Capacitors and Inductors with Current and Voltage Represented Graphically

Introduction

Each of the circuits in this problem set consists of a single capacitor or inductor and a single independent source. When the independent source is a current source, the current source current is equal to the current in the capacitor or inductor. Similarly, when the independent source is a voltage source, the voltage source voltage is equal to the voltage across the capacitor or inductor.

These problems can be solved using the element equations for the capacitor and inductor. (An "element equation" is the equation that describes the relationship between the element voltage and element current.) The element equations of capacitors and inductors involve derivatives and integrals. Since the voltages and currents in these problems are described graphically, it's useful to interpret derivatives as slopes and integrals as areas.

Capacitors and inductors are described in Sections 7.3 and 7.6 of *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda.

Worked Examples

Example 1:
Consider the circuit shown in Figure 1. Find the value of the capacitance *C*.

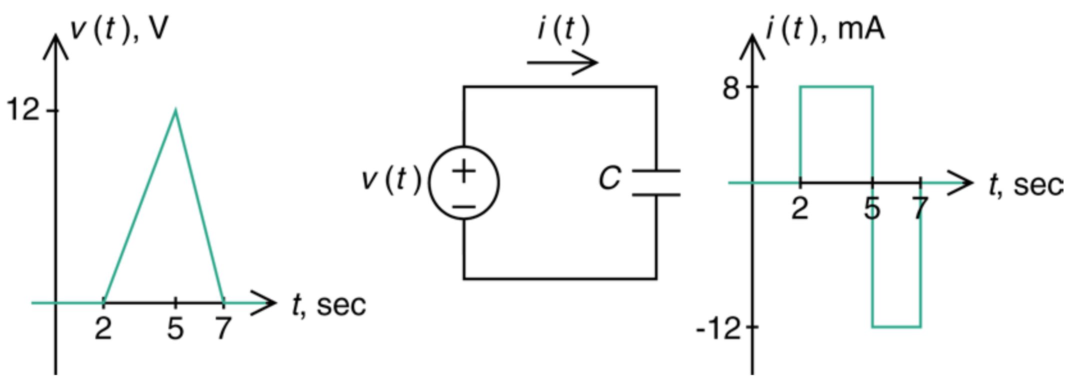

Figure 1 The circuit considered in Example 1.

Solution: The current and voltage of the capacitor are related by

$$i(t) = C\frac{d}{dt}v(t) \tag{1}$$

Since $i(t)$ and $v(t)$ are represented graphically, by plots rather than equations, it is useful to interpret Equation 1 as

$$\text{the value of } i(t) = C \times \text{ the slope of } v(t)$$

Pick a time when both the value of $i(t)$ and the slope $v(t)$ are easily determined. For example, at time t = 3 seconds, $i(3)$ = 8 mA = 0.008 A and

$$\frac{d}{dt}v(3) = \frac{0-12}{2-5} = 4\,\frac{\text{V}}{\text{s}}$$

(The notation $\frac{d}{dt}v(3)$ indicates that the derivative $\frac{d}{dt}v(t)$ is evaluated at time t = 3 s.) Using Equation 1 at time t = 3 s gives

$$0.008 = C(4) \quad \Rightarrow \quad C = 0.002\,\frac{\text{A}}{\text{V/s}} = 0.002 \text{ F} = 2 \text{ mF}$$

Example 2:
Consider the circuit shown in Figure 2. Find the value of the inductor voltage, $v(t)$, at time t = 3 ms. (That is, find $v(0.003)$.)

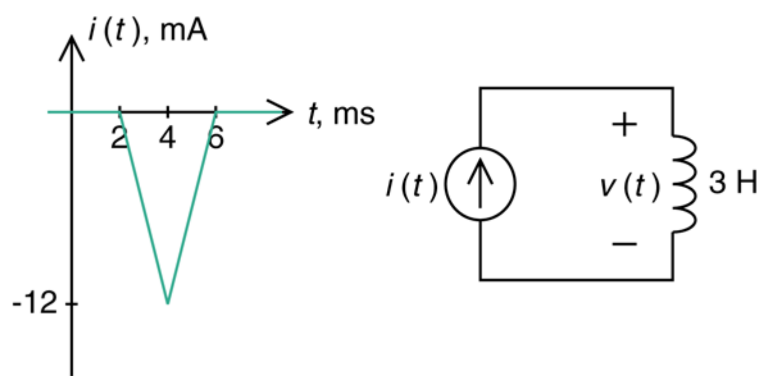

Figure 2 The circuit considered in Example 2.

Solution: The voltage and current of the inductor are related by

$$v(t) = L\frac{d}{dt}i(t) \tag{2}$$

Since $i(t)$ and $v(t)$ are represented graphically, by plots rather than equations, it is useful to interpret Equation 2 as

$$\text{the value of } v(t) = L \times \text{ the slope of } i(t)$$

The value of the voltage at time t = 3 ms = 0.003 s is required. We need to determine the slope of i(t) at time t = 0.003 s. That slope is

$$\frac{d}{dt}i(0.003) = \frac{0-(-0.012)}{0.002-0.004} = -6\,\frac{\text{A}}{\text{s}}$$

(The notation $\frac{d}{dt}i(0.003)$ indicates that the derivative $\frac{d}{dt}i(t)$ is evaluated at time t = 0.003 s.)

The inductance of the inductor in Figure 2 is L = 3 H. Using Equation 2 gives

$$v(0.003) = 3(-6) = -18\ \text{H}\frac{\text{A}}{\text{s}} = -18\ \text{V}$$

Example 3:
Consider the circuit shown in Figure 3. Find the value of the inductance L.

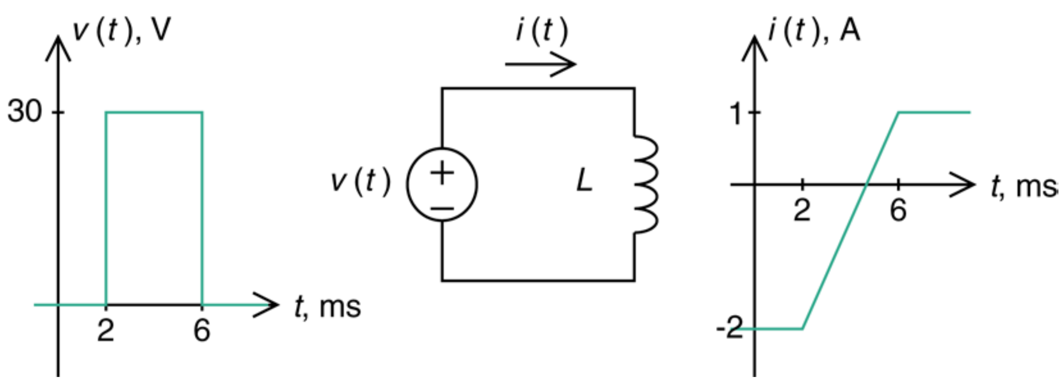

Figure 3 The circuit considered in Example 3.

Solution: The current and voltage of the inductor are related by

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0) \tag{3}$$

or

$$i(t) - i(t_0) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau \tag{4}$$

Since $i(t)$ and $v(t)$ are represented graphically, by plots rather than equations, it is useful to interpret Equation 4 using

$i(t)$ - $i(t_0)$ = the difference between the values of voltage at times t and t_0

and

$\int_{t_0}^{t} v(\tau)\,d\tau$ = the area under the plot of $v(t)$ versus t, for times between t and t_0

Pick convenient values t and t_0, for example, t_0 = 2 ms and t = 6 ms. Then

$$i(t) - i(t_0) = 1 - (-2) = 3 \text{ A}$$

and

$$\int_{t_0}^{t} v(\tau)\,d\tau = \int_{0.002}^{0.006} 30\,d\tau = (30)\,(0.006\text{-}0.002) = 0.12 \text{ V-s}$$

Using Equation 4 gives

$$3 = \frac{1}{L}(0.12) \quad \Rightarrow \quad L = 0.040\ \frac{\text{V-s}}{\text{A}} = 0.040 \text{ H} = 40 \text{ mH}$$

Example 4:
Consider the circuit shown in Figure 4. Find the value of the inductor current, $i(t)$, at time t = 3 ms. (That is, find $i(0.003)$.)

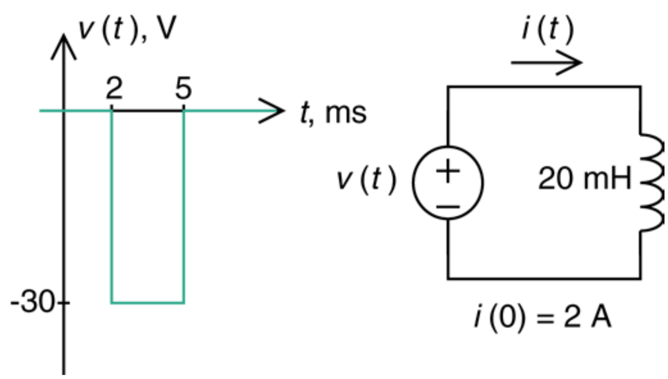

Figure 4 The circuit considered in Example 4.

Solution: The current and voltage of the inductor are related by

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0) \tag{5}$$

Since $i(t)$ and $v(t)$ are represented graphically, by plots rather than equations, it is useful to interpret Equation 5 using

$$\int_{t_0}^{t} v(\tau)\, d\tau = \text{the area under the plot of } v(t) \text{ versus } t\text{, for times between } t \text{ and } t_0$$

The values t and t_0 are specified to be $t_0 = 0$ ms and $t = 3$ ms. Then $i(t_0) = 2$ A and

$$\int_{t_0}^{t} v(\tau)\, d\tau = \int_{0}^{0.002} 0\, d\tau + \int_{0.002}^{0.003} -30\, d\tau = 0 + (-30)\,(0.003-0.002) = -0.03 \text{ V-s}$$

At time $t_0 = 0$ ms the inductor current is give to be $i(t_0) = 2$ A . The inductance of the inductor in Figure 4 is $L = 20$ mH = 0.02 H. Using Equation 5 gives

$$i(0.003) = \frac{1}{0.02}(-0.03) + 2 = 0.5 \text{ A}$$

Example 5:
Consider the circuit shown in Figure 5. Find the value of the capacitance *C*.

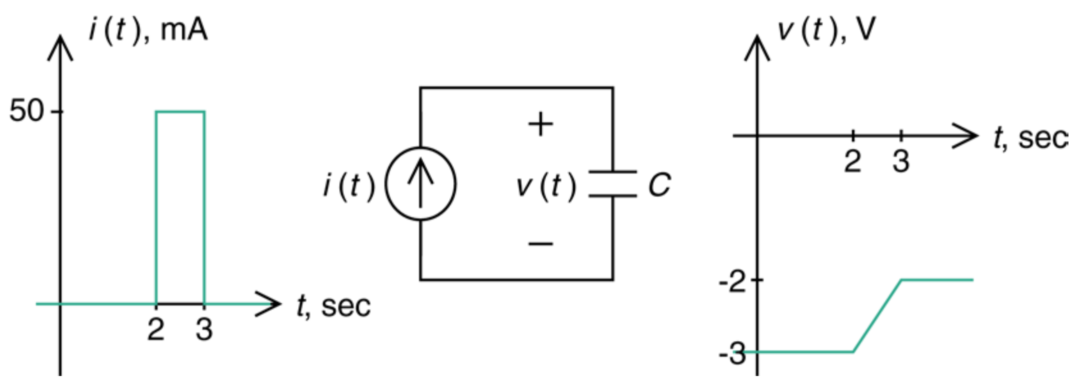

Figure 5 The circuit considered in Example 5.

Solution: The current and voltage of the capacitor are related by

$$i(t) = C\frac{d}{dt}v(t) \tag{6}$$

Since $i(t)$ and $v(t)$ are represented graphically, by plots rather than equations, it is useful to interpret Equation 6 as

$$\text{the value of } i(t) = C \times \text{ the slope of } v(t)$$

Pick a time when both the value of $i(t)$ and the slope $v(t)$ are easily determined. For example, at time $t = 2.5$ seconds, $i(2.5) = 50$ mA = 0.050 A and

$$\frac{d}{dt}v(2.5)=\frac{-3-(-2)}{2-3}=1\ \frac{\text{V}}{\text{s}}$$

(The notation $\frac{d}{dt}v(2.5)$ indicates that the derivative $\frac{d}{dt}v(t)$ is evaluated at time t = 2.5 s.)

Using Equation 6 at time t = 2.5 s gives

$$0.050=C\,(1)\quad\Rightarrow\quad C=0.050\ \frac{\text{A}}{\text{V/s}}=0.050\ \text{F}=50\ \text{mF}$$