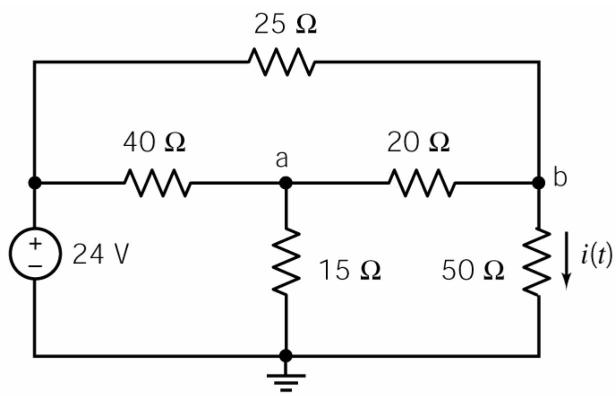
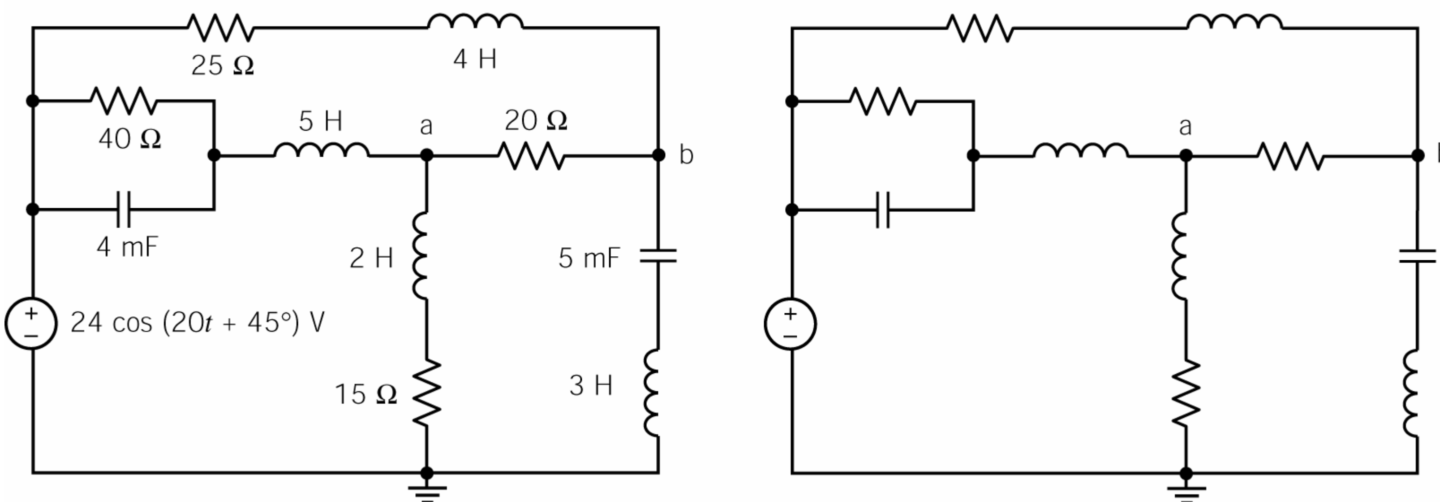


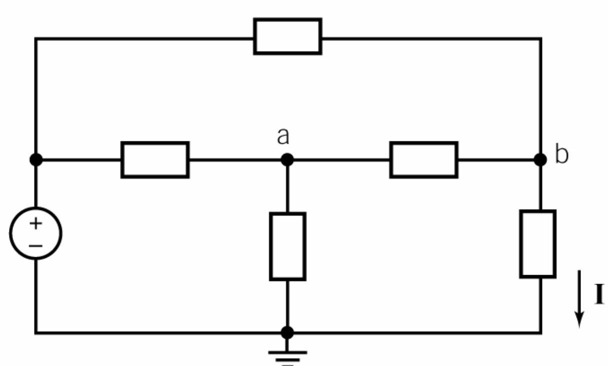
(a)



time domain

frequency domain

(b)



(c)

ELEMENT	TIME DOMAIN	FREQUENCY DOMAIN
Voltage source	$v(t) = B\cos(\omega t + \phi)$	$\mathbf{V}(\omega) = Be^{j\phi}$
Resistor	$i(t)$, R, $+\ v(t)\ -$; $v(t) = R\,i(t)$	$\mathbf{I}(\omega)$, R, $+\ \mathbf{V}(\omega)\ -$; $\mathbf{V}(\omega) = R\,\mathbf{I}(\omega)$
Capacitor	$i(t)$, C, $+\ v(t)\ -$; $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$	$\mathbf{I}(\omega)$, $\frac{1}{j\omega C}$, $+\ \mathbf{V}(\omega)\ -$; $\mathbf{V}(\omega) = \frac{1}{j\omega C}\,\mathbf{I}(\omega)$
Inductor	$i(t)$, L, $+\ v(t)\ -$; $v(t) = L\,\frac{d}{dt}$	$\mathbf{I}(\omega)$, $j\omega L$, $+\ \mathbf{V}(\omega)\ -$; $\mathbf{V}(\omega) = j\omega L\,\mathbf{I}(\omega)$

Complex Numbers

$$A\angle\theta = a + jb$$

where $A\angle\theta$ is the complex number in polar form and $a + jb$ is the complex number in rectangular form. The conversion from polar form to rectangular form and visa versa is described by

$$a = \text{the real part of } A\angle\theta = A\cos(\theta)$$

$$b = \text{the imaginary part of } A\angle\theta = A\sin(\theta)$$

$$A = \text{the magnitude of } a + jb = \sqrt{a^2 + b^2}$$

$$\theta = \text{the angle of } a + jb = \begin{cases} \tan^{-1}\left(\dfrac{b}{a}\right) & \text{when} \quad a > 0 \\ 180° - \tan^{-1}\left(\dfrac{b}{-a}\right) & \text{when} \quad a < 0 \end{cases}$$

Solution

(a) The node equations are

$$\frac{24-v_a}{40}=\frac{v_a-v_b}{20}+\frac{v_a}{15}$$

$$\frac{24-v_b}{25}+\frac{v_a-v_b}{20}=\frac{v_b}{50}$$

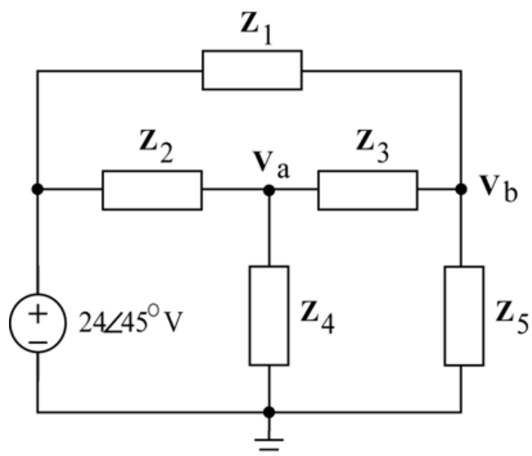
or

$$\begin{bmatrix} \frac{1}{40}+\frac{1}{20}+\frac{1}{15} & -\frac{1}{20} \\ -\frac{1}{20} & \frac{1}{25}+\frac{1}{20}+\frac{1}{50} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \end{bmatrix}=\begin{bmatrix} \frac{24}{40} \\ \frac{24}{25} \end{bmatrix}$$

Solving using MATLAB gives

$$v_a=8.713 \text{ V} \quad \text{and} \quad v_b=12.69 \text{ V}$$

(b, c) Use phasors and impedances to represent the circuit in the frequency domain as



where

$$\mathbf{Z}_1=25+j(20)4=25+j80=83.82\angle 72.7° \ \Omega$$

$$\mathbf{Z}_2=\left(40 \,||\, \frac{1}{j(20)(0.004)}\right)+j(20)5=3.56+j88.6=88.68\angle 87.7° \ \Omega$$

$$\mathbf{Z}_3=20 \ \Omega$$

$$\mathbf{Z}_4=15+j(20)2=15+j40=42.72\angle 69.4°$$

$$\mathbf{Z}_5=j(20)3+\frac{1}{j(20)(0.005)}=j50=50\angle 90° \ \Omega$$

The node equations are

$$\frac{24\angle 45° - \mathbf{V}_a}{\mathbf{Z}_2} = \frac{\mathbf{V}_a}{\mathbf{Z}_4} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3}$$

$$\frac{24\angle 45° - \mathbf{V}_b}{\mathbf{Z}_1} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3} = \frac{\mathbf{V}_b}{\mathbf{Z}_5}$$

$$\begin{bmatrix} \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4} & -\frac{1}{\mathbf{Z}_3} \\ -\frac{1}{\mathbf{Z}_3} & \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_5} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} \frac{24\angle 45°}{\mathbf{Z}_2} \\ \frac{24\angle 45°}{\mathbf{Z}_1} \end{bmatrix}$$

Solving using MATLAB gives

$$\mathbf{V}_a = 7.89\angle 44.0°$$

$$\mathbf{V}_b = 8.45\angle 45.1°$$

so

$$v_a(t) = 7.89\cos(20t + 44°) \text{ V}$$

$$v_a(t) = 8.45\cos(20t + 45.1) \text{ V}$$