Example:
Consider the circuit shown in Figure 1. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the 5 $\Omega$ resistor, $v_o(t)$. The network function that represents this circuit is

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = 0.208 \frac{j\omega}{1 + \frac{j\omega}{3}}$$

(1)

Determine the value of the capacitance, $C$.

![Figure 1](image1.png)

Figure 1 The circuit considered in this example.

Solution: The circuit has been represented twice, by a circuit diagram and also by a network function. The unknown capacitance, $C$, appears in the circuit diagram, but not in the given network function. We can analyze the circuit to determine its network function. This second network function will depend on the unknown capacitance. We will determine the value of the capacitance by equating the two network functions.

A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 2 shows the frequency domain representation of the circuit from Figure 1.

![Figure 2](image2.png)

Figure 2 The circuit from Figure 1, represented in the frequency domain, using impedances and phasors.
The impedances of the capacitor and the two resistors are connected in series in Figure 2. $V_i(\omega)$ is the voltage across these three series impedances and $V_o(\omega)$ is the voltage across one of the impedances. Apply the voltage division principle to get

$$V_o(\omega) = \frac{5}{5 + 3 + \frac{1}{j\omega C}} V_i(\omega)$$

Divide both sides of this equation by $V_i(\omega)$ to obtain the network function of the circuit

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{5}{5 + 3 + \frac{1}{j\omega C}}$$ (2)

We can simplify the algebra required to find $C$ by putting the network function in Equation 2 into the same form as the network function in Equation 1 before equating the two network functions. Let’s multiply the numerator and denominator by $j\omega C$ to get

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{5}{8 + \frac{1}{j\omega C}} \times \frac{j\omega C}{j\omega C} = 5 \frac{j\omega}{1 + j\omega C}$$ (3)

Equating the network functions given by Equations 1 and 3 gives:

$$5 \frac{j\omega}{1 + j\omega C} = 0.208 \frac{j\omega}{1 + j\omega/3}$$

Comparing corresponding parts of this equation indicates that:

$$5 C = 0.208 \quad \text{and} \quad 8 C = \frac{1}{3}$$

The values of $C$ obtained from these equations must agree. (If they do not, we’ve made an error.) Solving these equations gives

$$C = 41.60 \text{ mF} \quad \text{and} \quad C = 41.67 \text{ mF}$$

These values agree, but there is some uncertainty in the third significant figure. It’s appropriate to report our result with two significant figures:

$$C = 42 \text{ mF}$$