeling the element currents and voltages.

Example 4:

Consider the circuit shown in Figure 8. Find the value of gain, A, of the VCCS (**V**oltage **C**ontrolled **C**urrent **S**ource).

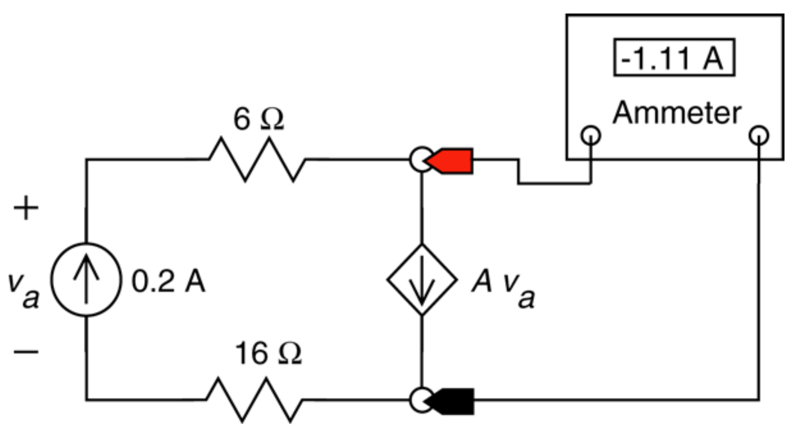

Figure 8 The circuit considered in Example 4.

Solution: Figure 9 shows the circuit from Figure 8 after replacing the ammeter by an equivalent short circuit and labeling the current measured by the ammeter. Also, the voltage across the short circuit is labeled.

Figure 10 shows the circuit after labeling the element currents and voltages as follows. Apply KCL at the bottom node of the current source and again at the top node of the current source to see that the current in the 6 Ω resistor and the 16 Ω resistor are both equal to 0.2 A as labeled in Figure 10. Apply Ohm's law to see that the voltage across the 6 Ω resistor is 1.2 V and that the voltage across the 16 Ω resistor is 3.2 V, as labeled in Figure 10. (Since Ohm's law was used, the reference direction of the voltage across each resistor was chosen so that the reference directions of the voltage and current of each resistor adhere to the passive convention.)

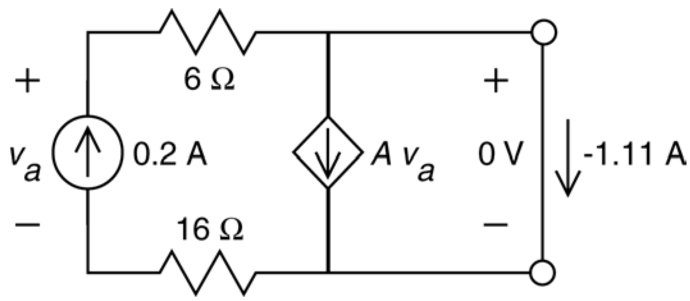

Figure 9 The circuit from Figure 8 after replacing the ammeter by a short circuit.

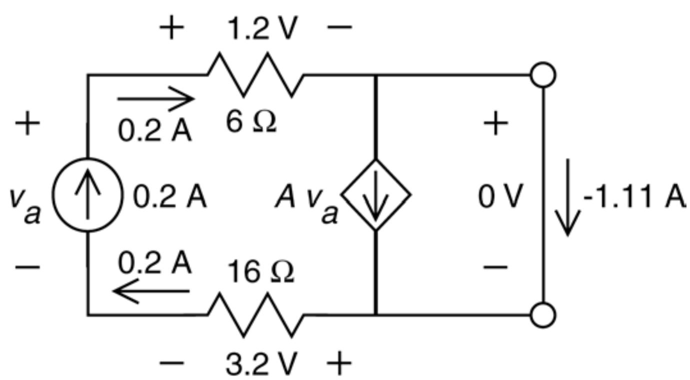

Figure 10 The circuit from Figure 9 after labeling the element currents and voltages.

Apply KVL to the loop consisting of the voltage source, the two resistors and the short circuit that replaced to ammeter to get

$$1.2 + 0 + 3.2 - v_a = 0 \quad \Rightarrow \quad v_a = 4.4 \text{ V}$$

Apply KCL at top node of the dependent current source to get

$$0.2 = A\,v_a + (-1.11) \quad \Rightarrow \quad A(4.4) = 1.31 \quad \Rightarrow \quad A = 0.3 \text{ A/V}$$

Example 5:
Consider the circuit shown in Figure 11. Find the value of the gain, *A*, of the VCVS (**V**oltage **C**ontrolled **V**oltage **S**ource).

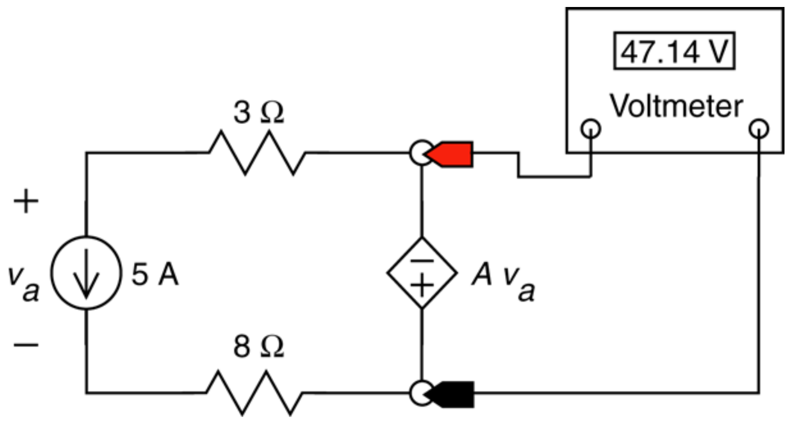

Figure 11 The circuit considered in Example 5.

Solution: Figure 12 shows the circuit from Figure 11 after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter.

Figure shows the circuit after labeling the element currents and voltages as follows. Apply KCL at the bottom node of the current source and again at the top node of the current source to see that the current in the 3 Ω resistor and the 8 Ω resistor are both equal to 5 A as labeled in Figure 13. Apply Ohm's law to see that the voltage across the 3 Ω resistor is 15 V and that the voltage across the 8 Ω resistor is 40 V, as labeled in Figure12. (Since Ohm's law was used, the reference direction of the voltage across each resistor was chosen so that the reference directions of the voltage and current of each resistor adhere to the passive convention.)

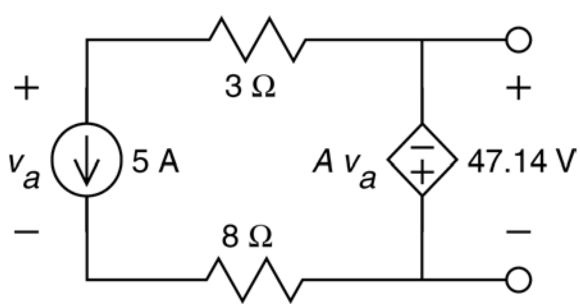

Figure 12 The circuit from Figure 11 after replacing the voltmeter by an open circuit.

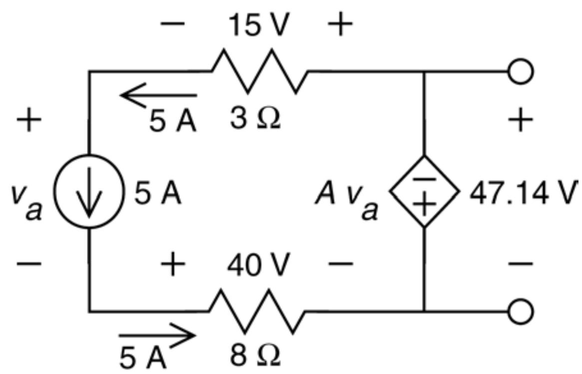

Figure 13 The circuit from Figure 12 after labeling the element currents and voltages.

Apply KVL to the loop consisting of the voltage source, the two resistors and the open circuit that replaced to voltmeter to get

$$-15+47.14-40-v_a=0 \quad \Rightarrow \quad v_a=-7.86 \text{ V}$$

Apply KVL to the loop consisting of the dependent voltage source and the open circuit that replaced to voltmeter to get

$$+47.14+A\,v_a=0 \quad \Rightarrow \quad A\,v_a=-47.14 \quad \Rightarrow \quad A\left(-7.86\right)=-47.14 \quad \Rightarrow \quad A=6 \text{ V/V}$$

Example 6:
Consider the circuit shown in Figure 14. Find the value current measured by the ammeter.

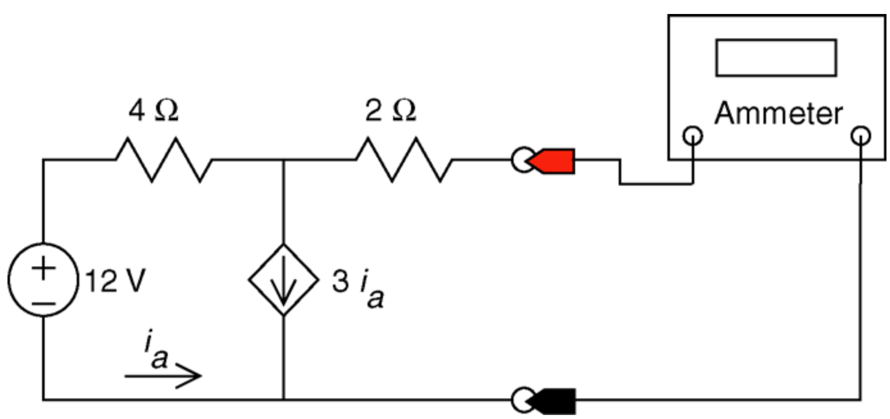

Figure 14 The circuit considered in Example 6.

Solution: See Example 3.3-4 in *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda