PROBLEM 8.3

KNOWN: Temperature and velocity of water flow in a pipe of prescribed dimensions.

FIND: Pressure drop and pump power requirement for (a) a smooth pipe, (b) a cast iron pipe with a clean surface, and (c) smooth pipe for a range of mean velocities 0.05 to 1.5 m/s.

SCHEMATIC:

ASSUMPTIONS: (1) Steady, fully developed flow.

PROPERTIES: Table A.6, Water (300 K): \( \rho = 997 \text{ kg/m}^3, \mu = 855 \times 10^{-6} \text{ N} \cdot \text{s/m}^2, \nu = \frac{\mu}{\rho} = 8.576 \times 10^{-7} \text{ m}^2/\text{s}. \)

ANALYSIS: From Eq. 8.22a and 8.22b, the pressure drop and pump power requirement are

\[
\Delta p = f \frac{\rho u_m^2}{2D} L \quad P = \Delta p \dot{V} = \Delta p \left( \pi D^2/4 \right) u_m
\]

The friction factor, \( f \), may be determined from Figure 8.3 or Eq. 8.20 for different relative roughness, \( e/D \), surfaces or from Eq. 8.21 for the smooth condition, \( 3000 \leq \text{Re}_D \leq 5 \times 10^6 \),

\[
f = \left( 0.790 \ln \left( \text{Re}_D \right) - 1.64 \right)^{-2} \quad (3)
\]

where the Reynolds number is

\[
\text{Re}_D = \frac{u_m D}{\nu} = \frac{1 \text{ m/s} \times 0.25 \text{ m}}{8.576 \times 10^{-7} \text{ m}^2/\text{s}} = 2.915 \times 10^5 \quad (4)
\]

(a) Smooth surface: from Eqs. (3), (1) and (2),

\[
f = \left( 0.790 \ln \left( 2.915 \times 10^5 \right) - 1.64 \right)^{-2} = 0.01451
\]

\[
\Delta p = 0.01451 \left( 997 \text{ kg/m}^3 \times 1 \text{ m}^2/\text{s}^2 / 2 \times 0.25 \text{ m} \right) 1000 \text{ m} = 2.89 \times 10^4 \text{ kg/s}^2 \cdot \text{m} = 0.289 \text{ bar} <
\]

\[
P = 2.89 \times 10^4 \text{ N/m}^2 \left( \pi \times 0.25^2 \text{ m}^2/4 \right) 1 \text{ m/s} = 1418 \text{ N} \cdot \text{m/s} = 1.42 \text{ kW} <
\]

(b) Cast iron clean surface: with \( e = 260 \mu \text{m} \), the relative roughness is \( e/D = 260 \times 10^{-6} \text{ m}/0.25 \text{ m} = 1.04 \times 10^{-3} \). From Figure 8.3 or Eq. 8.20 with \( \text{Re}_D = 2.92 \times 10^5 \), find \( f = 0.021 \). Hence,

\[
\Delta p = 0.402 \text{ bar} \quad P = 1.97 \text{ kW} <
\]

(c) Smooth surface: Using IHT with the expressions of part (a), the pressure drop and pump power requirement as a function of mean velocity, \( u_m \), for the range 0.05 \( \leq u_m \leq 1.5 \text{ m/s} \) are computed and plotted below.

Continued...
The pressure drop is a strong function of the mean velocity. So is the pump power since it is proportional to both $\Delta p$ and the mean velocity.

**COMMENTS:**

1. Note that $L/D = 4000 \gg (x_{fd,h}/D) \approx 10$ for turbulent flow and the assumption of fully developed conditions is justified.

2. Surface fouling results in increased surface roughness and increases operating costs through increasing pump power requirements.

3. The *IHT Workspace* used to generate the graphical results follows.

```
// Pressure drop:
deltap = f * rho * um^2 * L / ( 2 * D )  // Eq (1); Eq 8.22a
deltap_bar = deltap / 1.00e5  // Conversion, Pa to bar units
Power = deltap * ( pi * D^2 / 4 ) * um  // Eq (2); Eq 8.22b
Power_kW = Power / 1000  // Useful for scaling graphical result

// Reynolds number and friction factor:
ReD = um * D / nu  // Eq (3)
f = (0.790 * ln (ReD) - 1.64 ) ^ (-2)  // Eq (4); Eq 8.21, smooth surface condition

// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0  // Quality (0=sat liquid or 1=sat vapor)
rho = rho_Tx("Water",Tm,x)  // Density, kg/m^3
nu = nu_Tx("Water",Tm,x)  // Kinematic viscosity, m^2/s

// Assigned variables:
um = 1  // Mean velocity, m/s
Tm = 300  // Mean temperature, K
D = 0.25  // Tube diameter, m
L = 1000  // Tube length, m
```
PROBLEM 8.12

KNOWN: Internal flow with prescribed wall heat flux as a function of distance.

FIND: (a) Beginning with a properly defined differential control volume, the temperature distribution, $T_m(x)$, (b) Outlet temperature, $T_{m,o}$, (c) Sketch $T_m(x)$, and $T_s(x)$ for fully developed and developing flow conditions, and (d) Value of uniform wall flux $q_s''$ (instead of $q_s' = ax$) providing same outlet temperature as found in part (a); sketch $T_m(x)$ and $T_s(x)$ for this heating condition.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Incompressible liquid with negligible viscous dissipation.

PROPERTIES: Table A.6, Water (300 K): $c_p = 4.179 \text{ kJ/kg} \cdot \text{K}$.

ANALYSIS: (a) Applying energy conservation to the control volume above,

$$dq_{\text{conv}} = \dot{m}c_p dT_m$$

where $T_m(x)$ is the mean temperature at any cross-section and $dq_{\text{conv}} = q' \cdot dx$. Hence,

$$ax = \dot{m}c_p \frac{dT_m}{dx}.$$  \(1\)

Separating and integrating with proper limits gives

$$a \int_{x=0}^{x} x dx = \dot{m}c_p \left[ T_m(x) \right]_{T_m,i} - T_m \right]_{x=0}^{x} = \dot{m}c_p \left[ T_m(x) \right] - \frac{ax^2}{2\dot{m}c_p}.$$  \(3,4\)

(b) To find the outlet temperature, let $x = L$, then

$$T_m(L) = T_{m,o} = T_{m,i} + \frac{ax^2}{2\dot{m}c_p}.$$  \(5\)

Solving for $T_{m,o}$, we find

$$T_{m,o} = 27^\circ \text{C} + \frac{20 \text{ W/m}^2 \cdot \text{m}^2}{2 \left( \frac{450 \text{ kg/h}}{3600 \text{ s/h}} \right) \cdot \left( 4179 \text{ J/kg} \cdot \text{K} \right) \cdot 4179 \text{ J/kg} \cdot \text{K}} = 27^\circ \text{C} + 17.2^\circ \text{C} = 44.2^\circ \text{C}.$$  \(<\)

(c) For linear wall heating, $q_s' = ax$, the fluid temperature distribution along the length of the tube is quadratic as prescribed by Eq. (4). From the convection rate equation,

$$q_s' = h(x) \cdot \pi D \left( T_s(x) - T_m(x) \right)$$

For fully developed flow conditions, $h(x) = h$ is a constant; hence, $T_s(x) - T_m(x)$ increases linearly with $x$. For developing conditions, $h(x)$ will decrease with increasing distance along the tube eventually achieving the fully developed value.

Continued...
(d) For uniform wall heat flux heating, the overall energy balance on the tube yields

$$q = q'' \pi DL = \dot{m} c_p \left( T_{m,o} - T_{m,i} \right)$$

Requiring that $T_{m,o} = 44.2\degree C$ from part (a), find

$$q'' = \frac{(450/3600) \text{kg/s} \times 4179 \text{J/kg} \cdot \text{K}(44.2 - 27) \text{K}}{\pi D \times 30 \text{m}} = 95.3 / D \text{ W/m}^2$$

where $D$ is the diameter (m) of the tube which, when specified, would permit determining the required heat flux, $q''$. For uniform heating, Section 8.3.2, we know that $T_m(x)$ will be linear with distance. $T_s(x)$ will also be linear for fully developed conditions and appear as shown below when the flow is developing.

**COMMENTS:**
1. Note that $c_p$ should be evaluated at $T_m = (27 + 44)\degree C/2 = 309 \text{ K}$.
2. Why did we show $T_s(0) = T_m(0)$ for both types of history when the flow was developing?
3. Why must $T_m(x)$ be linear with distance in the case of uniform wall flux heating?
PROBLEM 8.13

KNOWN: Internal flow with constant surface heat flux, \( q_s'' \).

FIND: (a) Qualitative temperature distributions, \( T(x) \), under developing and fully-developed flow, (b) Exit mean temperature for both situations.

SCHEMATIC:

ASSUMPTIONS: (a) Steady-state conditions, (b) Constant properties, (c) Incompressible flow with negligible viscous dissipation.

ANALYSIS: Based upon the analysis leading to Eq. 8.39, note for the case of constant surface heat flux conditions,

\[
\frac{dT_m}{dx} = \text{constant}.
\]

Hence, regardless of whether the hydrodynamic or thermal boundary layer is fully developed, it follows that

\[ T_m(x) \quad \text{is linear and} \]

\[ T_{m,2} \quad \text{will be the same for all flow conditions}. \]

The surface heat flux can also be written, using Eq. 8.27, as

\[ q_s'' = h \left[ T_s(x) - T_m(x) \right]. \]

Under fully-developed flow and thermal conditions, \( h = h_{fd} \) is a constant. When flow is developing \( h > h_{fd} \). Hence, the temperature distributions appear as below.
PROBLEM 8.23

KNOWN: Thin-walled tube experiences sinusoidal heat flux distribution on the wall.

FIND: (a) Total rate of heat transfer from the tube to the fluid, \( q \), (b) Fluid outlet temperature, \( T_{m,o} \), (c) Axial distribution of the wall temperature \( T_s(x) \) and (d) Magnitude and position of the highest wall temperature, and (e) For prescribed conditions, calculate and plot the mean fluid and surface temperatures, \( T_{m,x}(x) \) and \( T_s(x) \), respectively, as a function of distance along the tube; identify features of the distributions; explore the effect of \( \pm 25\% \) changes in the convection coefficient on the distributions.

SCHEMATIC:

\[
q_s^o(x) = q_o^o \sin (\pi x / L)
\]

ASSUMPTIONS: (1) Steady-state conditions, (2) Applicability of Eq. 8.34, (3) Turbulent, fully developed flow.

ANALYSIS: (a) The total rate of heat transfer from the tube to the fluid is

\[
q = \int_0^L q_s^o Pdx = \int_0^D q_o^o \pi D \left( \int_0^\pi \sin (\pi x / L) dx \right) = q_o^o \pi D \left( -\cos (\pi x / L) \right)_0^L = 2DLq_o^o \quad (1)<
\]

(b) The fluid outlet temperature follows from the overall energy balance with knowledge of the total heat rate,

\[
q = \dot{m}c_p (T_{m,o} - T_{m,i}) = 2DLq_o^o \quad T_{m,o} = T_{m,i} + \left( 2DLq_o^o / \dot{m}c_p \right) \quad (2)<
\]

(c) The axial distribution of the wall temperature can be determined from the rate equation

\[
q_s^o = h \left( T_s(x) - T_m(x) \right) \quad T_{s,x} = T_{m,x}(x) + q_s^o / h \quad (3)
\]

where, by combining expressions of parts (a) and (b), \( T_{m,x}(x) \) is

\[
\int_0^x q_s^o Pdx = \dot{m}c_p \left( T_{m,x} - T_{m,i} \right) \quad T_{m,x} = T_{m,i} + \frac{q_o^o \pi D}{\dot{m}c_p} \left( -\cos (\pi x / L) \right)_0^x \quad (4)
\]

Hence, substituting Eq. (4) into (3), find

\[
T_s(x) = T_{m,i} - \int_0^x \frac{DLq_o^o}{\dot{m}c_p} \left[ 1 - \cos (\pi x / L) \right] + \frac{q_o^o}{h} \sin (\pi x / L) \quad (5)<
\]

(d) To determine the location of the maximum wall temperature \( x' \) where \( T_s(x') = T_{s,max} \), set

\[
\frac{dT_s(x)}{dx} = 0 = \frac{DLq_o^o}{\dot{m}c_p} \left[ 1 - \cos (\pi x / L) \right] + \frac{q_o^o}{h} \sin (\pi x / L) \quad (6)
\]

\[
\frac{DLq_o^o}{\dot{m}c_p} \cdot \frac{\pi}{L} \sin (\pi x' / L) + \frac{q_o^o}{h} \cdot \frac{\pi}{L} \cos (\pi x' / L) = 0 \quad \tan (\pi x' / L) = -\frac{q_o^o / h}{DLq_o^o / \dot{m}c_p} = -\frac{\dot{m}c_p}{DLh}
\]

Continued...
PROBLEM 8.23 (Cont.)

\[ x' = \frac{L}{\pi} \tan^{-1} \left( -\frac{mc_p}{DLh} \right) \]  \hspace{1cm} (6)

At this location, the wall temperature is

\[ T_{s,\max} = T_s(x') = T_{m,i} + \frac{DLd''_0}{mc_p} \left[ 1 - \cos \left( \pi x'/L \right) \right] + \frac{q''_0}{h} \sin \left( \pi x'/L \right) \]  \hspace{1cm} (7)

(e) Consider the prescribed conditions for which to compute and plot \( T_m(x) \) and \( T_s(x) \),

\[
\begin{align*}
D &= 40 \text{ mm} & \dot{m} &= 0.025 \text{ kg/s} & h &= 1000 \text{ W/m}^2 \text{K} \\
L &= 4 \text{ m} & c_p &= 4180 \text{ J/kg.K} & q''_0 &= 10,000 \text{ W/m}^2
\end{align*}
\]

Using Eqs. (4) and (5) in IHT, the results are plotted below.

The effect of a lower convection coefficient is to increase the wall temperature. The position of the maximum temperature, \( T_{s,\max} \), moves away from the tube exit with decreasing convection coefficient.

COMMENTS:  (1) Because the flow is fully developed and turbulent, assuming \( h \) is constant along the entire length of the tube is reasonable.

(2) To determine whether the \( T_s(x) \) distribution has a maximum (rather than a minimum), you should evaluate \( d^2T_s(x)/dx^2 \) to show the value is indeed negative.
**PROBLEM 8.31**

**KNOWN:** Thermal conductivity and inner and outer diameters of plastic pipe. Volumetric flow rate and inlet and outlet temperatures of air flow through pipe. Convection coefficient and temperature of water.

**FIND:** Pipe length and fan power requirement.

**SCHEMATIC:**

![Schematic Diagram]

**ASSUMPTIONS:** (1) Steady-state, (2) Negligible heat transfer from air in vertical legs of pipe, (3) Ideal gas with negligible viscous dissipation and pressure variation, (4) Smooth interior surface, (5) Constant properties.

**PROPERTIES:** Table A-4, Air (T_{m,i} = 29°C): \( \rho = 1.155 \text{ kg/m}^3 \), \( c_p = 1007 \text{ J/kg} \cdot \text{K} \), \( \mu = 183.6 \times 10^{-7} \text{ N} \cdot \text{s/m}^2 \), \( k_a = 0.0261 \text{ W/m} \cdot \text{K} \), Pr = 0.707.

**ANALYSIS:** From Eq. (8.45a)

\[
\frac{T_{\infty} - T_{m,o}}{T_{\infty} - T_{m,i}} = \exp \left( -\frac{UA_s}{mc_p} \right)
\]

where, from Eqs. (3.34) and (3.35),

\[
(UA_s)^{-1} = R_{\text{tot}} = \frac{1}{h_{\text{l}}\pi D_1 L} + \frac{\ln(D_0/D_1)}{2\pi L k} + \frac{1}{h_o\pi D_0 L}
\]

With \( m = \rho_i \dot{v}_i = 0.0289 \text{ kg/s} \) and \( Re_D = 4m/\pi D_1 \mu = 13,350 \), flow in the pipe is turbulent. Assuming fully developed flow throughout the pipe, and from Eq. (8.60),

\[
\bar{h}_i = \frac{k_n}{D_i^{0.23}} \left( \frac{Re_D}{Pr^{0.3}} \right)^{4/5} \left( \frac{0.0261 \text{ W/m} \cdot \text{K} \times 0.023}{0.15} \right) = 7.20 \text{ W/m}^2 \cdot \text{K}
\]

\[
(UA_s)^{-1} = \frac{1}{L} \left( \frac{1}{7.21 \text{ W/m}^2 \cdot \text{K} \times \pi^{0.15}} + \frac{\ln(0.17/0.15)}{2\pi \times 0.15 \text{ W/m} \cdot \text{K}} + \frac{1}{1500 \text{ W/m}^2 \cdot \text{K} \times \pi \times 0.17} \right)
\]

\[
UA_s = \frac{L}{(0.294 + 0.133 + 0.001)} = 2.335 \text{ L W/K}
\]

\[
\frac{T_{\infty} - T_{m,o}}{T_{\infty} - T_{m,i}} = \frac{17 - 21}{17 - 29} = 0.333 = \exp \left( -\frac{2.335 L}{0.0289 \text{ kg/s} \times 1007 \text{ J/kg} \cdot \text{K}} \right) = \exp(-0.0802 L)
\]

\[
L = -\ln \left( \frac{0.333}{0.0802} \right) = 13.7 \text{ m}
\]

From Eqs. (8.22a) and (8.22b) and with \( u_{m,i} = \dot{v}_i / \left( \pi D_1^2 / 4 \right) = 1.415 \text{ m/s} \), the fan power is

\[
P = (\Delta p) \dot{v} \approx f \frac{\rho_i u_{m,i}^2}{2 D_1} L \dot{v}_i = 0.0291 \left( \frac{1.155 \text{ kg/m}^3}{2 \left( 0.15 \text{ m} \right)} \right) \frac{(1.415 \text{ m/s})^2}{13.7 \text{ m} \times 0.025 \text{ m}^3 / \text{s}} = 0.077 \text{ W} <
\]

where \( f = (0.790 \ln \text{Re}_D - 1.64)^{-2} = 0.0291 \) from Eq. (8.21).

**COMMENTS:** (1) With \( L/D_1 = 91 \), the assumption of fully developed flow throughout the pipe is justified. (2) The fan power requirement is small, and the process is economical. (3) The resistance to heat transfer associated with convection at the outer surface is negligible.
**PROBLEM 8.74**

**KNOWN:** Inner and outer radii and thermal conductivity of a Teflon tube. Flowrate and temperature of confined water. Heat flux at outer surface and temperature and convection coefficient of ambient air.

**FIND:** Fraction of heat transfer to water and temperature of tube outer surface.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Fully-developed flow, (3) One-dimensional conduction, (4) Negligible tape contact and conduction resistances.

**PROPERTIES:** *Table A-6*, *Water* (*T_m = 290K*): $\mu = 1080 \times 10^{-6}$ kg/s-m, $k = 0.598$ W/m-K, $Pr = 7.56$.

**ANALYSIS:** The outer surface temperature follows from a surface energy balance

$$ q'' = h_o \left( T_{s,o} - T_\infty \right) + \frac{T_{s,o} - T_m}{(r_0 / k) \ln (r_0 / \eta_1) + (r_0 / \eta_1) / h_i} $$

With $Re_D = \frac{4 m}{\pi D \mu} = 4 \left( \frac{0.2kg/s}{0.02m} \right) \left[ \pi \left( \frac{0.02 m}{1080 \times 10^{-6} \text{ kg/s} \cdot \text{m}} \right) \right] = 11,789$ the flow is turbulent and Eq. 8.60 yields

$$ h_i = \frac{0.023Re_D^{4/5}Pr^{0.4}}{(0.02 m)0.023(11,789)^{4/5}(7.56)^{0.4}} = 2792 \text{ W/m}^2 \cdot \text{K}. $$

Hence

$$ 2000 \text{ W/m}^2 = 25 \text{ W/m}^2 \cdot \text{K} \left( T_{s,o} - 300K \right) + \frac{T_{s,o} - 290K}{(0.013m/0.35 \text{ W/m} \cdot \text{K})ln(1.3) + (1.3)/(2792 \text{ W/m}^2 \cdot \text{K})}. $$

and solving for $T_{s,o}$,

$$ T_{s,o} = 308.3 \text{ K}. $$

The heat flux to the air is

$$ q'' = h_o \left( T_{s,o} - T_\infty \right) = 25 \text{ W/m}^2 \cdot \text{K} \left( 308.3 - 300 \right) \text{K} = 207.5 \text{ W/m}^2. $$

Hence,

$$ q'' / q'' = \left( \frac{2000 - 207.5}{2000 \text{ W/m}^2} \right) = 0.90. $$

**COMMENTS:** The resistance to heat transfer by convection to the air substantially exceeds that due to conduction in the teflon and convection in the water. Hence, most of the heat is transferred to the water.
**PROBLEM 8.80**

**KNOWN:** Flow rate and inlet temperature of air passing through a rectangular duct of prescribed dimensions and surface heat flux.

**FIND:** Air and duct surface temperatures at outlet.

**SCHEMATIC:**

![Diagram of a rectangular duct with flow rate and temperatures indicated.]

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Uniform surface heat flux, (3) Constant properties, (4) Atmospheric pressure, (5) Fully developed conditions at duct exit, (6) Ideal gas with negligible viscous dissipation and pressure variation.

**PROPERTIES:** Table A-4, Air \( \bar{T}_m \approx 300 \text{K}, \text{1 atm} \): \( c_p = 1007 \text{J/kg} \cdot \text{K}, \mu = 184.6 \times 10^{-7} \text{N} \cdot \text{s}/\text{m}^2, k = 0.0263 \text{W/m} \cdot \text{K}, \text{Pr} = 0.707.\)

**ANALYSIS:** For this uniform heat flux condition, the heat rate is

\[
q = q_s^* A_s = q_s^* \left[ 2(L \times W) + 2(L \times H) \right]
\]

\[
q = 600 \text{W/m}^2 \left[ 2(1 \text{m} \times 0.016 \text{m}) + 2(1 \text{m} \times 0.004 \text{m}) \right] = 24 \text{ W}.
\]

From an overall energy balance

\[
T_{m,o} = T_{m,i} + \frac{q}{\dot{m} c_p} = 300 \text{K} + \frac{24 \text{ W}}{3 \times 10^{-4} \text{kg/s} \times 1007 \text{ J/kg} \cdot \text{K}} = 379 \text{ K}.
\]

The surface temperature at the outlet may be determined from Newton’s law of cooling, where

\[
T_{s,o} = T_{m,o} + \frac{q^*}{h}.
\]

From Eqs. 8.66 and 8.1

\[
D_h = \frac{4 A_c}{P} = \frac{4 \left(0.016m \times 0.004m\right)}{2 \left(0.016m + 0.004m\right)} = 0.0064 \text{ m}
\]

\[
Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m} D_h}{A_c \mu} = \frac{3 \times 10^{-4} \text{kg/s}(0.0064m)}{64 \times 10^{-6} \text{m}^2 \left(184.6 \times 10^{-7} \text{ N} \cdot \text{s}/\text{m}^2\right)} = 1625.
\]

Hence the flow is laminar, and from Table 8.1

\[
h = \frac{k}{D_h} = \frac{0.0263 \text{ W/m} \cdot \text{K}}{0.0064 \text{ m}} = 5.33 = 22 \text{ W/m}^2 \cdot \text{K}
\]

\[
T_{s,o} = 379 \text{ K} + \frac{600 \text{ W/m}^2}{22 \text{ W/m}^2 \cdot \text{K}} = 406 \text{ K}.
\]

**COMMENTS:** The calculations should be repeated with properties evaluated at \( \bar{T}_m = 340 \text{ K} \). The change in \( T_{m,o} \) would be negligible, and \( T_{s,o} \) would decrease slightly.
PROBLEM 8.84

KNOWN: Temperature and velocity of gas flow between parallel plates of prescribed surface temperature and separation. Thickness and location of plate insert.

FIND: Heat flux to the plates (a) without and (b) with the insert.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Gas has properties of atmospheric air, (4) Plates are of infinite width W, (5) Fully developed flow.

PROPERTIES: Table A-4, Air (1 atm, T_m = 1000 K): \( \rho = 0.348 \text{ kg/m}^3 \), \( \mu = 424.4 \times 10^{-7} \text{ kg/s \cdot m} \), \( k = 0.0667 \text{ W/m \cdot K} \), Pr = 0.726.

ANALYSIS: (a) Based upon the hydraulic diameter \( D_h \), the Reynolds number is

\[
\text{Re}_{D_h} = \frac{\rho u_m D_h}{\mu} = \frac{0.348 \text{ kg/m}^3 (60 \text{ m/s}) 0.08 \text{ m}}{424.4 \times 10^{-7} \text{ kg/s \cdot m}} = 39,360.
\]

Since the flow is fully developed and turbulent, use the Dittus-Boelter correlation,

\[
\text{Nu}_D = 0.023 \text{Re}_{D_h}^{4/5} \text{Pr}^{0.3} = 0.023(39,360)^{4/5} (0.726)^{0.3} = 99.1
\]

\[
h = \frac{k}{\text{Nu}_D} = \frac{0.0667 \text{ W/m \cdot K}}{99.1} = 82.6 \text{ W/m}^2 \cdot \text{K}
\]

\[
q^* = h(T_m - T_s) = 82.6 \text{ W/m}^2 \cdot \text{K}(1000 - 350) \text{K} = 53,700 \text{ W/m}^2.
\]

(b) From continuity,

\[
m = (\rho u_m A)_a = (\rho u_m A)_b, \quad u_m = u_m \quad (\rho A)_a / (\rho A)_b = 60 \text{ m/s}(40/20) = 120 \text{ m/s}.
\]

For each of the resulting channels, \( D_h = 0.02 \text{ m} \) and

\[
\text{Re}_{D_h} = \frac{\rho u_m D_h}{\mu} = \frac{0.348 \text{ kg/m}^3 (120 \text{ m/s}) 0.02 \text{ m}}{424.4 \times 10^{-7} \text{ kg/s \cdot m}} = 19,680.
\]

Since the flow is still turbulent,

\[
\text{Nu}_D = 0.023(19,680)^{4/5} (0.726)^{0.3} = 56.9
\]

\[
h = \frac{56.9(0.0667 \text{ W/m \cdot K})}{0.02 \text{ m}} = 189.8 \text{ W/m}^2 \cdot \text{K}
\]

\[
q^* = 189.8 \text{ W/m}^2 \cdot \text{K}(1000 - 350) \text{K} = 123,400 \text{ W/m}^2.
\]

COMMENTS: From the Dittus-Boelter equation,

\[
h_b / h_a = (u_m,b / u_m,a)^{0.8} (D_{h,a} / D_{h,b})^{0.2} = (2)^{0.8} (4)^{0.2} = 1.74 \times 1.32 = 2.30.
\]

Hence, heat transfer enhancement due to the insert is primarily a result of the increase in \( u_m \) and secondarily a result of the decrease in \( D_h \).
**PROBLEM 8.88**

**KNOWN:** Heat exchanger to warm blood from a storage temperature $10^\circ$C to $37^\circ$ at 200 ml/min. Tubing has rectangular cross-section $6.4 \text{ mm} \times 1.6 \text{ mm}$ sandwiched between plates maintained at $40^\circ$C.

**FIND:** (a) Length of tubing and (b) Assessment of assumptions to indicate whether analysis under- or over-estimates length.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Incompressible liquid with negligible viscous dissipation, (3) Blood flow is fully developed, (4) Blood has properties of water, and (5) Negligible tube wall and contact resistance.

**PROPERTIES:** *Table A-6*, Water ($\overline{T}_m \approx 300 \text{ K}$): $c_{p,f} = 4179 \text{ J/kg} \cdot \text{K}$, $\rho_f = 1/\nu_f = 997 \text{ kg/m}^3$, $\nu_f = \mu_f/\rho_f = 8.58 \times 10^{-7} \text{ m}^2/\text{s}$, $k = 0.613 \text{ W/m} \cdot \text{K}$, $Pr = 5.83$.

**ANALYSIS:** (a) From an overall energy balance and the rate equation,

$$q = m c_p \left( T_{m,o} - T_{m,i} \right) = \overline{h} A_s \Delta T_{LMTD}$$

where

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln \left( \Delta T_1 / \Delta T_2 \right)} = \frac{(40 - 15) - (40 - 37)}{\ln (25/3)} = 11.7^\circ \text{C}.$$  

To estimate $\overline{h}$, find the Reynolds number for the rectangular tube,

$$\text{Re}_D = \frac{u_m D_h}{\nu} = \frac{0.326 \text{ m/s} \times 0.00256 \text{ m}}{8.58 \times 10^{-7} \text{ m}^2/\text{s}} = 973$$

where

$$D_h = 4 A_c / P = 4 \left( 6.4 \text{ mm} \times 1.6 \text{ mm} \right) / 2 \left( 6.4 + 1.6 \right) \text{ mm} = 2.56 \text{ mm}$$

$$A_c = \left( 6.4 \text{ mm} \times 1.6 \text{ mm} \right) = 1.024 \times 10^{-5} \text{ m}^2$$

$$u_m = \dot{m} / \rho A_c = \dot{V} / A_c = 200 \text{ m}^3/60 \text{ s} \left( 10^{-6} \text{ m}^3 / \text{m}^3 \right) / 1.024 \times 10^{-5} \text{ m}^2 = 0.326 \text{ m/s}.$$  

Hence the flow is laminar, but assuming fully developed flow with an isothermal surface from *Table 8.1* with $b/a = 6.4/1.6 = 4$,

$$\text{Nu}_D = \frac{h D_h}{k} = 4.44 \quad h = \frac{4.44 \times 0.613 \text{ W/m} \cdot \text{K}}{0.00256 \text{ m}} = 1063 \text{ W/m}^2 \cdot \text{K}.$$  

Continued …
PROBLEM 8.88 (Cont.)

From Eq. (1) with

\[ A_S = PL = 2(6.4+1.6) \times 10^{-3} \text{ m} \times L = 1.6 \times 10^{-2} L \]

\[ \dot{m} = \rho A_c u_m = 997 \text{ kg/m}^3 \times 1.024 \times 10^{-5} \text{ m}^2 \times 0.326 \text{ m/s} = 3.328 \times 10^{-3} \text{ kg/s} \]

the length of the rectangular tubing can be found from Eq. (1) as

\[ 3.328 \times 10^{-3} \text{ kg/s} \times 4179 \text{ J/kg K} (37 - 10) \text{ K} = 1063 \text{ W/m}^2 \cdot \text{K} \times 1.6 \times 10^{-2} \text{ L m}^2 \times 11.7 \text{ K} \]

\[ L = 1.9 \text{ m}. \]

(b) Consider these comments with regard to whether the analysis under- or over-estimates the length,

\[ \Rightarrow \text{With } x_{fd,h} \approx 0.05D_hRe_D = 0.12 \text{ m and } x_{fd,t} = x_{fd,h}Pr = 0.73 \text{ m, the thermal development may not be negligible and would contribute to increasing heat transfer; the present analysis over predicts the length,} \]

\[ \Rightarrow \text{negligible tube wall resistance - depends upon materials of construction; if plastic, analysis under predicts length,} \]

\[ \Rightarrow \text{negligible thermal contact resistance between tube and heating plate - if present, analysis under predicts length.} \]
PROBLEM 8.98

KNOWN: Heat rate per unit length at the inner surface of an annular recuperator of prescribed dimensions. Flow rate and inlet temperature of air passing through annular region.

FIND: (a) Temperature of air leaving the recuperator, (b) Inner pipe temperature at inlet and outlet and outer pipe temperature at inlet.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform heating of recuperator inner surface, (4) Adiabatic outer surface, (5) Air is ideal gas with negligible viscous dissipation and pressure variation, (6) Fully developed air flow throughout.

PROPERTIES: Table A-4, Air (given): \( c_p = 1030 \text{ J/kg-K} \), \( \mu = 270 \times 10^{-7} \text{ N-s/m}^2 \), \( k = 0.041 \text{ W/m-K} \), \( Pr = 0.68 \).

ANALYSIS: (a) From an energy balance on the air

\[
\dot{q}_i L = \dot{m}_a c_{p,a} (T_{a,2} - T_{a,1})
\]

\[
T_{a,2} = T_{a,1} + \frac{\dot{q}_i L}{\dot{m}_a c_{p,a}} = 300 \text{ K} + \frac{1.25 \times 10^5 \text{ W/m} \times 7 \text{ m}}{2.1 \text{ kg/s} \times 1030 \text{ J/kg-K}} = 704.5 \text{ K}.
\]

(b) The surface temperatures may be evaluated from Eqs. 8.67 and 8.68 with

\[
Re_D = \frac{\rho \frac{u_m D_h}{\mu}}{\mu} = \frac{\dot{m}_a}{\mu} \left( \frac{D_0 - D_i}{\pi/4} \right) \mu = \frac{4 \dot{m}_a}{\pi (D_0 + D_i) \mu} = \frac{4(2.1 \text{ kg/s})}{\pi (4.05 \text{ m}) 270 \times 10^{-7} \text{ N-s/m}^2} = 24,452
\]

The flow is turbulent and from Eq. 8.60

\[
h_i \approx h_o \approx \frac{k}{D_h} 0.023 \text{ Re}_D^{4/5} \text{ Pr}^{0.4} = \frac{0.041 \text{ W/m-K}}{0.05 \text{ m}} 0.023(24,452)^{4/5} (0.68)^{0.4} = 52 \text{ W/m}^2 \cdot \text{K}.
\]

With \( q_i^* = \dot{q}_i / \pi D_1 = 1.25 \times 10^5 \text{ W/m} / \pi \times 2 \text{ m} = 19,900 \text{ W/m}^2 \)

Eq. 8.67 gives

\[
(T_{s,i} - T_m) = q_i^* / h_i = 19,900 \text{ W/m}^2 / 52 \text{ W/m}^2 \cdot \text{K} = 383 \text{ K}
\]

\[
T_{s,i,1} = 683 \text{ K} \quad T_{s,i,2} = 1087 \text{ K}.
\]

From Eq. 8.68, with \( q_o^* = 0 \), \( (T_{s,o} - T_m) = 0 \). Hence

\[
T_{s,o,1} = T_{a,1} = 300 \text{ K}.
\]