

CS447/CS547/EE667 Exam 2 - November 20, 2008

This is a closed-book, closed notes examination. There are four problems. Do not spend too much time on any problem. Read them all first and solve them in the order that allows you to make the most progress.

1. You do not have to show your work for this problem:

(A) Solve $T(n) = 8T(n/3) + n^2$

(B) Solve $T(n) = 9T(n/3) + n^2$

(C) Solve $T(n) = 10T(n/3) + n^2$

2. (A) Write a divide and conquer function called `sum_even` that takes an array, a beginning position and an ending position as arguments, and returns a boolean value: `true` if the sum of the numbers between those two positions of the array is even, `false` if the sum is odd. Notice that your function is required to return only a boolean value. You may not use any other functions or global variables. You will need to use the mod operator (or if you prefer, you can assume there exists a boolean function to tell you if a number is even).
(B) Write a recurrence equation for the running time of your program.
(C) Guess the solution to your recurrence equation, and prove it by induction.

3. Give a dynamic programming algorithm to find the length of the longest sorted subarray of an array which ends with the last number in the array. The subarray does not need to be consecutive. For example, if the array is $[5, 3, 2, 8, 6, 9, 4, 10, 1, 7]$ then the answer is 3, because $[2, 6, 7]$ is the longest sorted subarray ending in 7.

4. Consider a directed graph $G = (V, E)$, with $V = \{s, u, v, t\}$, and $E = \{(s, u), (s, v), (u, v), (u, t), (v, t)\}$. The capacities on the edges are $c(s, u) = 20, c(s, v) = 20, c(u, v) = 30, c(u, t) = 10, c(v, t) = 20$. Suppose the network flow algorithm has already come up with the following flow: $f(s, u) = 20, f(u, v) = 20, f(v, t) = 20$.
- (A) Draw this graph.
 - (B) Draw the residual graph.
 - (C) Draw the augmented graph.
 - (D) Draw the new residual graph
 - (E) Give the value of the max-flow.
 - (F) What is the resulting min-cut? (not just the value)