Example:
Consider the circuit shown in Figure 1. The input to the circuit is the voltage of the voltage source, \( v_i(t) \). The output is the voltage across the capacitor, \( v_o(t) \). The network function that represents this circuit is

\[
H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{5}{j \omega \left(1 + \frac{j \omega}{10}\right)}
\]

Determine the value of the inductance, \( L \), and of the gain, \( A \), of the Current Controlled Current Source (CCCS).

\[\text{Figure 1} \quad \text{The circuit considered in this example.}\]

Solution: The circuit has been represented twice, by a circuit diagram and also by the given network function. The unknown parameters, \( L \) and \( A \), appear in the circuit diagram, but not in the given network function. We can analyze the circuit to determine its network function. This version of the network function will depend on the unknown parameters. We will determine the value of these parameters by equating the two version of the network function.

A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 2 shows the frequency domain representation of the circuit from Figure 1.

\[\text{Figure 2} \quad \text{The circuit from Figure 1, represented in the frequency domain, using impedances and phasors.}\]
The circuit in Figure 2 consists of two meshes. The mesh current of the left-hand mesh is the same current the controlling current of the CCCS, \( I_a(\omega) \). Apply Kirchhoff’s Voltage Law (KVL) to the left-hand mesh to get

\[
20I_a(\omega) + j\omega LI_a(\omega) - V_i(\omega) = 0
\]

Solve for \( I_a(\omega) \) to get

\[
I_a(\omega) = \frac{V_i(\omega)}{20 + j\omega L} = \frac{0.05}{1 + j\omega L/20} V_i(\omega) \tag{2}
\]

The mesh current of the right-hand mesh is the same current the controlled current of the CCCS, \( A I_a(\omega) \). The output voltage is obtained by multiplying this mesh current by the impedance of the capacitor

\[
V_o(\omega) = \frac{1}{j\omega} A I_a(\omega) \tag{3}
\]

Substituting the expression for \( I_a(\omega) \) from Equation 2 into Equation 3 gives

\[
V_o(\omega) = \frac{1}{j\omega} A \times \frac{0.05}{1 + j\omega L/20} V_i(\omega) = \frac{0.05 A}{j\omega \left(1 + j\omega L/20\right)} V_i(\omega)
\]

Divide both sides of this equation by \( V_i(\omega) \) to obtain the network function of the circuit

\[
H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{0.05 A}{j\omega \left(1 + j\omega L/20\right)} \tag{4}
\]

Comparing the network functions given by Equations 1 and 14 gives \( A = 100 \text{ V/V} \) and \( L = 2 \text{ H} \).