Example:
Consider the circuit shown in Figure 1. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the 10 kΩ resistor, $v_o(t)$. The network function that represents this circuit is given to be

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = -20 \frac{1 + j\omega}{2500} \frac{1 + j\omega}{2500}$$

(1)

Determine the values of the capacitance, $C_1$, and of the resistances, $R_1$ and $R_2$.

![Figure 1](image)

**Figure 1** The circuit considered in this example.

Solution: The circuit has been represented twice, by a circuit diagram and also by the given network function. The unknown capacitance and resistances, $C_1$, $R_1$ and $R_2$, appear in the circuit diagram, but not in the given network function. We can analyze the circuit to determine its network function. This second network function will depend on the unknown capacitance and resistances. We will determine the value of the capacitance and resistances by equating the two network functions.

A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 2 shows the frequency domain representation of the circuit from Figure 1.
Figure 2 The circuit from Figure 1, represented in the frequency domain, using impedances and phasors.

To analyze the circuit in Figure 2, we write a node equation at the node labeled as node \( a \). (The node voltage at node \( a \) is zero volts because the voltages at the input nodes of an ideal op amp are equal. The current entering the inverting input of the op amp is zero, so there are four currents in this node equation, the currents in the impedances corresponding to \( R_1, R_2, C_1 \) and \( C_2 \).)

\[
\frac{V_i(\omega)}{R_1} + \frac{V_i(\omega)}{j \omega C_1} + \frac{V_o(\omega)}{R_2} + \frac{V_o(\omega)}{j \omega C_2} = 0
\]

Doing a little algebra gives

\[
\left( \frac{1}{R_1} + j \omega C_1 \right) V_i(\omega) = - \left( \frac{1}{R_2} + j \omega C_2 \right) V_o(\omega)
\]

Then,

\[
\left( \frac{1 + j \omega C_1 R_1}{R_1} \right) V_i(\omega) = - \left( \frac{1 + j \omega C_2 R_2}{R_2} \right) V_o(\omega)
\]

Finally, the network function of the circuit is

\[
H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{1 + j \omega C_1 R_1}{R_1} \frac{R_2}{1 + j \omega C_2 R_2} = - \left( \frac{R_2}{R_1} \right) \frac{1 + j \omega C_1 R_1}{1 + j \omega C_2 R_2} \tag{2}
\]

The network functions given in Equations 1 and 2 must be equal. That is
\[-20 \frac{1 + j \frac{\omega}{25000}}{1 + j \frac{\omega}{2500}} = H(\omega) = \left( \frac{R_2}{R_1} \right) \frac{1 + j \omega C_1 R_1}{1 + j \omega C_2 R_2} \]

Equating coefficients gives:

\[20 = \frac{R_2}{R_1}, \quad 25000 = \frac{1}{C_1 R_1}, \quad \text{and} \quad 2500 = \frac{1}{C_2 R_2} \]

Now using \( C_2 = 2 \text{ nF} = 2 \times 10^{-9} \text{ F} \) gives

\[R_2 = \frac{1}{2500 \times (2 \times 10^{-9})} = \frac{10^9}{5000} = 200 \text{ k} \Omega, \quad R_1 = \frac{R_2}{20} = 10 \text{ k} \Omega \]

and

\[C_1 = \frac{1}{25000 R_1} = \frac{1}{(2.5 \times 10^4)(1 \times 10^4)} = 0.4 \times 10^{-8} = 4 \text{ nF} \]