Integer Partitions:

The number of partitions of the non-negative whole number $n$ is simply the number of ways that counting numbers sum to $n$. For example, the partitions of 4 are

$$4, \ 3+1, \ 2+2, \ 2+1+1, \ \text{and} \ 1+1+1+1$$

and we write $p(4) = 5$.

The restricted partition function $p(n, m)$ enumerates the partitions of $n$ into exactly $m$ parts. The relationship between $p(n)$ and $p(n, m)$ is:

$$p(n) = p(n, 1) + p(n, 2) + \cdots + p(n, n-1) + p(n, n)$$

In this summer session we have the option to explore several open questions regarding $p(n, m)$. The open questions include divisibility properties of $p(n, m)$; combinatorial witnesses to divisibility; asymptotic behaviour including log concavity; etc.

The research involved when studying integer partitions is a mixed bag of classical and modern number theory and combinatorics. Creativity is at premium. In addition to classical techniques of $q$-series, we will explore integer partitions with techniques from the relatively new fields of polyhedral geometry and Ehrhart theory. Be warned: the classical and modern techniques overlap in surprising ways that are still being researched.

Suggested background: For the classical techniques all that is required is a bit of elementary number theory, familiarity with the geometric series, and some basic combinatorics. For the polyhedral geometry a strong background in topology, abstract algebra and combinatorics is required.

$$p(6k, 3) = 0\binom{k+2}{2} + 3\binom{k+1}{2} + 3\binom{k}{2}$$
$$p(6k+1, 3) = 0\binom{k+2}{2} + 4\binom{k+1}{2} + 2\binom{k}{2}$$
$$p(6k+2, 3) = 0\binom{k+2}{2} + 5\binom{k+1}{2} + 1\binom{k}{2}$$
$$p(6k+3, 3) = 1\binom{k+2}{2} + 4\binom{k+1}{2} + 1\binom{k}{2}$$
$$p(6k+4, 3) = 1\binom{k+2}{2} + 5\binom{k+1}{2} + 0\binom{k}{2}$$
$$p(6k+5, 3) = 2\binom{k+2}{2} + 4\binom{k+1}{2} + 0\binom{k}{2}$$