Example:
Consider the circuit shown in Figure 1. Find the value of the gain, $A$, of the VCCS.

Figure 1 The circuit considered in this example.

Solution: Figure 2 shows the circuit from Figure 1 after replacing the ammeter by an equivalent short circuit and labeling the current measured by the ammeter. We will analyze this circuit by writing and solving node equations. Figure 3 shows the circuit after selecting a reference node and numbering the other nodes. Let $v_1$, $v_2$ and $v_3$ denote the node voltages at nodes 1, 2 and 3, respectively.

Figure 2 The circuit from Figure 1 after replacing the ammeter by a short circuit.

Figure 3 The circuit from Figure 2 after labeling the nodes.
The voltage of the 36 V voltage source can be expressed in terms of the node voltages as

\[ 36 = v_1 - 0 \quad \Rightarrow \quad v_1 = 36 \text{ V} \]

The controlling voltage of the dependent source, \( v_a \), is the voltage across an open circuit. This voltage can be expressed in terms of the node voltages at the nodes of the open circuit. Hence

\[ v_a = v_2 - 0 = v_2 \]

Node 3 is connected to the reference node by the short circuit that replaced the ammeter. The voltages across a short circuit is 0 V. Consequently

\[ v_3 - 0 = 0 \text{ V} \quad \Rightarrow \quad v_3 = 0 \text{ V} \]

Apply KCL at node 2 to get

\[ \frac{v_1 - v_2}{20} = \frac{v_2 - v_3}{8} \quad \Rightarrow \quad \frac{36 - v_2}{20} = \frac{v_2 - 0}{8} \quad \Rightarrow \quad v_2 = 10.3 \text{ V} \]

Apply KCL at node 3 to get

\[ \frac{v_2 - v_3}{8} = A v_a + (-39.9) \quad \Rightarrow \quad A v_a = 41.2 \text{ V} \]

Finally,

\[ A = \frac{A v_a}{v_a} = \frac{41.2}{10.3} = 4 \text{ A/V} \]