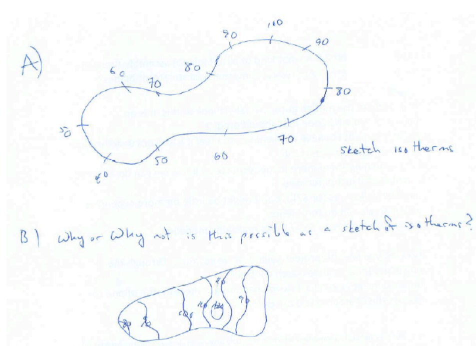
Homework 1: 1.1[2,3,4, just read 5], 1.2[2a-d,4a (but don't bother with the part carried over to page 19),7,10,11], 1.3[4abcd,6(hint-see triangle ineq-1.1.9),9a,10(hint-see 1.2.9-10),just read 7]

Homework 2: 1.3[11,12], 2.1[1a,c,2a,b,3b,e]
***on 3b, I want you to resort to the Cauchy Riemann equations theorem. However, for 3e, it is not my intention for you to refer to CR, not because they do not apply, which they do, but because it is technically very hard to work 3e in the forward argument by CR. Instead, I am intending for you to simply name the singular points, which in turn are points of non-analyticity.

1) Prove that the complex conjugate function, $f(z) = \bar{z}$, is nowhere differentiable. (Hint: consider limits of the difference quotient at any point z_0, by approaching z_0 from different cardinal directions.) (Note...later, in a different problem set, you will be asked the same, but to use CR eqns. Do not use this method today.)
2) Does the sequence, $a_n = i^n, n = 1,2,...$ converge as, $n \to \infty$? If yes, what does it converge to, and if no, then prove why not. (Hint: start by writing out several terms of the sequence, and compare to the definition of a limit on page 20).

Homework 3:

1) Prove that $f(z) = \bar{z}$ is nowhere differentiable using the CR equations.
2) Check that $f(z) = z^3$, and $f(z) = 1/z$, are analytic. Use Maple to plot level curves of each of the real and imaginary parts of the functions $f(z) = z^3$, and $f(z) = 1/z$, and discuss the consequence of of analyticity in terms of how the level curves of these harmonic conjugates cross.
3) Show that if f is analytic in a domain D, and either Re(f(z)) or Im(f(z)) is constant in D, then f(z) must be a constant in D.
4) Verify that the real and imaginary parts of the following functions each satisfy Laplace's equation (and are thus harmonic conjugates): $f(z) = z^2 + 2z + 1$, and $g(z) = 1/z$.
5) Show that if v(x,y) is a harmonic conjugate for u(x,y), then –u is a harmonic conjugate for v.
6) Sketch the isotherms for the following edge – temp. distribution. Does this configuration violate the maximum principle?



Homework 4: 2.2[1b,c], 2.2[2a-d,3a, and READ 3b], 2.4[1,2abd,3ac,5,9], 2.5[1abc,2ab,3]

Homework 5: 2.4[5] (NOTE!!! I made an error…this is NOT meant to be 2.5-5..but given the late date, I will take either…but advice: 2.4-5 is MUCH easier), 2.6 [1a,2a,3], 3.1[4a,5ab]

1) Write out the first 5 terms, and find the sum of the following: $\sum_{j=0}^{\infty}(\frac{i}{3})^j$, and also $\sum_{j=0}^{\infty}\frac{3}{(1+i)^j}$.
2) Use the ratio test to determine convergence of the following: $\sum_{k=17}^{\infty}(\frac{(3+i)^k}{k!}$,
3) Prove that if the sequence $\{z_n\}_{n=1}^{\infty}$ converges, then $(z_n - z_{n-1}) \to 0$ as $n \to \infty$.
4) Use the ratio test to determine the domain of convergence for each the following power-series functions: $f(z) = \sum_{j=1}^{\infty} jz^j$, $g(z) = \sum_{k=0}^{\infty}\frac{(z-i)^k}{2^k}$, and $h(z) = \sum_{j=0}^{\infty}\frac{z^j}{j!}$.

Homework 6: 3.3 [1,2,3]

1) Prove the Ratio Test Theorem: Suppose that the terms of the series $\sum_{j=0}^{\infty} c_j$ have the property that the ratios $|c_{j+1}/c_j|$ approach a Limit L as $j \to \infty$. Then the series converges if $L<1$ and diverges if $L>1$.

(Hint: Look carefully at the Weierstrass-M test. Then look carefully at the theorem concerning convergence of the geometric series. The trick is to show that given the assumed hypothesis, allows you to compare the given series $\sum_{j=0}^{\infty} c_j$ to a judiciously chosen modification of a geometric series, $\sum_{j=0}^{\infty} L_j$. I.e., if $L<1$, choose $\varepsilon > 0$ and show that $|c_{j+1}/c_j| < L + \varepsilon < 1$ for $j \geq J$. Then show that $|c_k| \leq |c_J|(L+\varepsilon)^{k-J}$ for $k>J$ and use the comparison test.

2) Use the Taylor series to verify that:
   a. $\sin(-z) = -\sin(z)$,
   b. $\frac{de^z}{dz} = e^z$
   c. $e^{-iz} = \cos(z) - i\sin(z)$
3) Determine the convergence regions for the Taylor series for the following:

a. $\dfrac{1+z}{1-z}$, around $z_0 = i$.
b. $\dfrac{1+z}{1-z}$, around $z_0 = 0$.
c. e^{-z^2} around $z_0 = 0$.

Homework 7: 3.5 [1a, 2a,b, 3a], 4.1 [1a]

1. Use the unilateral z-transform to solve the following difference equation:

   $a_{n+1} + 2a_n = 1,$
   $a_0 = 1.$
2. Let f(z) have an isolated singularity at z0 and suppose that f(z) is bounded in some punctured neighborhood of z0. Prove directly from the integral theorem formula for the Laurent coefficients that $a_{-j} = 0$, for all j=1, 2, ….; that is, f(z) must have a removable singularity.
3. Without appeal to Picard's theorem, prove the Casorati and Weierstrass theorem. If f(z) has an essential singularity at z0, then in any punctured neighborhood of z0, the function f(z) comes arbitrarily close to any specified complex number. [Hint: Let the specified number be c and assume to the contrary that $| f(z) - c | \geq \delta > 0$ in every small punctured neighborhood of z0. Then using the previous problem, show that f(z)-c (and hence f(z) itself) must either have a pole or a removable singularity.

Homework 8: Congratulations!!! You made it! It's the end of the semester. The birds are out. The sun is out. MOST of the snow storms are behind of…I think. This LAST homework is short and sweet. It is not to hand in, but rather a few prep problems of fair game topics for the final exam:

1. This is a reprise, since it was delayed from the last homework: Use the unilateral z-transform to solve the following difference equation:

   $a_{n+1} + 2a_n = 1,$
   $a_0 = 1.$

2. 4.2 1a, 2a.