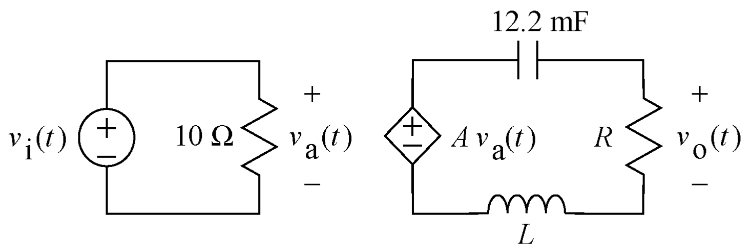
# Additional Practice Problems for EE221 Final Exam

**1.** The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage $v_o(t)$. The step response is $v_o(t) = 6e^{-4t}\sin(5t)u(t)$.




Determine the values of the gain, *A*, of the VCVS, the resistance, *R*, and the inductance, *L*.

*A* = ______3.75______ V/V, *R* = _____16_____ Ω and *L* = ______2______ H .

Equating the transfer function of the circuit to Laplace transform of the given step response yields:

$$\frac{\frac{AR}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{6(5)s}{(s+4)^2 + 25} = \frac{30s}{s^2 + 8s + 41}$$

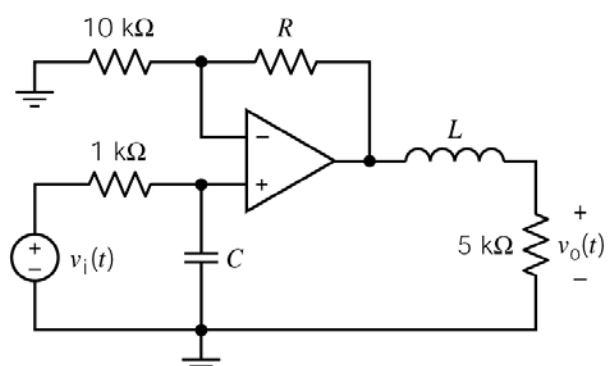
Equating coefficients:

$$\frac{1}{LC} = \frac{1}{L(0.0122)} = 41 \quad \Rightarrow \quad L = 2 \text{ H},$$

$$\frac{R}{L} = \frac{R}{2} = 8 \quad \Rightarrow \quad R = 16\ \Omega \qquad \text{and} \qquad \frac{AR}{L} = \frac{A16}{2} = 30 \quad \Rightarrow \quad A = 3.75 \text{ V/V}$$

**2.** The input to this circuit is the voltage source voltage, $v_i(t)$. The output is the voltage, $v_o(t)$. The transfer function of this circuit is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{15\times10^6}{(s+2000)(s+5000)}$$



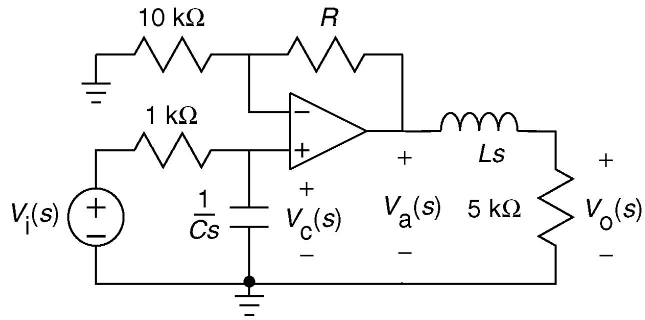

Determine the values of *R*, *L* and *C*:

*R* = ___5___kΩ, *L* = ___1___H and *C* = ___0.5___**μF**.

or

*R* = ___5___kΩ, *L* = __2.5__H and *C* = ___0.2___**μF**.

The transfer function can also be calculated form the circuit itself. The circuit can be represented in the frequency domain as




We can save ourselves some work be noticing that the 10000 ohm resistor, the resistor labeled R and the op amp comprise a non-inverting amplifier. Thus

$$V_a(s) = \left(1 + \frac{R}{10000}\right) V_c(s)$$

Now, writing node equations,

$$\frac{V_c(s) - V_i(s)}{1000} + CsV_c(s) = 0 \quad \text{and} \quad \frac{V_o(s) - V_a(s)}{Ls} + \frac{V_o(s)}{5000} = 0$$

Solving these node equations gives

$$H(s) = \frac{\frac{1}{1000C}\left(1 + \frac{R}{10000}\right)\frac{5000}{L}}{\left(s + \frac{1}{1000C}\right)\left(s + \frac{5000}{L}\right)}$$

Comparing these two equations for the transfer function gives

$$\left(s + \frac{1}{1000C}\right) = (s + 2000) \quad \text{or} \quad \left(s + \frac{1}{1000C}\right) = (s + 5000)$$

$$\left(s + \frac{5000}{L}\right) = (s + 2000) \quad \text{or} \quad \left(s + \frac{5000}{L}\right) = (s + 5000)$$

$$\frac{1}{1000C}\left(1 + \frac{R}{10000}\right)\frac{5000}{L} = 15 \times 10^6$$

The solution isn't unique, but there are only two possibilities. One of these possibilities is

$$\left(s+\frac{1}{1000C}\right)=\left(s+2000\right) \quad \Rightarrow \quad C=0.5\,\mu\text{F}$$

$$\left(s+\frac{5000}{L}\right)=\left(s+5000\right) \quad \Rightarrow \quad L=1\text{ H}$$

$$\frac{1}{1000\left(0.5\times10^{6}\right)}\left(1+\frac{R}{10000}\right)\frac{5000}{1}=15\times10^{6} \quad \Rightarrow \quad R=5\text{ k}\Omega$$

The other is

$$\left(s+\frac{1}{1000C}\right)=\left(s+5000\right) \quad \Rightarrow \quad C=0.2\,\mu\text{F}$$

$$\left(s+\frac{5000}{L}\right)=\left(s+2000\right) \quad \Rightarrow \quad L=2.5\text{ H}$$

$$\frac{1}{1000\left(0.2\times10^{6}\right)}\left(1+\frac{R}{10000}\right)\frac{5000}{2.5}=15\times10^{6} \quad \Rightarrow \quad R=5\text{ k}\Omega$$

**3.** The transfer function of a circuit is $H(s)=\dfrac{12}{s^2+8s+16}$. The step response of this circuit is: $\textit{step response}=\left[b-e^{-at}\left(c+d\,t\right)\right]u(t)$. Determine the values of the constant coefficients *a*, *b*, *c* and *d*:

*a* = ___4__1/s, *b* = ___0.75___V, *c* = ___0.75___V and *d* = ___3___V.

The Laplace transform of the step response is:

$$\frac{H(s)}{s}=\frac{12}{s\left(s^2+8s+16\right)}=\frac{12}{s\left(s+4\right)^2}=\frac{\frac{3}{4}}{s}+\frac{-3}{\left(s+4\right)^2}+\frac{k}{s+4}$$

The constant *k* is evaluated by multiplying both sides of the last equation by $s\left(s+4\right)^2$.

$$12=\frac{3}{4}\left(s+4\right)^2-3s+ks\left(s+4\right)=\left(\frac{3}{4}+k\right)s^2+\left(3+4k\right)s+12 \quad \Rightarrow \quad k=-\frac{3}{4}$$

The step response is

$$\mathfrak{L}^{-1}\left[\frac{H(s)}{s}\right]=\left(\frac{3}{4}-e^{-4t}\left(3t+\frac{3}{4}\right)\right)u(t)\ \text{ V}$$

**4.** The transfer function of a circuit is $H(s)=\dfrac{80\,s}{s^2+8\,s+25}$. The step response of this circuit is: $\textit{step response}=\left[b\,e^{-at}\sin\left(c\,t\right)\right]u(t)$. Determine the values of the constant coefficients *a*, *b*, *c* and *d*:

*a* = ___4___1/s, *b* = ___26.67___V, and *c* = ___3___V.

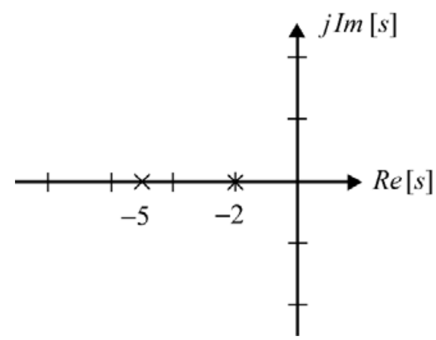
The step response is given by

$$v_o(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{80\,s}{s\left(s^2+8s+25\right)}\right] = \mathcal{L}^{-1}\left[\frac{80}{s^2+8s+25}\right] = \mathcal{L}^{-1}\left[\frac{80}{3}\times\frac{3}{(s+4)^2+3^2}\right]$$

$$= \frac{80}{3}e^{-4t}\sin(3t)u(t) \text{ V}$$

**5.** The input to a linear circuit is the voltage, $v_i$. The output is the voltage, $v_o$. The transfer function of the circuit is

$$H(s) = \frac{V_o(s)}{V_i(s)}$$




The poles and zeros of $H(s)$ are shown on this pole-zero diagram. (There are no zeros.) The dc gain of the circuit is

$$\mathbf{H}(0) = 5$$

The step response of the circuit is $v_o(t) = \left(a + b\,e^{-5t} - c\,e^{-2t}\right)u(t)$ V. Determine the values of the constants *a*, *b* and *c*.

*a* = _____5____ V, *b* = ___10/3__ V and *c* = ___25/3__ V.

The transfer function of the circuit is

$$H(s) = \frac{a}{(s+2)(s+5)}$$

where *a* is a constant to be determined. The circuit is stable because all its poles lie in the right half of the *s*-plane. Consequently,

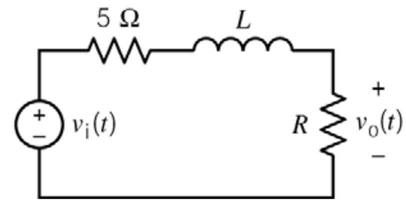
$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{a}{(2+j\omega)(5+j\omega)} = \frac{\frac{a}{10}}{\left(1+j\frac{\omega}{2}\right)\left(1+j\frac{\omega}{5}\right)}$$

At dc ($\omega = 0$) $\quad 5 = \mathbf{H}(0) = \frac{a}{10} \Rightarrow a = 50$

The step response is given by

$$v_o(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{50}{s(s+2)(s+5)}\right] = \mathcal{L}^{-1}\left[\frac{5}{s} + \frac{\frac{10}{3}}{s+5} - \frac{\frac{25}{3}}{s+2}\right] = \left(5 + \frac{10}{3}e^{-5t} - \frac{25}{3}e^{-2t}\right)u(t) \text{ V}$$

**6**. The input to a circuit is the voltage source voltage, $v_i$. The step response of the circuit is




$$v_o(t) = \frac{3}{4}\left(1 - e^{-100t}\right)u(t) \text{ V}$$

Determine the value of the inductance, *L*, and of the resistance, *R*

*R* = ___15___ Ω and *L* = ___0.2___ H.

From the step response: $$\frac{H(s)}{s} = \mathcal{L}\left[\frac{3}{4}\left(1 - e^{-100t}\right)u(t)\right] = \frac{75}{s(s+100)}$$

From the circuit $$H(s) = \frac{R}{R+5+Ls} \Rightarrow \frac{H(s)}{s} = \frac{\frac{R}{L}}{s\left(s + \frac{R+5}{L}\right)}$$

Comparing gives

$$\left.\begin{array}{l} \frac{R}{L} = 75 \\ \frac{R+5}{L} = 100 \end{array}\right\} \Rightarrow \begin{array}{l} R = 15\ \Omega \\ L = 0.2\ \text{H} \end{array}$$

**7.** The input to a circuit is the voltage source voltage, $v_i$. The step response of the circuit is

$$v_o(t) = 5\left(1 - (1+2t)e^{-2t}\right)u(t) \text{ V}$$

When the input is $$v_i(t) = 5\cos(2t + 45°) \text{ V}$$

the steady-state response is $$v_i(t) = A\cos(2t + \theta) \text{ V}$$

Determine the values of *A* and *θ*.

*A* =___12.5__ V and *θ* = ___–45___°.

The transfer function of this circuit is given by

$$\frac{H(s)}{s} = \mathcal{L}\left[\left(5 - 5e^{-2t}(1+2t)\right)u(t)\right] = \frac{5}{s} + \frac{-5}{s+2} + \frac{-10}{(s+2)^2} = \frac{20}{(s+2)^2} \Rightarrow H(s) = \frac{20}{(s+2)^2}$$

This transfer function is stable so we can determine the network function as

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \left.\frac{20}{(s+2)^2}\right|_{s=j\omega} = \frac{20}{(2+j\omega)^2}$$

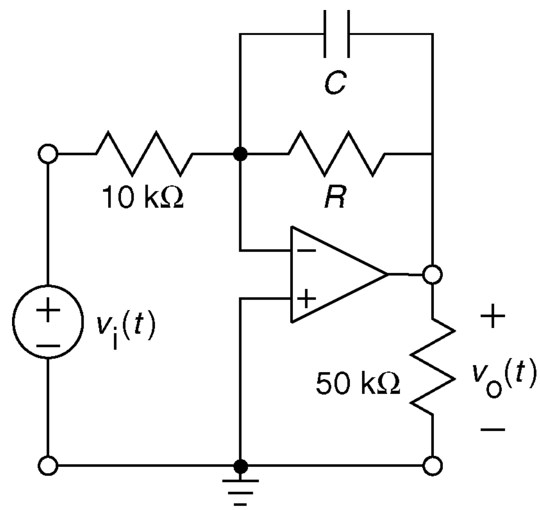
The phasor of the output is

$$\mathbf{V}_o(\omega) = \frac{20}{(2+j2)^2}(5\angle 45°) = \frac{20}{\left(2\sqrt{2}\angle 45°\right)^2}(5\angle 45°) = 12.5\angle -45° \text{ V}$$

The steady-state response is

$$v_o(t) = 12.5\cos(2t - 45°) \text{ V}$$

**8.** The input to a circuit is the voltage $v_i(t)$. The output is the voltage $v_o(t)$.




When the input is:

$$v_i(t) = 2 + 4\cos(100t) + 5\cos(200t + 45°) \quad \text{V}$$

the corresponding output is:

$$v_o(t) = -5 + 7.071\cos(100t + 135°) + c_2\cos(200t + \theta_2) \quad \text{V}$$

Determine the value of $R$, $C$, $c_2$, and $\theta_2$:

$R$ = ___25___ kΩ , $C$ = ___0.4___ μF, $c_2$ = __5.59__V and $\theta_2$ = __161.6__°

The network function of the circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)} = -\frac{\dfrac{R}{1+j\omega RC}}{10^4} = -\frac{\dfrac{R}{10^4}}{1+j\omega RC}$$

At dc:

$$\frac{-5}{2} = -\frac{R}{10^4} \quad \Rightarrow \quad R = 25 \text{ k}\Omega$$

At ω = 100 rad/s

$$-\frac{2.5}{1+j(100)(25\times 10^3)C} = \frac{7.071\angle 135°}{4\angle 0°}$$

$$180 - \tan^{-1}\left((100)(25\times 10^3)C\right) = 135° \quad \Rightarrow \quad C = \frac{\tan(45°)}{(100)(25\times 10^3)} = 0.4\ \mu\text{F}$$

Finally, at ω = 200 rad/s

$$-\frac{2.5}{1+j(200)(25\times 10^3)\left(0.4\times 10^{-6}\right)} = -\frac{2.5}{1+j2} = 1.118\angle 116.6°$$

so

$$c_2 = (1.118)(5) = 5.59 \text{ and } \theta_2 = 45° + 116.6° = 161.6°$$

**9.** The transfer function of a circuit is $H(s) = \dfrac{20}{s+8}$. When the input to this circuit is sinusoidal, the output is also sinusoidal. Let $\omega_1$ be the frequency at which the output sinusoid is twice as large as the input sinusoid and let $\omega_2$ be the frequency at which output sinusoid is delayed by one tenth period with respect to the input sinusoid. Determine the values of $\omega_1$ and $\omega_2$.

$\omega_1$ = ______6_____ rad/s and $\omega_2$ = ____5.8123____ rad/s

The circuit is stable so $\mathbf{H}(\omega) = H(s)\big|_{s \leftarrow j\omega} = \dfrac{20}{8 + j\omega}$.

The gain is 2 at the frequency $\omega_1$ so $2 = \dfrac{20}{\sqrt{8^2 + \omega_1^{\,2}}}$ and $\omega_1 = \sqrt{\left(\dfrac{20}{2}\right)^2 - 8^2} = 6$ rad/s.

When the frequency is $\omega_2$, the period is $\dfrac{2\pi}{\omega_2}$. Also a delay $t_o$ corresponds to a phase shift $-\omega_2 t_o$.

In this case, $t_0 = 0.1\left(\dfrac{2\pi}{\omega_2}\right)$ so the phase shift is -0.2 $\pi$. Then $-0.2\pi = -\tan^{-1}\left(\dfrac{\omega_2}{8}\right)$ so

$\omega_2 = 8\tan(0.2\pi) = 5.8123$ rad/s.