Algebraic Graph Theory (C. Tamon)
Optional Problem 0

For a positive real-valued $\alpha$, consider the following weighted graph called $K_2(\alpha)$:

whose adjacency matrix is given by the following $2 \times 2$ matrix

$$A = \begin{bmatrix} 0 & 1 \\ 1 & \alpha \end{bmatrix}. $$

1. Find the eigenvalues of $A$.
2. Find the orthonormal eigenvectors of $A$.
3. Write the eigenprojectors\(^1\) of $A$.
4. (Spectral Theorem) Write the spectral decomposition of $A$ in terms of its eigenvalues and eigenprojectors.
5. Compute the matrix exponential $U(t) = e^{itA}$, where $t \in \mathbb{R}$.
6. (True/False) For any $t > 0$, the entries of $U(t)$ are nonzero.

References


\(^1\)These are the projectors onto the eigenspaces of $A$. They are called constituent matrices in Lancaster (1969).