Calculus ABCs:
A Gateway for Freshman Calculus

Scott R. Fulton

Final revision for PRIMUS

3 January 2003

ADDRESS: Department of Mathematics and Computer Science, Clarkson University, Potsdam, NY 13699–5815 USA, fulton@clarkson.edu.

ABSTRACT: This paper describes a gateway testing program designed to ensure that students acquire basic skills in freshman calculus. Students must demonstrate they have mastered standards for "Absolutely Basic Competency"—the Calculus ABCs—in order to pass the course with a grade of C or better. We describe the background, standards, and testing program. Based on three years of objective data, together with subjective impressions, we conclude that the program is effective in encouraging many students to master the basics. We also describe improvements currently in place, including web-based test generation.

KEYWORDS: Calculus, gateway tests, assessment

1 Introduction

At any level of mathematical education there is a core body of knowledge which may be regarded as absolutely basic: for any given course there are certain facts and skills which each student should know before starting—or completing—the course. For example, a student starting high school algebra should be able to compute $\frac{1}{2}+\frac{1}{3}$ (correctly!). Likewise, a student finishing a graduate course in analysis should be able to state the definition of the limit of a real function at a point. While the *extent* of this core knowledge will be vigorously debated, the fact that it *exists* should be clear: we expect students to know something when they come to our classes—and to learn something before they are done.

In particular, freshman calculus covers a wide range of topics. As students struggle to learn concepts, techniques, and applications, it is possible for certain basic skills to get lost in the shuffle. In addition, some students enter the course with weak preparation in algebra and may limp through without correcting this deficiency. Consequently, some students may pass the course without being able to do some of the basics. For example, a student who passes freshman calculus should be able to solve a quadratic equation or differentiate e^{2x}—but some who can't may pass anyway.

Megginson [2] described "gateway testing" as a means of isolating and assessing these basic skills. Many universities include such tests as part of either precalculus or calculus courses. Current examples include Virginia Tech, Harvard, and the Universities of Michigan, Nebraska (Lincoln), Wisconsin (Milwaukee), and North Texas. The purpose, content, grading, and other details of such tests vary significantly. Some programs treat the test as a "barrier" (must pass to get a certain course grade), while others give students a grade bonus for passing the tests (or a penalty for not passing) or simply include a "gateway component" in the course grade. Similarly, some programs test students on prerequisite material, while others assess learning of material in the current course. Standards for passing a gateway test are typically around 80%.

For our freshman calculus program we have instituted a gateway testing program we call the *Calculus ABCs*, standing for *Absolutely Basic Competency*. The ABCs program consists of a set of standards for basic skills, a testing program to certify that students have mastered them, and resources for aiding students who need some review work. The *Calculus ABCs Tests* function as gateway tests in the sense described by Megginson [2]: students must demonstrate mastery of the Calculus ABCs by passing the test at a high level (90% or better) in order to pass freshman calculus with a grade of C or better. While the ABCs were originally run as a pilot program for three years in Calculus II only, the successes documented here have led us to expand the program to Calculus I also.

This paper describes our Calculus ABCs program and evaluates the results. The remainder of the paper is organized as follows. To establish the context, Section 2 briefly describes the freshman calculus course at Clarkson. Section 3 details the ABCs standards, testing program, and resources. Objective and subjective results from the three-year pilot program are summarized in Section 4. Section 5 describes improvements introduced starting in Fall 2002, and Section 6 summarizes our conclusions.

2 Background

Freshman calculus is currently taught at Clarkson in a relatively traditional manner. Calculus I covers functions, limits, derivatives, applications, exponential and logarithmic functions, the definite integral, the Fundamental Theorem of Calculus, and integration by substitution. Calculus II covers techniques and applications of integration (including first-order differential equations), infinite sequences and series, and power and Taylor series, plus some odds and ends. Both courses are taught in a lecture/recitation format, with three hours of lecture by the professor (sections of 60–100 students) and one hour of recitation with a graduate teaching assistant (sections of about 20–30 students) per week. Each course receives three hours of credit.

In the past we have used technology in the course in different ways, including computer-based recitations, locally-produced take-home computer projects [1], and smaller projects from a supplement to a standard text. Currently, the computer algebra system (CAS) Maple is used as a tool primarily in lectures. Basic instruction in using Maple is provided, and students are encouraged (but not required) to use it where appropriate.

Approximately 70% of incoming freshmen take the standard Calculus I/II sequence in their Freshman fall and spring, respectively (another 15% begin directly in Calculus II or above, and the rest take a two-semester applied calculus/finite mathematics or statistics sequence). Of the 450–550 students starting the standard sequence each fall, about 60% are engineering majors and 20% science and mathematics majors, with the rest from business, interdisciplinary programs, and liberal arts. With this high percentage of engineering majors (and the highly structured engineering curriculum) it is important for students both to learn the material solidly (of course!) and to stay on track.

Course grades are reported on the usual A/B/C/D/F scale (with + grades but no − grades). A grade of D or above is considered passing; however, a C or better is regarded as essential for going on to subsequent courses. Students who receive a grade below C are strongly urged to repeat the course; when they do so, the second grade replaces the first in computing their GPA. Thus, we want to ensure that every student who earns a C or better is in fact ready to go on to other courses.

3 Program Description

In the Spring semester in 2000, 2001, and 2002, all students enrolled in Calculus II were required to demonstrate mastery of the Calculus ABCs by passing an ABCs Test. This was a *necessary* condition—not a *sufficient* condition—for getting a grade of C or better in the course. That is, students who passed the ABCs could earn any grade in the course (A to F) based on their performance on the other measures (homework, quizzes, hour exams, and final exam), while those who did not pass the ABCs could earn no higher than a D+.

The material tested covered basic algebra, trigonometry, derivatives, and basic integrals, as specified in detail in the standards listed in Table 1 (these were posted for students on the web). Note that all of this material should have been learned *before* enrolling in Calculus II—either in Calculus I or in high school. This choice of approach (testing only prerequisite material) was made primarily for fairness: for example, testing derivatives near the end of Calculus I would have given an advantage to the many students who had studied some calculus in high school.

The specific standards we chose (knowledge and skills we regard as absolutely basic at this level) reflect the expectations of our faculty and the needs of the engineering program, which is our largest customer. Each of the items listed is either something we view as critical for success in Calculus II or something needed by most students in subsequent courses. For example, factoring quadratics is on the list, since it is needed for some integrals treated in Calculus II. Similarly, computing the derivative of a function like $\sin(5t - 4)$ is on the list, since this skill is needed to understand derivations and work problems in physics and engineering courses. In contrast, implicit differentiation (which we teach in Calculus I and some students need in advanced engineering courses) is not on the list, since it is a more advanced skill that appears less essential. At other institutions a similar list of standards would likely be different; indeed, the range of what one might consider "absolutely basic" is probably large.

Each ABCs Test consisted of 20 problems, which were graded right/wrong (essentially no partial credit), with all answers written out (no "multiple guessing"). A score of 90% or

1. Algebra, Trigonometry, Exponentials, and Logarithms (35%):
    - Find the equation of a given line (in slope-intercept or point-slope form)
    - Simplify an expression or solve an equation using basic algebra
    - Factor a quadratic polynomial (real roots)
    - Evaluate $\sin(x)$, $\cos(x)$, and $\tan(x)$ at integer multiples of $\pi/6$ and $\pi/4$
    - Evaluate $\arcsin(x)$, $\arccos(x)$, and $\arctan(x)$ at arguments corresponding to integer multiples of $\pi/6$ and $\pi/4$
    - Simplify an expression or solve an equation using basic properties of exponentials and logarithms
    - Sketch a graph of $\sin(x)$, $\cos(x)$, $\tan(x)$, e^x, e^{-x}, or $\ln(x)$ (with axis scaling labeled)
2. Derivatives (35%):
    - Find the derivative of any of the following functions:
        - powers, roots, and polynomials
        - trig functions [$\sin(x)$, $\cos(x)$, and $\tan(x)$]
        - exponentials [e^x]
        - logarithms [$\ln(x)$]
        - any of these functions evaluated at $(ax + b)$, where a and b are constants
        - any *linear combination* of these (i.e., multiplied by constants and added)
    - Find the derivative of any combination of the above functions using one or two applications of the product, quotient, and/or chain rules
3. Integrals (30%):
    - Find the antiderivative (indefinite integral) of any of the following functions:
        - powers, roots, and polynomials
        - trig functions [$\sin(x)$ and $\cos(x)$]
        - exponentials [e^x]
        - any of these functions evaluated at $(ax + b)$, where a and b are constants
        - any *linear combination* of these (i.e., multiplied by constants and added)
    - Evaluate integrals by substitution (where a single substitution $u = f(x)$ transforms the integrand to one on the list above)
    - Evaluate definite integrals using the Fundamental Theorem of Calculus (where the antiderivative is either given or on the above list)

Table 1: Test Content: The Calculus ABCs.

better was required to pass, and students could take the test as often as desired (subject to space limitations). In the first two years, new versions of the test were given approximately once per week during the semester: 14 times in Spring 2000 and 12 times in Spring 2001. Students signed up in advance via the web to reserve a space. In most cases the test was given in a lecture hall with seating for 80 students; this required two proctors (graduate TAs) who subsequently graded the test. The standards for grading were posted on the web, so students knew exactly what was expected. Scores in borderline cases were reviewed by a professor before returning the tests.

In Spring 2002 we changed the approach slightly, offering the test only five times during the semester (in the first few weeks) but with enough room to accommodate all students each time. The few students who had not passed after these five chances were given one more chance during final exam week. This change helped students to master the basics early (thus giving them a better base for the rest of the semester), cut down on the administrative work, and allowed us to focus completely on the course material after the first few weeks.

In principle, since the ABCs covered material students already knew, our only role should have been to assess student performance. In practice, of course, many students had not mastered all topics, so we provided help in many ways:

- Review material and problems in the textbook were identified for each of the ABCs topics, so students could work on their weaker areas on their own.
- Practice tests and answers were made available (paper copies and online).
- When ABCs topics came up in the course of lectures they were explicitly identified, stressed, and reviewed (quickly!).
- ABCs topics were reviewed in recitations as needed.
- The usual support network of professors, graduate TAs (both with office hours), undergraduate tutors (both group sessions with required attendance and private tutoring), and drop-in help sessions was available throughout the semester.
- Optional ABCs review sessions were given twice each semester; attendance for these was poor in the first year but better in subsequent years.
- Beginning in Fall 2002, a more aggressive approach was put in place for students who needed significantly more help (see Section 5).

Overall, a student determined to do nothing could do so (as will always be the case); however, those who made the effort to succeed generally did so, as will be shown in the next section.

4 Results

The program ran smoothly all three years. The primary results were:

- In three years (956 students), only *one* student had his grade lowered due to the ABCs[1]
- Objective and anecdotal evidence suggests that many students significantly improved their basic mathematical skills.

4.1 Pass rates

Since not all students took the tests on the same dates, it is most revealing to group the test scores for each student in order (i.e., the first time he/she took the test, the second, etc.). This data is reported in Table 2 for the three years of the pilot program. Note that this table includes data for all students who stuck with the course through at least the first of three hour exams, including those who subsequently dropped the course. Pass rates for the first test are low (except in 2002, where 40% passed the first test), but generally increase for each subsequent test; this is true in spite of the fact that each test includes only those students who have not yet passed—generally those with weaker skills.

Spring 2000	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Total students	313	248	171	107	55	28	13	4	1
Students passed	55	71	58	43	23	13	6	3	1
Median score	75	80	83	83	84	85	83	90	90
Minimum score	15	18	25	40	40	46	70	80	90

Spring 2001	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Total students	310	223	145	83	45	9	2	2	1	1
Students passed	73	63	52	29	26	6	0	1	0	1
Median score	75	77	80	78	90	90	73	80	70	100
Minimum score	15	15	15	30	50	62	68	70	70	100

Spring 2002	1st	2nd	3rd	4th	5th	6th
Total students	333	194	140	72	23	1
Students passed	134	52	63	45	20	1
Median score	83	78	85	90	95	90
Minimum score	23	35	40	53	70	90

Table 2: Student counts and scores for Calculus ABCs tests.

[1] This student passed the ABCs in Spring 2000 but earned only a low D+ for the course; when he repeated the course in Spring 2001 he never registered for the test, showed up for the last test in the last week and scored only 75, thereby lowering his course grade from a low C to a D+.

Figure 1 shows the cumulative fraction of the class which had passed the ABCs test as a function of the test number. While more students than expected had to take the test many times to pass, in the first two years we found that three tries was enough for about 60% and five tries enough for about 80% of the students; very few students who took the test more than this passed it. Based on this result, the test was offered fewer times in the third year as discussed above, and the pass rate was actually better. With the exception of the one student previously mentioned, all those who never passed the ABCs (5–20% of the students who started the class) did not earn a grade above D+ anyway (or dropped the course). Note that values shown in Fig. 1 do not take into account students who dropped the course; of those who completed the course, the percentage of students who passed the ABCs was 92%, 90%, and 97% for the years 2000, 2001, and 2002, respectively.

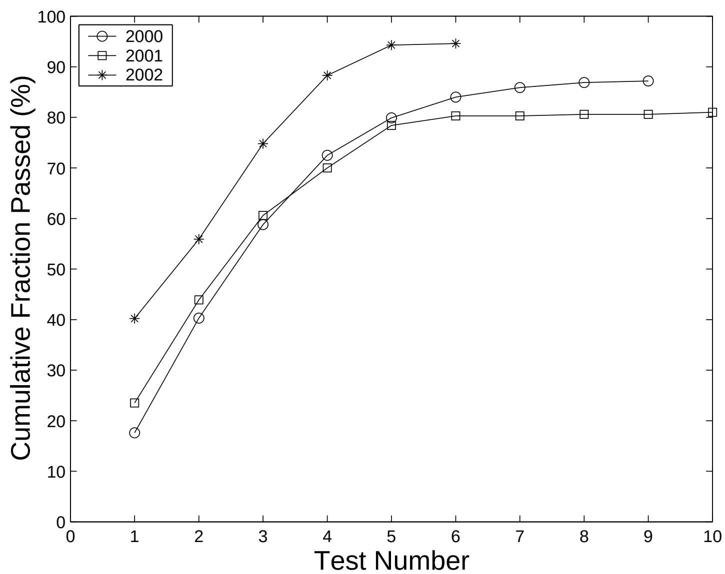

Figure 1: Cumulative fraction of the class having passed the ABCs test.

There was a definite trend of increased scores as students took the test multiple times. Figure 2 shows the median improvement in score from one test to the next. Here, the improvement at test n for a given student is defined as his/her score on test n minus his/her score on test $n-1$; the quantity plotted is the median of these improvements (tests with less than three students are omitted). Thus, for example, in Spring 2000 at least half of the students taking the test a second time increased their score (over their score on their first try) by at least 10 points. All three years showed a steady improvement in scores as students took the test multiple times, with a typical improvement of 5–15 points per try. In short: students who could not at first pass the test generally improved and usually ended up passing. While there may be some effect here of simply "learning how to take the test," given the large number of problems (twenty) and their open-ended nature (no multiple choice), it seems clear that many students were being challenged to learn how to do the basics—and succeeding. This conclusion is reinforced by anecdotal evidence: many students would get something wrong (say, applying the chain rule or evaluating an inverse trig function), discover they didn't understand the topic, ask for help, study, practice, and master it.

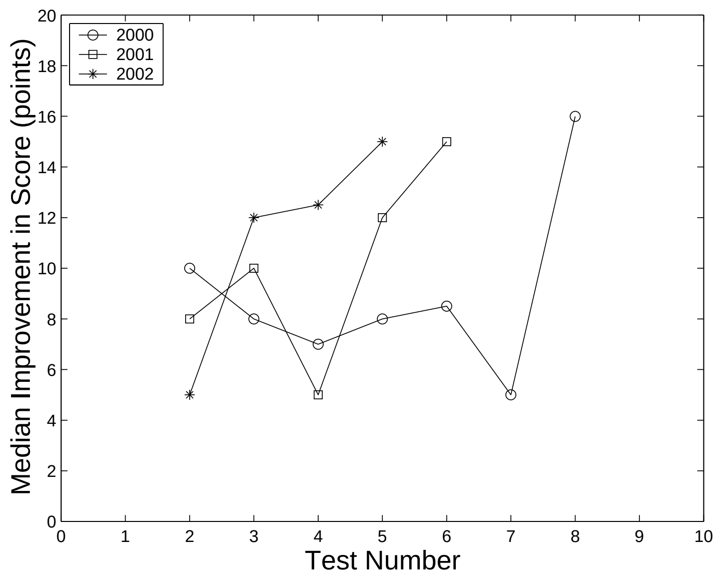

Figure 2: Median improvement in score from one test to the next.

4.2 Additional effects

We found that surprisingly many students were not accustomed to such a high standard: for many, this was the first time anybody had required them to get most answers right (not just to get answers mostly right). Students were forced to get essentially all of the details right for once, rather than just the basic idea. While this came as a shock to some, by the time they had succeeded (as the vast majority did) they had learned important lessons: that such standards exist and that they are capable of meeting them.

On the other hand, some students apparently concluded they would never succeed and gave up, dropping the course. While some students will drop the course in any semester, there was an impression on our part (especially in Spring 2001) that sometimes this was due primarily to not being able to pass an ABCs Test.

4.3 Student reaction

Student reaction was mixed. Predictably, nobody was overly enthusiastic about having to take another test. However, few students voiced complaints directly to the professors during the semester. When asked their opinions individually, more often than not students agreed with the goals of the program and felt it was fair.

To provide a more objective measure, we added four "questions" to the standard course evaluations in Spring 2000. The questions—and the mean response on the scale of 1 (strongly disagree) to 5 (strongly agree)—were:

- The Calculus ABCs Tests covered essential mathematics.
  (response: 4.1)
- Preparing for ABCs Tests was a useful learning experience.
  (response: 3.3)
- Passing an ABCs Test is a reasonable requirement for freshman calculus.
  (response: 3.1)
- Registering for ABCs Tests in advance on the web was relatively easy.
  (response: 4.4)

These responses are generally positive or at worst neutral. However, additional student comments written on the course evaluations were mainly negative. In part, the ABCs provided a focal point for complaints.

4.4 Reaction from other departments

At the beginning of Spring 2000 a memo was sent to the provost and to the dean and department chairs in the School of Engineering outlining the ABCs program and asking for feedback. While few responded, those who did respond were uniformly positive: they liked the whole idea. Subsequently, several faculty from other departments have expressed interest in this program and have requested copies of the tests (indeed, some have used tests of similar material at the beginning of engineering courses as a wake-up call for students). Again, all feedback from other faculty to date has been supportive of our efforts.

5 Improvements

Based on our experiences during the first two years of the pilot program, our department decided to expand the ABCs program and make it a standard part of freshman calculus. Some changes were implemented in the third year of the pilot (Spring 2002) as described in Section 3, e.g., giving the test five times early in the semester. Beyond that, we have extended the ABCs to cover both semesters of freshman calculus. The ABCs for Calculus I now cover only material students should have learned in high school: algebra, trigonometry, exponents and base-10 logarithms, and geometry (areas, perimeters, and volumes of simple regions and solids). The ABCs for Calculus II remain nearly the same as before. The new standards (similar to those in Table 1) are available at the web site `www.clarkson.edu/class/calcabcs`.

Another improvement developed for Fall 2002 is automatic generation of ABCs tests. This uses a test generator (written by Brendan Johnson and soon to be available as open-source software) to construct a test by combining problems from a large database according to a user-supplied template. Each problem in the database is stored as a separate file (a few lines of LaTeX source), and the output is available in PDF format (by default) or Postscript or LaTeX (if desired). With at least 20–50 problems in each of the many categories (e.g., "solve an algebraic equation", "evaluate a trig function", "compute a derivative using the chain rule", etc.), the supply of different tests is enormous. Consequently, we can make them *all* available to students via the web to be used as practice tests; instructors can print the same tests (with modifications if desired) to administer to their classes. The interested reader is directed to the web site noted above to see the actual tests and the test generation process in action.

Also new in Fall 2002 is a more formal mechanism for providing additional help to those students who need it. In addition to the resources listed in Section 3, students who did not pass the ABCs in the five chances given early in the semester continue in calculus but are also placed in a companion course MA041 (Co-Calculus Mathematics). Designed to review and reinforce the prerequisite material for calculus, this course meets two hours per week for the remainder of the semester. In past years many of these students would have struggled through a full semester before failing (or dropping) calculus. We hope that with the current system many will be able to succeed in their first semester.

One area which still needs improvement is in "public relations": some students see the Calculus ABCs as just another way to fail. In contrast, we see the program as a way to identify weaknesses early and to help students solidify skills they will need for subsequent classes. We believe the program has done just that; our challenge is now to communicate this clearly to the students.

6 Conclusion

After three years of experience with the Calculus ABCs program, it appears that it is running smoothly and having a positive impact on student learning. The program gives students a strong incentive to learn basic skills, and allows faculty to certify that each student who passes freshman calculus with a grade of C or better has mastered these skills at a high level of competency (at least at one point in time). While this by no means addresses all of the issues of learning and mastery involved in calculus, we view it as a positive step.

Acknowledgments

Implementing the Calculus ABCs would not have been possible without the help and support of my colleagues Mike Felland, Christos Xenophontos, Ray Toland, Radica Sipcic, and Rodney Croos-Dabrera, and our graduate TAs. I would also like to thank our former and current Department Chairs (Professors David Powers and Peter Turner, respectively) for their enthusiastic support.

References

[1] Fulton, S. R., 1994. Can Maple help in teaching calculus with limited time and staff? In *Maple V: Mathematics and Its Application*, R. J. Lopez, ed. Boston MA: Birkhäuser. 63–70.

[2] Megginson, R. E., 1994. A gateway testing program at the University of Michigan. In *Preparing for a New Calculus*, Anita Solow, ed. MAA Notes Number 36. Washington DC: Mathematical Association of America. 85-88.

Biographical Sketch

Scott Fulton earned a B.A. degree from Kalamazoo College and M.S. and Ph.D. degrees from Colorado State University. Since 1986 he has been at Clarkson University, where he is currently Associate Professor in the Department of Mathematics and Computer Science. Besides teaching, he conducts research in computational and applied mathematics, including numerical methods for PDEs and applications to atmospheric dynamics, hurricanes, and climate modeling.