Commentary on
Emergence in Biomolecular Networks?

James F. Lynch

Department of Mathematics
Clarkson University

December 30, 2008 / APA Eastern Division 105th Annual Meeting

A Strict Definition of Emergence

Let $\langle \mathcal{S}, R \rangle$ be a dynamical system:

- $\mathcal{S}$ is the set of states.
- R is the rule defining state transitions. R can be:
    - deterministic, e.g., Kauffman's Boolean networks
    - nondeterministic
    - probabilistic, e.g., Derrida & Pomeau's Boolean networks

Let $\mathcal{P}$ be a proof system.

Definition

A property of $\langle \mathcal{S}, R \rangle$ is emergent (with respect to $\mathcal{P}$) if it is not a logical consequence of the definition of $\langle \mathcal{S}, R \rangle$.

Existence of Emergent Properties

Using automata theoretic arguments, it can be shown that emergent properties exist for certain dynamical systems.

But these kinds of properties are not interesting to biologists.

What about biologically interesting “emergent" properties? Automata theoretic arguments don’t seem useful here.

In some cases, e.g., random Boolean networks, properties that were formerly called emergent have been (partially) explained mathematically.

Possible Responses

- A weaker notion of emergence.
- A critique of the mathematics.

Grimm and Railsback, *Individual-based Modeling and Ecology*:

- Emergent properties are not just the sum of the properties of the agents.
- Emergent properties are of a different type than the properties of the agents.
- Emergent properties cannot be easily predicted from the properties of the agents.

Measures of Order for Random Boolean Networks

- Fraction of nodes that eventually stabilize. Almost all nodes stabilize $\Rightarrow$ ordered behavior.
- Limit cycle size. Small limit cycle (with respect to N) $\Rightarrow$ ordered behavior.

Analysis of Random Boolean Networks

- Methods of statistical physics.
  - Approximation by thermodynamic limit.
  - Mean-field approximation.

  Neither approach is mathematically rigorous.
- Combinatorial methods—random graph theory.

Conjectures Compared to Theorems

Conjectures

1. $K \leq 2 \Rightarrow$ most nodes stabilize quickly.
2. $K \leq 2 \Rightarrow$ limit cycle size $< \sqrt{N}$.
3. $K > 2 \Rightarrow$ many nodes do not stabilize.

Theorems

1. $K \leq 2 \Rightarrow$ most nodes stabilize quickly.
2. $K \leq 2 \Rightarrow$ limit cycle size is superpolynomial.
3. $K > 2 \Rightarrow$ many nodes do not stabilize quickly.

Questions

- Can a weak version of emergence be formalized?
- Is it a useful notion?
- Will progress in analyzing complex dynamical systems require the development of new mathematical techniques?