Example:
Consider the circuit shown in Figure 1. Find the value of the gain, $A$, of the CCVS.

![Figure 1](image1.png)

Figure 1 The circuit considered in this example.

Solution: Figure 2 shows the circuit from Figure 1 after replacing the voltmeter by an equivalent open circuit and labeling the voltage measured by the voltmeter. This circuit can be analyzed using node equations. Figure 3 shows the circuit after selecting a reference node and numbering the other nodes. Let $v_1$, $v_2$ and $v_3$ denote the node voltages at nodes 1, 2 and 3, respectively.

![Figure 2](image2.png)

Figure 2 The circuit from Figure 1 after replacing the voltmeter by an open circuit.

![Figure 3](image3.png)

Figure 3 The circuit from Figure 2 after labeling the nodes.
The voltage of the 12 V voltage source can be expressed in terms of the node voltages as

\[ 12 = v_1 - 0 \quad \Rightarrow \quad v_1 = 12 \text{ V} \]

The voltmeter measures the node voltage at node 3 so

\[ v_3 = 3.27 \text{ V} \]

This circuit contains a dependent source. When analyzing such a circuit using node equations, it is important to express the controlling current or voltage of the dependent source in terms of the node voltages. In this circuit, the controlling current of the dependent source is the current \( i_a \). The controlling current of the dependent source is equal to the current directed from right to left in the 8 \( \Omega \) resistor. Hence \( i_a \) is related to node voltages by

\[ i_a = \frac{v_2 - v_1}{8} = \frac{3.27 - 12}{8} = -1.09 \text{ A} \]

The dependent source voltage is related to the node voltages by

\[ A i_a = 0 - v_2 = -v_2 \]

The value of the current in an open circuit is always zero so applying KCL at node 3 gives

\[ \frac{v_2 - v_3}{7} = 0 \]

Substituting for the node voltages gives

\[ -A i_a - \frac{3.27}{7} = 0 \quad \Rightarrow \quad -A(-1.09) - 3.27 = 0 \quad \Rightarrow \quad A = 3 \text{ V/A} \]