Example 1:
Consider the circuit shown in Figure 1. The input to the circuit is the current of the current source, $i_s(t)$. The output is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$.

![Figure 1](image1.png)

**Figure 1** The circuit considered in Example 1.

**Solution:** The input current is sinusoidal and the circuit is at steady state. The output voltage is also a sinusoid and has the same frequency as the input current. Consequently, the circuit can be represented in the frequency domain, using phasors and impedances. Figure 2 shows the frequency domain representation of the circuit from Figure 1.

![Figure 2](image2.png)

**Figure 2** The circuit from Figure 1, represented in the frequency domain, using impedances and phasors.

The coil currents, $I_1(\omega)$ and $I_2(\omega)$, and the coil voltages, $V_1(\omega)$ and $V_2(\omega)$, are labeled in Figure 2. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that one coil current, $I_1(\omega)$, enters the undotted end of its coil while the other coil current, $I_2(\omega)$, enters the dotted of its coil.

The device equations for coupled coils are:

$$V_1(\omega) = j16 I_1(\omega) - j8 I_2(\omega)$$  \hspace{1cm} (1)

and

$$V_2(\omega) = -j8 I_1(\omega) + j12 I_2(\omega)$$  \hspace{1cm} (2)
The coils are connected in parallel, consequently $V_1(\omega) = V_2(\omega)$. Equating the expressions for $V_1(\omega)$ and $V_2(\omega)$ in Equations 1 and 2 gives

\[
j 16 I_1(\omega) - j 8 I_2(\omega) = -j 8 I_1(\omega) + j 12 I_2(\omega)
\]

\[
j 24 I_1(\omega) = j 20 I_2(\omega)
\]

\[
I_1(\omega) = \frac{5}{6} I_2(\omega)
\]

Apply Kirchhoff’s Current Law (KCL) to the top node of the coils to get

\[
0.121 \angle 140^\circ = I_x(\omega) = I_1(\omega) + I_2(\omega) = \frac{5}{6} I_2(\omega) + I_2(\omega) = \frac{11}{6} I_2(\omega)
\]

\[
I_2(\omega) = \frac{6}{11} (0.121 \angle 140^\circ) = 0.066 \angle 140^\circ \text{ A}
\]

Now the output voltage can be calculated using Equation 2:

\[
V_o(\omega) = V_2(\omega) = -j 8 I_1(\omega) + j 12 I_2(\omega)
\]

\[
= -j 8 \left( \frac{5}{6} I_2(\omega) \right) + j 12 I_2(\omega) = j \frac{-8(5) + 12(6)}{6} I_2(\omega)
\]

\[
= j 5.333 I_2(\omega)
\]

\[
= (5.333 \angle 90^\circ)(0.066 \angle 140^\circ) = 0.352 \angle 230^\circ \text{ V}
\]

In the time domain, the output voltage is given by

\[
v_o(t) = 0.352 \cos(4t + 230^\circ) \text{ V}
\]
Example 2:
Consider the circuit shown in Figure 3. The input to the circuit is the current of the current source, $i_s(t)$. The output is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$.

![Figure 3](image1.png)

*Figure 3* The circuit considered in Example 2.

Solution: The circuit shown in Figure 3 is very similar to the circuit shown in Figure 1. There is only one difference: the dot of the left-hand coil is located at the bottom of the coil in Figure 1 and at the top of the coil in Figure 3. As in Example 1, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 4 shows the frequency domain representation of the circuit from Figure 3.

![Figure 4](image2.png)

*Figure 4* The circuit from Figure 3, represented in the frequency domain, using impedances and phasors.

The coil currents, $I_1(\omega)$ and $I_2(\omega)$, and the coil voltages, $V_1(\omega)$ and $V_2(\omega)$, are labeled in Figure 4. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that the coil currents, $I_1(\omega)$ and $I_2(\omega)$, both enter the dotted of their respective coils.

The device equations for coupled coils are:

$$V_1(\omega) = j16 I_1(\omega) + j8 I_2(\omega)$$  \hspace{1cm} (3)

and

$$V_2(\omega) = j8 I_1(\omega) + j12 I_2(\omega)$$  \hspace{1cm} (4)
The coils are connected in parallel, consequently \( V_1(\omega) = V_2(\omega) \). Equating the expressions for \( V_1(\omega) \) and \( V_2(\omega) \) in Equations 3 and 4 gives

\[
j16 I_1(\omega) + j8 I_2(\omega) = j8 I_1(\omega) + j12 I_2(\omega)
\]

\[
j8 I_1(\omega) = j4 I_2(\omega)
\]

\[
I_1(\omega) = \frac{1}{2} I_2(\omega)
\]

Apply Kirchhoff’s Current Law (KCL) to the top node of the coils to get

\[
0.121\angle140^\circ = I_s(\omega) = I_1(\omega) + I_2(\omega) = \frac{1}{2} I_2(\omega) + I_2(\omega) = \frac{3}{2} I_2(\omega)
\]

\[
I_2(\omega) = \frac{2}{3} (0.121\angle140^\circ) = 0.08067\angle140^\circ \text{ A}
\]

Now the output voltage can be calculated using Equation 4:

\[
V_o(\omega) = V_2(\omega) = j8 I_1(\omega) + j12 I_2(\omega)
\]

\[
= j8 \left( \frac{1}{2} I_2(\omega) \right) + j12 I_2(\omega)
\]

\[
= j16 I_2(\omega)
\]

\[
= (16\angle90^\circ)(0.04033\angle140^\circ) = 1.291\angle230^\circ \text{ V}
\]

In the time domain, the output voltage is given by

\[
v_o(t) = 1.291 \cos(4t + 230^\circ) \text{ V}
\]