Computer Calculus Project 3

Prof Erik Bollt, Due Fri. December 7: Last Day of Class

No Extensions. Start Early!

This represents a serious part of your semester grade. As such, please work it carefully and seriously. A typed and well spelled presentation, with title page, figures, codes, tables, etc, is expected.. In addition you must fill-out and sign the enclosed statement of work and participation sheet.

1. Here we will evaluate, $I = \int_{1/2}^{1} \frac{1}{\sqrt{x + x^4}} dx$
   a. Considering this as an area problem, $f(x) = \frac{1}{\sqrt{x + x^4}}$, graph this function, and an appropriate region, and color the region with your pen to indicate the area.
   b. Consider g(x)=4/3. Using your picture from 1, prove that $\int_{1/2}^{1} \frac{1}{\sqrt{x + x^4}} dx < \int_{1/2}^{1} g(x) dx$.
   c. Estimate $\int_{1/2}^{1} g(x) dx$ by means of the area of one simple Trapezoid, using what you learned about areas of trapezoids back in the day (5^{th}? grade).
   d. Use MATLAB to estimate $I = \int_{1/2}^{1} \frac{1}{\sqrt{x + x^4}} dx$ using the trapezoid rule and Simpson's rule each with 10, 20, 40 and 80 subdivisions.
   e. Use the trapezoid rule and Simpson's rule to obtain $I = \int_{1/2}^{1} \frac{1}{\sqrt{x + x^4}} dx$ with error less than 10^{-6}. Use the error formulas to obtain the numbers of subdivisions required. Print this information and the value of the integral for each method.
   f. Considering convexity, do you expect the trapezoid rule estimate T_n to be an overestimate or an underestimate of the true answer $I = \int_{1/2}^{1} \frac{1}{\sqrt{x + x^4}} dx$. Complete this argument.
   g. Compare your answers in 1e to the back-of-the envelope computation in 1c

2. Consider the function, $I(x) = \int_{x}^{1} \frac{1}{\sqrt{s + s^4}} ds$.

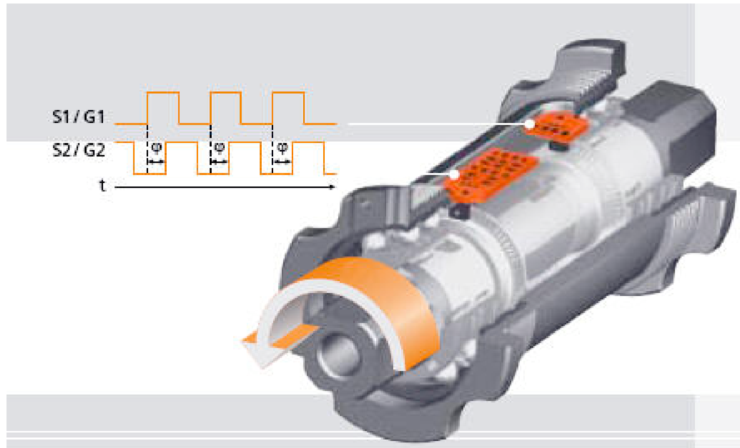
   (Hint, this problem will be easier if you have built a subroutine in part 1f which takes as an input, x and eps (desired precision), and computes the necessary grid size, and outputs I(x).)

a. Make a picture, indicating how it relates to area for, for several indicative values for s.
b. Form the difference quotient, as part of the definition of the derivative,

$$I'(x) \approx \frac{I(x+h)-I(x)}{h} = \frac{\int_{x+h}^{1} \frac{1}{\sqrt{s+s^4}}\,ds - \int_{x}^{1} \frac{1}{\sqrt{s+s^4}}\,ds,}{h}.$$

   i. Compute these estimates for several values of $h = 10^{-i}$,i=1,2,3,4,5., when x=3/4, and use an appropriate grid from 1g to be sure that your estimates of each integral are of order at least 10^{-7}.
   ii. Also compute the EXACT value of I'(3/4) using the fundamental theorem of calculus.
   iii. Make a table, with LABELLED columns, i, h, I'(3/4) estimates, I'(3/4) exact, and error.

c.
   i. Make a plot of I(x), for x in the range ½ to 1.
   ii. Make a plot also of I'(x), (using FTC is okay here).
   iii. Use bisection method to compute x such that I(x)=0.15. (Using bisection method includes the justifications necessary to prove that there is a root by IVT)..
   iv. Use Newton's method (this is a new code for you), to find the same root as in 2c-iii.

3. A Bicycle Time Trial Race. The latest and greatest in the excersise physiology of bicycle racing is to measure power, in watts, during a time trial, the goal being to ride as close as possible to the limit, but not to explode. On the left is a famous American bicycle racer. There are a number of devices for collecting power of a rider on the road. One popular method, based on strain guages leads to the SRM device. On the right, however, is another device called an ergomo, http://www.bicyclepowermeters.com/ It works as a bottom bracket (the bearing spindle around which the crank spins that the rider pedals). As power is applied, the bottom bracket torques. A bar code – like the lines at the supermarket code reader, is written on the side of the spindle as black lines (inside the device), and a laser reads the small changes in timing due to minute wobbles occuring at varying torques. Consider, power=force * speed. Please download the file 'ITT_8_14_2007.txt', in which you will find power and speed data from a bicycle racer in a 10 mile time trial..on 8-14-2007. The the columns of 'ITT_8_14_2007.txt', are

Minutes, Km/h, Watts, Km, Cadence, Hrate, ID

where you can see that in this data set, some of the data is not collected. It is an out and back course, as is the normal practice to eliminate any atmospheric advantages/disadvantages of the day.

a. Plot, a) speed vs. time, b) cadence vs time, and d) heart rate vs time. Now find the turn around point, and plot velocity vs time.

b. Use the trapezoid rule, and the fixed time grid forced by the data set given. Compute the displacement function, using $s(t) = \int_0^t v(q)dq$, and plot it. Is this so-called "10 mile time trial" really a 10 miler?

c. What was the average speed on this out and back course? What was the average velocity on this out and back course?

d. Compute the energy expended as a function of time, where $E(t) = \int_0^t P(q)dq$. Include units!