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EXPERIMENTS WITH AN ADAPTIVE MULTIGRID SHALLOW-WATER TROPICAL CYCLONE MODEL

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1. INTRODUCTION

Accurate modeling of tropical cyclone motion and intensity change requires resolving the flow both within and around the storm. Since the spatial scales in these two regions differ substantially, uniform resolution is inherently inefficient: the grid should be refined only near the storm. This idea motivates conventional nested-grid methods such as used in VICBAR (DeMaria et al., 1992) and the GFDL model (Kurihara et al., 1998). Adaptive multigrid methods also achieve nonuniform resolution by superimposing uniform grids of different mesh sizes, but they combine this idea with multigrid processing (Brandt, 1977) to achieve optimum solution speed and provide accurate truncation error estimates. The latter can be used in an adaptive mesh refinement scheme to provide just the resolution needed at each point.

The MUDBAR model of Fulton (2000) demonstrates the potential of adaptive multigrid methods in the context of a nondivergent barotropic model. We now consider the extension of these techniques to the next level of dynamical complexity, i.e., the shallow-water equations.

2. MODEL DESCRIPTION

In this section we briefly describe the model; further details can be found in Mitchell and Fulton (2000).

2.1 *Governing Equations*

We work with the shallow-water equations in the form

$$\begin{aligned} u_t + uu_x + vu_y - fv + \phi_x &= 0, \\ v_t + uv_x + vv_y + fu + \phi_y &= 0, \\ \phi_t + (\Phi + \phi)(u_x + v_y) + u\phi_x + v\phi_y &= 0, \end{aligned} \quad (1)$$

where u and v are the velocity components in x and y, respectively, ϕ is the deviation of the geopotential gh from a constant reference value Φ, and f is the Coriolis parameter. The model domain is a rectangle on a β-plane. At the boundaries we use open boundary conditions, specifying the quantity $v_n - \phi/c$ (where v_n is the outward normal velocity component and $c = \sqrt{\Phi}$); the tangential velocity component is also specified where there is inflow.

2.2 *Time Discretization*

To permit using larger time steps than would be allowed by the CFL condition of an explicit method, we use a semi-implicit (leapfrog/trapezoidal) time discretization. The resulting implicit equations for the solution (u, v, ϕ) at a new time level take the form

$$\begin{aligned} u + \Delta t\phi_x &= F, \\ v + \Delta t\phi_y &= G, \\ \phi + \Phi\Delta t(u_x + v_y) &= H, \end{aligned} \quad (2)$$

where Δt is the time step and the right-hand side depends on values at the previous time levels. The initial time step (second-order Runge-Kutta) takes a similar form. Eliminating u and v leads to a Helmholtz problem

$$\phi - (c\Delta t)^2(\phi_{xx} + \phi_{yy}) = Q = H - \Phi\Delta t(F_x + G_y) \quad (3)$$

to be solved for the geopotential; the corresponding velocity components can then be obtained from (2).

2.3 *Space Discretization*

The model is discretized in space using second-order centered finite differences on an Arakawa C-grid with mesh spacing h. The discretization of (2) is straightforward (but requires one-sided differences on the right-hand side when discretized at boundary points). The discretization of (3) takes the form

$$L^h\phi^h = Q^h, \quad (4)$$

where the discrete operator L^h has the difference stencil

$$L^h = \frac{1}{h^2}\begin{bmatrix} 0 & -\gamma^2 & 0 \\ -\gamma^2 & 1 + 4\gamma^2 & -\gamma^2 \\ 0 & -\gamma^2 & 0 \end{bmatrix}. \quad (5)$$

with $\gamma = c\Delta t/h$ being the Courant number.

To apply the boundary conditions we average the normal velocity across the boundary (using a ghost point value) and eliminate the velocity components using the discretized momentum equations to obtain a modified form of (4) applied at the boundary. For example, (5) is replaced by

$$L^h = \frac{1}{h^2}\begin{bmatrix} 0 & -\gamma^2 & 0 \\ 0 & 1 + 2\gamma + 4\gamma^2 & -2\gamma^2 \\ 0 & -\gamma^2 & 0 \end{bmatrix} \quad (6)$$

at the west boundary and by

$$L^h = \frac{1}{h^2}\begin{bmatrix} 0 & 0 & 0 \\ -2\gamma^2 & 1 + 4\gamma + 4\gamma^2 & 0 \\ 0 & -2\gamma^2 & 0 \end{bmatrix} \quad (7)$$

at the northeast corner; the right-hand side Q^h is modified similarly. Note that while the system (2) has a staggered discretization, the discrete Helmholtz problem (4) involves only values at the gridpoints.

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2.4 *Multigrid Solution*

To solve (4) efficiently we use a multigrid method. This uses Gauss-Seidel relaxation, full weighting of residuals, and bilinear interpolation of corrections in a Full Multigrid (FMG) algorithm with bicubic initial interpolation. Since the boundary conditions are combined with the interior equations as described above, proper residual transfers must be used at the boundaries. For example, the standard full weighting stencil

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \tag{8}$$

is replaced by

$$\frac{1}{16}\begin{bmatrix} 0 & 2 & 2 \\ 0 & 4 & 4 \\ 0 & 2 & 2 \end{bmatrix} \quad \text{and} \quad \frac{1}{16}\begin{bmatrix} 0 & 0 & 0 \\ 4 & 4 & 0 \\ 4 & 4 & 0 \end{bmatrix} \tag{9}$$

at the west boundary and northwest corner, respectively. For large time steps ($\gamma \gg 1$) the operator L^h approaches the Laplacian operator, and the multigrid smoothing factor approaches $\bar{\mu} = 0.25$ (with red-black ordering); for smaller time steps the smoothing factor actually *decreases*, so the performance of the solver improves.

2.5 *Local Mesh Refinement*

The resolution is increased near the vortex by superimposing nested uniform grids of different mesh sizes, in the same way as in the MUDBAR model (Fulton, 2000). Transfers between grids are more complicated due to the staggering of u and v. The only conditions applied at the grid interfaces are the same as the open boundary conditions applied at the domain boundaries, with the specified quantities (e.g., $v_n - \phi/c$) obtained by interpolating in time and space from the next coarser grid. For the results presented here we use patches of fixed size which are moved to follow the vortex over each time step.

3. NUMERICAL RESULTS

For a simple test case we use a small-scale vortex (Gaussian geopotential with e-folding width 112 km) embedded the sinusoidal zonal current of DeMaria (1985). The model domain is a square of side length 4096 km on a β-plane at 20° N with $c = 100$ m/s, using a base grid with $h = 32$ km and one patch with $h = 16$ km. Figure 1 shows the solution (contours of geopotential and wind vectors) at $t = 4$ hours and $t = 48$ hours; the smaller square shows the boundary of the grid patch. Since the initial condition is in geostrophic but not gradient balance, the initial unbalance generates a significant gravity wave front which propagates cleanly through the grid interface by 4 hours and out of the domain long before 48 hours. There is no evidence of false reflections or other problems at the grid interface.

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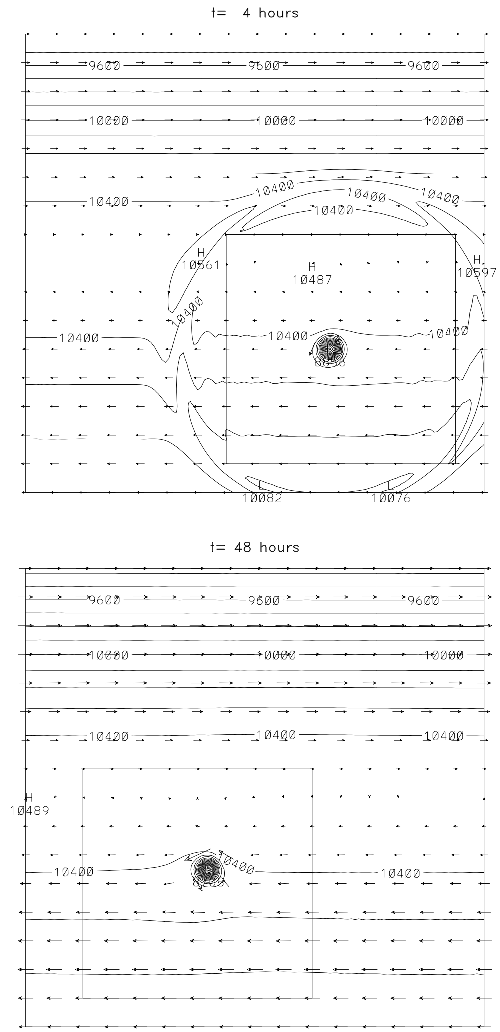

Figure 1. Sample solution

REFERENCES

Brandt, A., 1977: Multi-level adaptive solutions to boundary-value problems. *Math. Comput.*, **31**, 333–390.

DeMaria, M., 1985: Tropical cyclone track prediction with a barotropic model. *Mon. Wea. Rev.*, **113**, 1199–1210.

DeMaria, M., S. D. Aberson, K. V. Ooyama, and S. J. Lord, 1992: A nested spectral model for hurricane track forecasting. *Mon. Wea. Rev.*, **120**, 1628–1643.

Fulton, S. R., 2000: An adaptive multigrid barotropic tropical cyclone track model. Submitted to *Mon. Wea. Rev.*.

Kurihara, Y. M., R. E. Tuleya, and M. A. Bender, 1998: The GFDL hurricane prediction system and its performance in the 1995 hurricane season. *Mon. Wea. Rev.*, **126**, 1306–1322.

Mitchell, B. L., and S. R. Fulton, 2000: Adaptive Multigrid Solution of the Shallow Water Equations. *Technical Report No. 2000-02*, Dept. of Math and Comp. Sci., Clarkson University, `www.clarkson.edu/~mcs`.