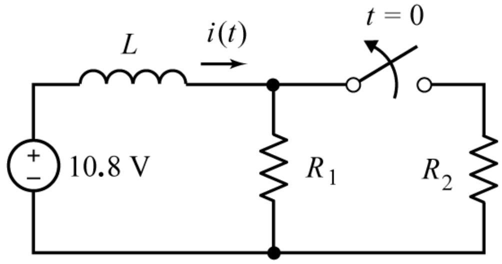
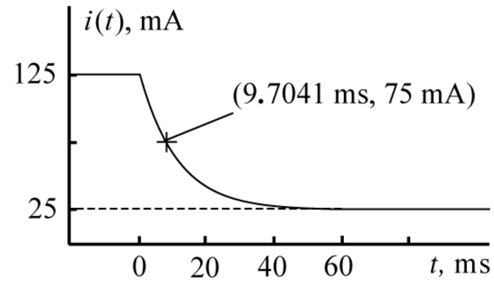
EE221 1st Midterm Exam - Spring 2010

Name ___________________________________ Student #__________

1.





a. Determine the values of R_1 and R_2.

R_1 = ____432___ Ω and R_2 = ___108_____ Ω.

b. Determine the value of the time constant, τ, of the this circuit after the switch opens.

τ = ____14___ ms.

a. $0.025 = \frac{10.8}{R_1} \quad \Rightarrow \quad R_1 = 432\ \Omega$ and $0.125 = \frac{10.8}{R_1 \parallel R_2} \quad \Rightarrow \quad 432 \parallel R_2 = 86.4\ \Omega \quad \Rightarrow \quad R_2 = 108\ \Omega$

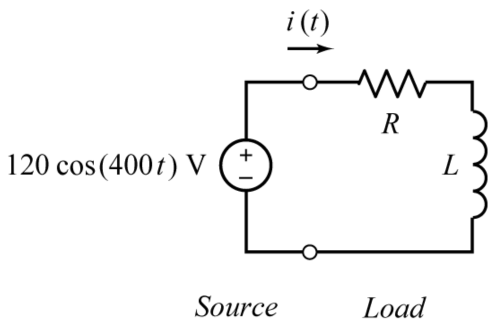
b. After the switch closes, the plot can be represented by the equation $i(t) = 25 + 100\,e^{-at}$ mA so

$$75 = 25 + 100\,e^{-a(0.0097041)} \quad \Rightarrow \quad a(0.0097041) = -\ln\left(\frac{75-25}{100}\right) \quad \Rightarrow \quad a = 71.4286\ \frac{1}{\text{s}}$$

$$\tau = \frac{1}{a} = 0.0140\ \text{s} = 14\ \text{ms}$$

Also, after the switch closes, the Thevenin resistance of the part of the circuit connected to the inductor is $R_1 = 432\ \Omega$. Consequently, the inductance is $L = \tau\,R_1 = 0.014(432) = 6.048\ \text{H}$.

2. An AC source is connected to a load:



a) Suppose that the voltage source supplies

$$\mathbf{S} = 11.686\angle 35.75° = 9.4837 + j\,6.8282 \text{ VA}$$

And the current is represented as

$$i(t) = B\cos(400t + \theta) \text{ mA}$$

Determine values of B and θ.

$B =$ ___194.8____mA and $\theta =$ ___–35.75___°

b) Suppose **instead** that

$$i(t) = 274.54\cos(400t - 29.6°) \text{ mA}$$

Determine values of the resistance and inductance.

$R =$ ____380___Ω and $L =$ ___540____mH

c) Suppose **instead** that $i(t) = 332.8\cos(400t - 33.7°)$ mA. Determine the values of the real and reactive powers supplied by the source to the load.

$P =$ ____16.61__ W and $Q =$ ___11.08___ VAR

d) Suppose **instead** that the voltage source supplies 20.43 W at a power factor of 0.8423 lagging. Determine the values of the apparent and reactive powers supplied by the source to the load.

$|\mathbf{S}| =$ __24.255__ VA and $Q =$ __13.074__ VAR

Solution:

a)

$$\mathbf{I} = \left(\frac{2\mathbf{S}}{\mathbf{V}}\right)^* = \left(\frac{2(11.686\angle 35.75°)}{120\angle 0°}\right)^* = 0.19477\angle -35.75° \text{ A}$$

b)

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle 0°}{0.27454\angle -29.6°} = 437.095\angle 29.6° = 420 + j\,90.04 = 380 + j(400)(0.5400)\ \Omega$$

c)

$$\mathbf{S} = \frac{(120\angle 0°)(0.3328\angle -33.7°)^*}{2} = 19.968\angle 33.7° = 16.61 + j11.08 \text{ VA}$$

d)

$$20.43 = P = |\mathbf{S}|\,pf = |\mathbf{S}|0.8423 \quad \Rightarrow \quad |\mathbf{S}| = 24.255 \text{ VA}$$

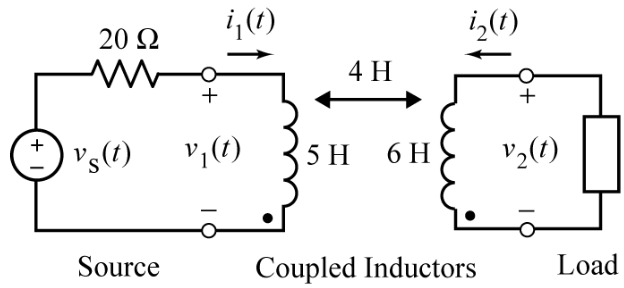
$$Q = |\mathbf{S}|\sin\left(\cos^{-1}(pf)\right) = 24.255\sin\left(\cos^{-1}(0.8423)\right) = 13.074 \text{ VAR}$$

3. The input to this circuit shown is

$$v_s(t) = 12\cos(8t) \ \text{V}$$

The impedance of the load is $12 + j\,16\ \Omega$.

Noticing that $i_1(t)$ and $i_2(t)$ are mesh currents, we can represent this circuit by the mesh equations

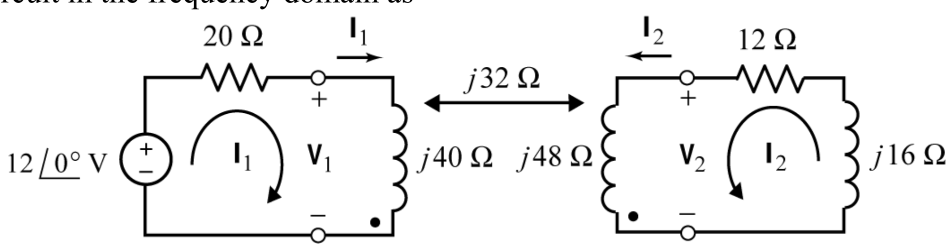


$$\begin{bmatrix} 20 + j\,a & j\,b \\ j\,c & 12 + j\,d \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 12\angle 0° \\ 0 \end{bmatrix}$$

where *a*, *b*, *c*, and *d* are real constants. Determine the values of *a*, *b*, *c*, and *d.*

$a =$ ____40____ Ω, $b =$ ___32____ Ω, $c =$ ___32____ Ω, and $d =$ ____64___ Ω

Represent the circuit in the frequency domain as



The coil voltages are given by

$$\mathbf{V}_1 = j\,40\,\mathbf{I}_1 + j\,32\,\mathbf{I}_2 \quad \text{and} \quad \mathbf{V}_2 = j\,48\,\mathbf{I}_2 + j\,32\,\mathbf{I}_1$$

Using KVL

$$20\,\mathbf{I}_1 + \mathbf{V}_1 - 12\angle 0° = 0 \quad \text{and} \quad 12\,\mathbf{I}_2 + j16\,\mathbf{I}_2 + \mathbf{V}_2 = 0$$

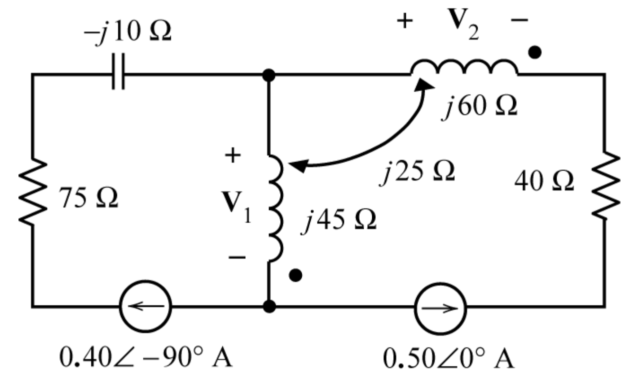
Substituting the coil voltages:

$$20\,\mathbf{I}_1 + j\,40\,\mathbf{I}_1 + j\,32\,\mathbf{I}_2 = 12\angle 0°$$

$$12\,\mathbf{I}_2 + j16\,\mathbf{I}_2 + j\,48\,\mathbf{I}_2 + j\,32\,\mathbf{I}_1 = 0$$

Solving gives $\quad \mathbf{I}_1 = 0.3575\angle -47°\ \text{A} \quad \text{and} \quad \mathbf{I}_2 = 0.1757\angle 143.6°\ \text{A}$

4. Here is an AC circuit represented in the frequency domain. This circuit contains a pair of coupled coils.



Represent the coil voltages as $\mathbf{V}_1 = A\angle\theta$ V and $\mathbf{V}_2 = B\angle\varphi$ V. Determine the values of the amplitude and angle of $\mathbf{V}_1$.

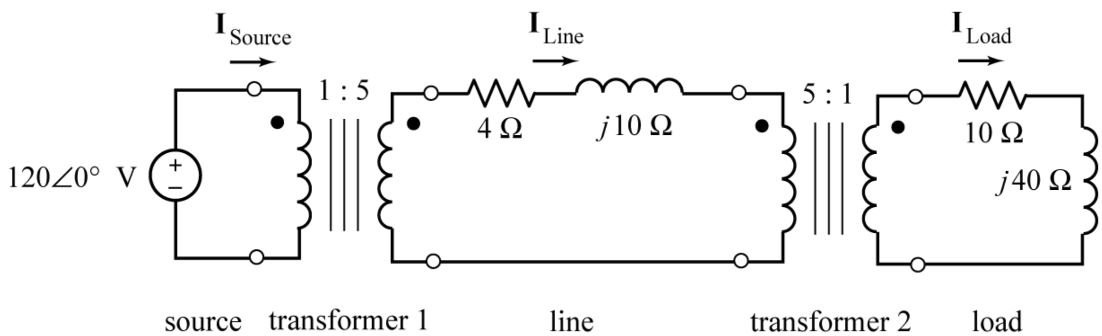
$A =$ ___20.6___ V, $\theta =$ ___29.1___ °

Solution:

$$\mathbf{V}_1 = j45(0.5 - j0.4) + j25(-0.5) = 18 + j10 = 20.59\angle 29.1° \text{ V},$$

$$\mathbf{V}_2 = j60(-0.5) + j25(0.5 - j0.4) = 10 - j17.5 = 20.16\angle -60.26° \text{ V}$$

5. Consider this circuit, represented in the frequency domain.



Given that the line current is $\mathbf{I}_{\text{Line}} = 0.5761\angle -75.88°$ A, determine P_{Source}, the average power supplied by the source, P_{Line}, the average power delivered to the line and P_{Load}, the average power delivered to the load.

$P_{\text{Source}} =$ ___42.15___ W, $P_{\text{Line}} =$ ___0.6638___ W and $P_{\text{Load}} =$ ___41.49___W

(***Hint:*** Use conservation of (average) power to check your answers.)

$$\mathbf{I}_{\text{Source}} = \left(\frac{5}{1}\right)\mathbf{I}_{\text{Line}} = 2.8805\angle -75.88° \text{ A} \quad \text{so} \quad P_{\text{Source}} = \frac{(120)(2.8805)}{2}\cos\left(0 - (-75.88°)\right) = 42.16 \text{ W}$$

$$P_{\text{Line}} = \frac{0.5761^2}{2}(4) = 0.6638 \text{ W}$$

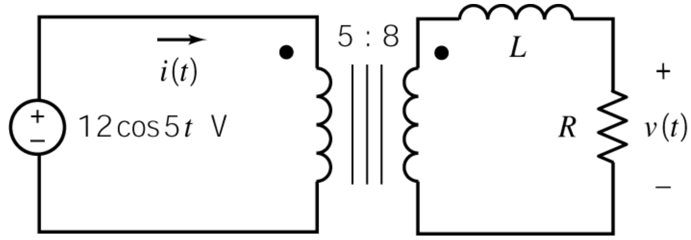
$$\mathbf{I}_{\text{Load}} = \left(\frac{5}{1}\right)\mathbf{I}_{\text{Line}} = 2.8805\angle -75.88° \text{ A} \quad \text{so} \quad P_{\text{Load}} = \frac{2.8805^2}{2}(10) = 41.49 \text{ W}$$

6. In this circuit

$$i(t) = 0.135\cos(5t - 81.16°) \text{ A}$$

and

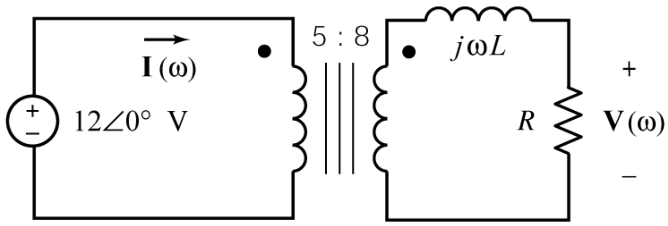
$$v(t) = 2.951\cos(5t - 81.16°) \text{ V}$$



Determine the resistance and inductance values:

$R =$ ____35____ Ω and $L =$ ____45____ H

Solution:
Represent the circuit in the frequency domain as



The impedance seen by the voltage source is

$$\left(\frac{5}{8}\right)^2 (R + j5L) = \frac{12\angle 0°}{\mathbf{I}} = \frac{12\angle 0°}{0.135\angle -81.16°} = 88.889\angle 81.16°$$

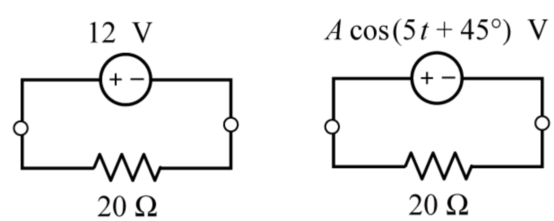
so

$$R + j5L = 227.556\angle 81.16° = 34.970 + j224.853 = 34.970 + j(5)44.971$$

Consequently $L = 44.971 \cong 45$ H and $R = 34.970 \cong 35$ Ω.

Notice also that
$$R = \frac{\mathbf{V}(\omega)}{\frac{5}{8}\mathbf{I}(\omega)} = \frac{2.951\angle -81.16°}{\left(\frac{5}{8}\right)0.135\angle -81.16°} = 34.975 \cong 35\ \Omega$$

7. The sources in these two circuits supply equal values of average power, P_{ave}. Determine the values of A, the amplitude of the sinusoidal voltage, and P_{ave}.



$A =$ __16.97___ V and $P_{\text{ave}} =$ ____7.2___ W

$$\frac{A}{\sqrt{2}} = 12 \quad \Rightarrow \quad A = 16.97 \simeq 17 \text{ W and } P_{\text{ave}} = 12\left(\frac{12}{20}\right) = 7.2 \text{ W}.$$