

# Chapter 9

## Fracture mechanics:

Show figures 9.1 through 9.6. Materials that would fail by ductile failure are obviously preferable for most engineering applications, but this does not occur for ceramics.

Stress concentration:

Statics analysis shows that when microscopic cracks occur in a solid material, the stress at the crack tip is concentrated, or in other words magnified above the level of the applied stress. Show figure 9.8. The maximum stress occurs at the crack tip according to:

$$\sigma_m = \sigma_0 \left[ 1 + 2 \left( \frac{a}{\rho_t} \right)^{1/2} \right]$$

Here  $\sigma_m$  is the maximum stress at the crack tip,  $\sigma_0$  is the nominal applied tensile stress,  $\rho_t$  is the radius of curvature at the crack tip, and  $a$  is the length of a surface crack, or half the length of an internal crack. For a long enough crack, this reduces to:

$$\sigma_m = 2\sigma_0 \left( \frac{a}{\rho_c} \right)^{1/2}$$

The ratio  $\sigma_m/\sigma_0$  is often referred to as the stress concentration factor,  $K_t$

$$K_t = 2 \left( \frac{a}{\rho_c} \right)^{1/2}$$

**Omit Griffith theory of brittle fracture, and stress analysis of cracks.**

## Fracture toughness:

Due to the disastrous nature of fracture, much effort has been expended to understand fracture mechanics. From a combination of fundamental and empirical reasons, brittle fracture will occur when the fracture toughness ( $K_C$ ) of a material is exceeded, where

$$K_C = Y(a/w) \sigma_c \sqrt{\pi a}$$

Here  $Y(a/w)$  is a geometrical factor that depends on the crack dimensions, where  $a$  is the crack length and  $w$  is the specimen thickness;  $\sigma_c$  is the critical stress for crack propagation, and subsequent failure; and  $a$  is again the length of a surface crack or half the length of an internal crack. If  $a \rightarrow 0$  or  $w \rightarrow \infty$ , then  $Y \rightarrow 1$ . As the sample thickness is increased, the fracture toughness declines, until the plane strain region is obtained.

The fracture toughness ( $K_{IC}$ ) is the critical value of the stress intensity factor at a crack tip needed to produce catastrophic failure under simple uniaxial loading. The subscript I stands for Mode I loading (uniaxial), illustrated in figure 9.10a while the subscript C stands for critical. The fracture toughness is given by:

$$K_{IC} = Y\sigma_c\sqrt{\pi a}$$

where Y is a dimensionless geometry factor on the order of 1,  $\sigma_f$  is the stress applied at failure, and a is the length of a surface crack (or one-half the length of an internal crack). The MKS units of  $K_{IC}$  are  $\text{MPa}\cdot\text{m}^{1/2}$ . Show table 9.1, which provides values for  $K_{IC}$  under “plane strain” conditions, meaning that  $t \geq 2.5\left(K_{IC} / \sigma_y\right)^2$ , where t is the sample thickness. For thin specimens, or “plane stress” conditions, the fracture toughness becomes thickness dependent.

This type of approach predicts that flaws larger than a certain size will cause fracture. This provides a lower detection limit for manufacturing flaws. The analysis given here applies to brittle fracture rather than to ductile failure. For design problems, we can use the equation for plane strain fracture toughness in two different manners, as illustrated in equations (9.6) and (9.7):

$$\sigma_c \leq \frac{K_{IC}}{Y\sqrt{\pi a}}$$

$$a_c = \frac{1}{\pi} \left( \frac{K_{IC}}{\sigma} \right)^2$$

**Example:**

Estimate the flaw size responsible for the failure of a turbine motor made from partially stabilized zirconia that fractures at a stress level of 300 MPa.

From above, we know that:

$$K_{IC} = Y\sigma_f\sqrt{\pi a}$$

Rearranging:

$$a_c = \frac{1}{\pi} \left( \frac{K_{IC}}{\sigma} \right)^2$$

From table 9.1, but missing in this textbook,  $K_{IC} = 9 \text{ MPa}\cdot\text{m}^{1/2}$ . Substituting:

$$a_c = \frac{1}{\pi} \left( \frac{9 \text{ MPa}\cdot\text{m}^{1/2}}{300 \text{ MPa}} \right)^2$$

$$a_c = 2.86 \times 10^{-4} \text{ m} = 286 \mu\text{m}$$

Note that were we to substitute electrical porcelain ( $K_{IC} = 1 \text{ MPa}\cdot\text{m}^{1/2}$ ) for partially stabilized zirconia, the critical flaw size would be reduced by 81x to about  $3.54 \mu\text{m}$ . Clearly, preventing flaw formation during fabrication becomes much more difficult for electrical porcelain.

### **Impact Fracture Testing:**

Since fracture occurs abruptly in some materials, tensile stress-strain test are inadequate, since the stress is applied gradually. This particularly applies to failure by brittle fracture. Impact testing techniques have been developed to test for abrupt failure arising from a sharp impact, the two most common being the Charpy and Izod tests. These are used to measure the impact energy, sometimes called the notch toughness. Show Figure 9.18. Note that a V-notch is machined into the sample for these impact testing techniques. Impact testing is more qualitative than fracture toughness, with only the latter characterized by a materials constant (plain strain fracture toughness).

One of the important observations during Charpy and Izod tests is the possible existence of ductile-to-brittle transitions. During low temperature operation, some materials become dramatically more brittle, and therefore subject to failure when exposed to impact forces. At high temperature, where the impact energy absorbed before failure is high, ductile failure occurs, with extensive elastic and plastic deformation before failure. At low temperature, where the impact energy absorbed before failure is low, brittle fracture occurs, with little deformation before failure. Show Figure 9.22, which illustrates the dramatic effect of temperature on impact energy that is observed in some metals, such as low-strength steels. Typical results for different types of metals are shown in Figure 9.21.

### **Fatigue:**

We have discussed methods for testing materials under a constant load (tensile stress-strain test) and under rapid loading (Charpy impact test). However, some materials undergo brittle failure by fatigue when exposed to a cyclic load. A typical fatigue curve is shown in figure 9.25a for materials, such as ferrous (Fe) and Ti alloys, that show a fatigue strength or endurance limit. However, most nonferrous alloys do not exhibit a horizontal region in the fatigue curve, as shown in figure 9.25b. These S-N curves are obtained using fatigue testing apparatus, as shown in figure 9.24.

The magnitude of the cyclic stress can be calculated as the mean stress ( $\sigma_m$ ), the stress range ( $\sigma_r$ ), the stress amplitude ( $\sigma_a$ ), or the stress ratio ( $R$ ), as given below:

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\sigma_r = \sigma_{\max} - \sigma_{\min}$$

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

The reason for the decay in strength of materials under cyclic stress is crack formation, followed by gradual

crack growth. Repeated stress application creates numerous local regions of plastic deformation, forming sharp discontinuities (extrusions and intrusions) that act as stress concentrators. In addition, cracks are gradually lengthened by similar processes. Fracture mechanics (toughness) can be used to predict the crack size at which brittle fracture will occur. However, additional theory is needed to model the evolution of crack growth during cyclic stress.

### Creep:

Materials that are exposed to stress at elevated temperatures often deform (and eventually fail) by creep. Show figure 9.35, which illustrates the three stages of creep. The instantaneous, elastic deformation is followed by primary creep, where the creep rate diminishes with time, possibly due to strain hardening. Secondary creep, often called steady-state creep, is characterized by a constant creep rate and is usually the longest stage in duration. Tertiary creep is characterized by an acceleration of the creep rate to failure. For engineering design purposes, the magnitude of creep can be described by the steady state creep rate, since this is of the longest duration and is most predictable.

The steady-state creep rate is strongly affected by temperature, as shown by equations (9.20) and (9.21):

$$\dot{\epsilon}_s = K_1 \sigma^n$$

$$\dot{\epsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

Obviously,  $K_1$  and  $K_2$  are related.

### Example

Steady-state creep data for an alloy at 200°C yield:

$\epsilon_s$ (hr <sup>-1</sup> )	$\sigma$ (MPa)
$2.5 \times 10^{-3}$	55
$2.4 \times 10^{-2}$	69

The activation energy for creep is known to be 140 kJ/mol. What is the steady-state creep rate at 250°C and 48 MPa?

We eventually need to use equation (9.21) above:

$$\dot{\epsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

However, we first need to determine the constants  $K_2$  and  $n$ . We can do that from the two data points that are given, which allow us to setup a 2x2 system of equations. Taking the ln of both sides of this equation:

$$\ln \dot{\epsilon}_{s1} = \ln K_2 + n \ln \sigma_1 - \frac{Q_c}{RT_1}$$

$$\ln \dot{\epsilon}_{s2} = \ln K_2 + n \ln \sigma_2 - \frac{Q_c}{RT_2}$$

Now we can subtract these to yield:

$$\ln \left( \frac{\dot{\epsilon}_{s1}}{\dot{\epsilon}_{s2}} \right) = n \ln \left( \frac{\sigma_1}{\sigma_2} \right)$$

Notice that because  $T_1 = T_2$ , the last term cancels out. Substituting in the data that was given:

$$\ln \left( \frac{2.4 \times 10^{-2}}{2.5 \times 10^{-3}} \right) = n \ln \left( \frac{69}{55} \right)$$

$$n = 9.97$$

Substituting into either of the two equations above yields  $K_2 = 3.27 \times 10^{-5} \text{ hr}^{-1}$ . Now we can substitute back into the original equation for the steady state strain:

$$\dot{\epsilon}_s = \left( 3.27 \times 10^{-5} \text{ hr}^{-1} \right) (48 \text{ MPa})^{9.97} \exp \left[ - \frac{(140,000 \text{ J / mole})}{(8.314 \text{ J / mole} - ^\circ\text{K})(523^\circ\text{K})} \right]$$

$$\dot{\epsilon}_s = 1.97 \times 10^{-2} \text{ hr}^{-1}$$