Example:
Consider the circuit shown in Figure 1. The input to the circuit is the voltage of the voltage source, 20 V. The output of this circuit is the voltage across the capacitor, $v_o(t)$. Determine the value of the capacitor voltage 2 seconds after the switch closes.

![Figure 1 The circuit considered in this example.](image)

Solution: Before the switch closes, the circuit will be at steady state. Because the only input to this circuit is the constant voltage of the voltage source, all of the element currents and voltages, including the capacitor voltage, will have constant values. Closing the switch disturbs the circuit by shorting out the 40 $\Omega$ resistor. Eventually the disturbance dies out and the circuit is again at steady state. All the element currents and voltages will again have constant values, but probably different constant values than they had before the switch closed.

During the disturbance the element voltages and currents are not constant. We can find these voltages and currents by analyzing the circuit using Laplace transforms. The Laplace transform model of the capacitor consists of two parts, an impedance and either a series voltage source or a parallel current source. The voltage of the voltage source or the current of the current source depend on the initial condition of the capacitor, i.e. the capacitor voltage at time $t = 0$. Before we can model the capacitor using Laplace transforms, we must determine $v_o(0)$.

Figure 2 shows the circuit before $t = 0$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit. A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the capacitor voltage, $v_o(t)$.

Because the circuit in Figure 2 is at steady state, the value of the capacitor voltage will be constant. This constant is the value of the capacitor voltage just before the switch closes. In the absence of unbounded currents, the voltage of a capacitor must be continuous. The value of the capacitor voltage immediately after the switch closes is equal to the value immediately before the switch closes. This value is the initial condition of the capacitor and has been labeled as $v_o(0)$ in Figure 2. The circuit in Figure 2 is analyzed by writing mesh equations:
The current in the right mesh, $i_2$, is zero because it is equal to the element the current of an open circuit. Letting $i_2 = 0$ in the mesh equations gives

$$40 i_1 + 10 i_1 - 20 = 0 \quad \Rightarrow \quad i_1 = \frac{20}{50} = 0.4 \ A$$

and

$$v_o(0) - 10 i_1 = 0 \quad \Rightarrow \quad v_o(0) = 10 i_1 = 4 \ V$$

Figure 2 shows the Laplace transform representation of the circuit. We can analyze the circuit in Figure 3 by writing and solving two mesh equations. Apply KVL to the left mesh to get

$$10\left(I_1(s) - I_2(s)\right) - \frac{20}{s} = 0$$
Solving for \( I_1(s) \) gives

\[
I_1(s) = I_2(s) + \frac{2}{s}
\]  

(1)

Apply KVL to the right mesh to get

\[
10I_2(s) + \frac{4}{s}I_2(s) + \frac{4}{s} - 10(I_1(s) - I_2(s)) = 0
\]

Collecting the terms involving \( I_2(s) \) gives

\[
\left(20 + \frac{4}{s}\right)I_2(s) = 10I_1(s) - \frac{4}{s} = 0
\]

(2)

Substituting \( I_1(s) \) from Equation 1 into Equation 2 gives

\[
\left(20 + \frac{4}{s}\right)I_2(s) = 10\left(\frac{2}{s} + \frac{2}{s}\right) - \frac{4}{s} = 0
\]

Collecting the terms involving \( I_2(s) \) gives

\[
\left(10 + \frac{4}{s}\right)I_2(s) = \frac{20}{s} - \frac{4}{s} = \frac{16}{s}
\]

Multiplying both sides of this equation by \( \frac{s}{10} \) gives

\[
\left(s + \frac{4}{10}\right)I_2(s) = \frac{16}{10}
\]

Solving for \( I_2(s) \) gives

\[
I_2(s) = \frac{1.6}{s + 0.4}
\]

(3)

Referring to Figure 3, we see that the capacitor voltage is related to the mesh current of the right mesh by

\[
V_o(s) = \frac{4}{s}I_2(s) + \frac{4}{s}
\]

Substituting the expression for \( I_2(s) \) from Equation 3 gives

\[
V_o(s) = \frac{4}{s}\left(\frac{1.6}{s + 0.4}\right) + \frac{4}{s} = \frac{6.4}{s(s + 0.4)} + \frac{4}{s}
\]
Partial fraction expansion gives

\[ V_o(s) = \frac{16}{s} - \frac{16}{s + 0.4} + \frac{4}{s} - \frac{16}{s + 0.4} \]

Taking the inverse Laplace transform gives

\[ v_o(t) = 20 - 16e^{-0.4t} \text{ for } t > 0 \]

The value of the capacitor voltage 2 seconds after the switch closes is

\[ v_o(2) = 20 - 16e^{-0.4(2)} = 12.81 \text{ V} \]