**Example:** Consider the circuit shown in Figure 1. The input to the circuit is the voltage of the voltage source, 20 V. The output of this circuit is the voltage across one of the 50 Ω resistors, $v_o(t)$. Determine the value of the output voltage 3 seconds after the switch closes.

![Figure 1](image)

**Figure 1** The circuit considered in this example.

**Solution:** Before the switch closes, the circuit will be at steady state. Because the only input to this circuit is the constant voltage of the voltage source, all of the element currents and voltages, including the inductor current, will have constant values. Closing the switch disturbsthe circuit by shorting out the 100 Ω resistor. Eventually the disturbance dies out and the circuit is again at steady state. All the element currents and voltages will again have constant values, but probably different constant values than they had before the switch closed.

During the disturbance the element voltages and currents are not constant. We can find these voltages and currents by analyzing the circuit using Laplace Transforms. The Laplace Transform model of the inductor consists of two parts, an impedance and either a series voltage or a parallel current source. The voltage of the voltage source or the current of the current source will depend on the initial condition of the inductor, i.e. the inductor current at time $t = 0$. Before we can model the capacitor using Laplace Transforms, we must determine $i(0)$.

Figure 2 shows the circuit before $t = 0$, when the switch is open and the circuit is at steady state. The open switch is modeled as a open circuit. An inductor in a steady-state dc circuit acts like a short circuit, so a short circuit replaces the inductor. The current in that short circuit is the inductor current, $i(t)$.

Because the circuit in Figure 2 is at steady state, the value of the inductor current will be constant. This constant is the value of the inductor current just before the switch closes. In the absence of unbounded currents, the current of an inductor must be continuous. The value of the inductor current immediately after the switch closes is equal to the value immediately before the switch closes. This value is the initial condition of the inductor and has been labeled as $i(0)$ in Figure 2. The initial condition can be calculated in two steps. First, notice that the two 50 Ω resistors are connected in parallel and use Ohm’s Law to calculate $i_R$ as
Next, use the current division principle to calculate the initial inductor current as

\[ i(0) = \frac{50}{50 + 50} i_R = \frac{1}{2} (0.16) = 0.08 \text{ A} \]

Figure 3 shows the Laplace Transform representation of the circuit. The Laplace Transform model of the inductor consists of two parts, an impedance and a voltage source. The voltage of the voltage source depends on the initial condition of the inductor as

\[ L i(0) = (62.5)(0.08) = 5 \text{ V} \]

Figure 2 The steady-state circuit before \( t = 0 \)

Figure 3 The circuit represented in the frequency domain, using the Laplace Transform.

We can analyze the circuit in Figure 3 by writing and solving two mesh equations. Apply KVL to the left mesh to get
\[ 50\left(I_1(s) - I_2(s)\right) - \frac{20}{s} = 0 \]

Hence
\[ 50\left(I_1(s) - I_2(s)\right) = \frac{20}{s} \] (1)

Apply KVL to the right mesh to get
\[ (62.5s)I_2(s) - 5 + (50)I_2(s) - 50\left(I_1(s) - I_2(s)\right) = 0 \] (2)

Substituting from Equation 1 into Equation 2 gives
\[ (62.5s)I_2(s) - 5 + (50)I_2(s) - \frac{20}{s} = 0 \]

Collecting the terms involving \( I_2(s) \) gives
\[ (62.5s + 50)I_2(s) = \frac{20}{s} + 5 \]

Dividing both sides by 62.5 gives
\[ (s + 0.8)I_2(s) = \frac{0.32}{s} + 0.08 \]

Solving for \( I_2(s) \) gives
\[ I_2(s) = \frac{0.08s + 0.32}{s(s + 0.8)} \]

Referring to Figure 3, we see that the output voltage is related to the mesh current of the right mesh by
\[ V_o(s) = (50)I_2(s) = \frac{4s + 16}{s(s + 0.8)} \]

Partial fraction expansion gives
\[ V_o(s) = \frac{20}{s} - \frac{16}{s + 0.8} \]

Taking the inverse Laplace transform gives
\[ v_o(t) = 20 - 16e^{-0.8t} \text{ for } t > 0 \]

The value of the capacitor voltage 3 seconds after the switch opens is
\[ v_o(3) = 20 - 16e^{-0.8(3)} = 18.5 \text{ V} \]