Example 1:
Consider the circuit shown in Figure 1. The input to the circuit is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the capacitor, $v_o(t)$. Determine amplitude, $A$, of $v_o(t)$.

![Figure 1](image1.png)

The circuit considered in Example 1.

Solution: The input voltage is a sinusoid. The output voltage is also a sinusoid and has the same frequency as the input voltage. Apparently the circuit has reached steady state. Consequently, the circuit in Figure 1 can be represented in the frequency domain, using phasors and impedances. Figure 2 shows the frequency domain representation of circuit from Figure 1. The impedance of the capacitor was calculated as

$$-j \frac{1}{\omega C} = -j \frac{1}{(1)(0.933)} = -j 1.07 \Omega$$

Apply the voltage divider principle to the circuit in Figure 2 to get

$$A \angle 64^\circ = \frac{-j 1.07}{4 - j 1.07} (7.2 \angle 139^\circ) = 1.864 \angle 64^\circ$$

Therefore $A = 1.864$ V.

![Figure 2](image2.png)

The circuit from Figure 1, represented in the frequency domain, using impedances and phasors
Example 2:
Consider the circuit shown in Figure 3. The input to the circuit is the voltage of the voltage source, \( v_s(t) \). The output is the voltage across the capacitor, \( v_o(t) \). Determine resistance, \( R \), of the resistor.

\[
v_s(t) = 7.83 \cos (3t + 126^\circ) \text{ V}
\]

\[
v_o(t) = 1.89 \cos (3t + 230^\circ) \text{ V}
\]

**Figure 3** The circuit considered in Example 2.

**Solution:** The input voltage is a sinusoid. The output voltage is also a sinusoid and has the same frequency as the input voltage. Apparently the circuit has reached steady state. Consequently, the circuit in Figure 3 can be represented in the frequency domain, using phasors and impedances. Figure 4 shows the frequency domain representation of circuit from Figure 3. The voltages \( V_s(\omega) \) and \( V_o(\omega) \) in Figure 4 are the phasors corresponding to \( v_s(t) \) and \( v_o(t) \). The capacitor and the resistor are represented as impedances in Figure 4. The impedance of the capacitor was calculated as

\[
-j \frac{1}{\omega C} = -j \frac{1}{3(0.3342)} = -j 0.997 \approx -j \Omega
\]

\[
V_s(\omega) = 7.83 \angle 126^\circ \text{ V}
\]

\[
V_o(\omega) = 1.89 \angle 230^\circ \text{ V}
\]

**Figure 4** The circuit from Figure 3, represented in the frequency domain, using impedances and phasors.

The current \( I(\omega) \) in Figure 3 is given by

\[
I(\omega) = \frac{V_o(\omega)}{-j} = \frac{1.89 \angle 230^\circ}{1 \angle -90^\circ} = 1.89 \angle 320^\circ \text{ A}
\]
The resistor voltage, $V_R(\omega)$, in Figure 4 is given by

$$V_R(\omega) = -V_s(\omega) - V_o(\omega) = -7.83\angle 126^\circ - 1.89\angle 230^\circ = 7.597\angle -40^\circ$$

The impedance of the resistor is given by

$$R = \frac{V_R(\omega)}{I(\omega)} = \frac{7.597\angle -40^\circ}{1.89\angle 320^\circ} = 4.02\angle -360^\circ \approx 4$$

Therefore $R = 4 \, \Omega$.

**Alternate Solution:** Apply the voltage divider principle to the circuit in Figure 4 to get

$$1.89\angle 230^\circ = \frac{-j}{-j + R} (-7.83\angle 126^\circ)$$

Doing some algebra gives

$$\frac{-j}{-j + R} = \frac{1.89\angle 230^\circ}{-7.83\angle 126^\circ} = \frac{1.89\angle 230^\circ}{(1\angle 180^\circ)(7.83\angle 126^\circ)} = 0.241\angle -76^\circ$$

Solving for $L$ gives

$$R = \frac{j((0.241\angle -76^\circ) - 1)}{0.241\angle -76^\circ} = 4.02 - j0.0022 \approx 4 \, \Omega$$