**Example 1:**
Consider the circuit shown in Figure 1. The input to the circuit is the voltage of the voltage source, \( v_s(t) \). The output is the voltage across the right-hand coil, \( v_o(t) \). Determine the steady-state output voltage, \( v_o(t) \).

![Figure 1](image1.png)

**Figure 1** The circuit considered in Example 1.

**Solution:** The input voltage is a sinusoid and the circuit is at steady state. The output voltage is also a sinusoid and has the same frequency as the input voltage. Consequently, the circuit can be represented in the frequency domain, using phasors and impedances. Figure 2 shows the frequency domain representation of the circuit from Figure 1.

The coil currents, \( I_1(\omega) \) and \( I_2(\omega) \), and the coil voltages, \( V_1(\omega) \) and \( V_2(\omega) \), are labeled in Figure 2. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that both coil currents, \( I_1(\omega) \) and \( I_2(\omega) \), enter the undotted ends of their respective coils.

![Figure 2](image2.png)

**Figure 2** The circuit from Figure 1, represented in the frequency domain, using impedances and phasors.

The device equations for coupled coils are:

\[
V_1(\omega) = j16 I_1(\omega) + j8 I_2(\omega)
\]  

(1)

and
\[ V_2(\omega) = j8I_1(\omega) + j20I_2(\omega) \]  

(2)

The coils are connected in parallel, consequently \( V_1(\omega) = V_2(\omega) \). Equating the expressions for \( V_1(\omega) \) and \( V_2(\omega) \) in Equations 1 and 2 gives

\[
j16I_1(\omega) + j8I_2(\omega) = j8I_1(\omega) + j20I_2(\omega)
\]

\[
j8I_1(\omega) = j12I_2(\omega)
\]

\[
I_1(\omega) = \frac{3}{2}I_2(\omega)
\]

Apply Kirchhoff’s Current Law (KCL) to the top node of the coils to get

\[
I(\omega) = I_1(\omega) + I_2(\omega) = \frac{3}{2}I_2(\omega) + I_2(\omega) = \frac{5}{2}I_2(\omega)
\]

Therefore

\[
I_1(\omega) = \frac{3}{5}I(\omega) \quad \text{and} \quad I_2(\omega) = \frac{2}{5}I(\omega)
\]

(3)

Substituting the expressions for \( I_1(\omega) \) and \( I_2(\omega) \) from Equation 3 into Equation 1 gives

\[
V_1(\omega) = j16 \left( \frac{3}{5}I(\omega) \right) + j8 \left( \frac{2}{5}I(\omega) \right) = \frac{16(3) + 8(2)}{5}I(\omega) = j12.8I(\omega)
\]

(4)

Apply KVL to the mesh consisting of the voltage source, resistor and left-hand coil to get

\[
4I(\omega) + V_1(\omega) − 5.7\angle158° = 0
\]

Using Equation 4 gives

\[
4I(\omega) + j12.8I(\omega) − 5.7\angle158° = 0
\]

Solving for \( I(\omega) \) gives

\[
I(\omega) = \frac{5.7\angle158°}{4 + j12.8} = \frac{5.7\angle158°}{13.41\angle73°} = 0.425\angle85° \ \text{A}
\]

Now the output voltage can be calculated using Equation 4:

\[
V_o(\omega) = V_1(\omega) = j12.8I(\omega) = j12.8(0.425\angle85°) = (12.8\angle90°)(0.425\angle85°) = 5.44\angle175° \ \text{V}
\]
In the time domain, the output voltage is given by

\[ v_o(t) = 5.44 \cos(4t + 175^\circ) \text{ V} \]

**Example 2:**
Consider the circuit shown in Figure 3. The input to the circuit is the voltage of the voltage source, \( v_s(t) \). The output is the voltage across the right-hand coil, \( v_o(t) \). Determine the steady-state output voltage, \( v_o(t) \).

![Figure 3](image)

**Solution:** The circuit shown in Figure 3 is very similar to the circuit shown in Figure 1. There is only one difference: the dot of the right-hand coil is located at the bottom of the coil in Figure 1 and at the top of the coil in Figure 3. Figure 4 shows the frequency domain representation of the circuit from Figure 3.

![Figure 4](image)

The coil currents, \( I_1(\omega) \) and \( I_2(\omega) \), and the coil voltages, \( V_1(\omega) \) and \( V_2(\omega) \), are labeled in Figure 4. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that one of coil currents, \( I_1(\omega) \),
enters the undotted end of the coil while the other coil current, $I_2(\omega)$, enters the dotted end of the coil.

The device equations for coupled coils are:

$$V_1(\omega) = j16 I_1(\omega) - j8 I_2(\omega)$$ (5)

and

$$V_2(\omega) = -j8 I_1(\omega) + j20 I_2(\omega)$$ (6)

The coils are connected in parallel, consequently $V_1(\omega) = V_2(\omega)$. Equating the expressions for $V_1(\omega)$ and $V_2(\omega)$ in Equations 5 and 6 gives

$$j16 I_1(\omega) - j8 I_2(\omega) = -j8 I_1(\omega) + j20 I_2(\omega)$$

$$j24 I_1(\omega) = j28 I_2(\omega)$$

$$I_1(\omega) = \frac{28}{24} I_2(\omega) = \frac{7}{6} I_2(\omega)$$

Apply Kirchhoff’s Current Law (KCL) to the top node of the coils to get

$$I(\omega) = I_1(\omega) + I_2(\omega) = \frac{7}{6} I_2(\omega) + I_2(\omega) = \frac{13}{6} I_2(\omega)$$

Therefore

$$I_1(\omega) = \frac{7}{13} I(\omega) \quad \text{and} \quad I_2(\omega) = \frac{6}{13} I(\omega)$$ (7)

Substituting the expressions for $I_1(\omega)$ and $I_2(\omega)$ from Equation 7 into Equation 5 gives

$$V_1(\omega) = j16 \left( \frac{7}{13} I(\omega) \right) - j8 \left( \frac{6}{13} I(\omega) \right) = j \frac{16(7) - 8(6)}{13} I(\omega) = j4.9 I(\omega)$$ (8)

Apply KVL to the mesh consisting of the voltage source, resistor and left-hand coil to get

$$4 I(\omega) + V_1(\omega) - 5.7 \angle 158^\circ = 0$$

Using Equation 8 gives

$$4 I(\omega) + j4.9 I(\omega) - 5.7 \angle 158^\circ = 0$$

Solving for $I(\omega)$ gives

$$I(\omega) = \frac{5.7 \angle 158^\circ}{4 + j4.9} = \frac{5.7 \angle 158^\circ}{6.325 \angle 51^\circ} = 0.901 \angle 107^\circ \ A$$
Now the output voltage can be calculated using Equation 8:

$$V_s(\omega) = V_1(\omega) = j \cdot 4.9 \cdot I(\omega)$$

$$= j \cdot 4.9 \cdot (0.901 \angle 107^\circ)$$

$$= (4.9 \angle 90^\circ)(0.901 \angle 107^\circ) = 4.41 \angle 197^\circ \text{ V}$$

In the time domain, the output voltage is given by

$$v_o(t) = 4.41 \cos (4t + 197^\circ) \text{ V}$$