CS447/CS547/EE667 Exam 1 - October 21, 2008

This is a closed-book, closed notes examination. There are three problems. Do not spend too much time on any problem. Read them all first and solve them in the order that allows you to make the most progress.

1. (A) Prove that $\sum_{i=1}^{n} i^{10} = \Theta(n^{11})$

   (B) Prove that $2^n = o(4^n)$

2. Suppose an airline only flies between certain pairs of cities, and you are given a list of the pairs of cities it flies between, along with the cost for each flight.

   (A) Give an algorithm to give a trip from city A to city B with the fewest cities possible. If there is more than one trip with the fewest cities possible, then you prefer a trip with the cheapest total cost.

   (B) Give an algorithm to give a trip from city A to city B with the cheapest total cost. If there is more than one trip with the cheapest total cost, then you prefer a trip with the fewest cities possible.

3. Suppose you are given an amount N of money and you want to determine how to give change for that amount, with the fewest coins. There are four values of coins: quarter (25 cents), dime (10 cents), nickel (5 cents) and penny (1 cent). Here is a greedy algorithm to solve the problem:

   1. Give quarters until $N < 25$

   2. Give dimes until $N < 10$

   3. Give nickels until $N < 5$

   4. Give pennies until $N < 1$

   In other words, you continually give the biggest coin you can, until you can't anymore.

   (A) Prove that algorithm is optimal (gives the fewest coins).

   (B) Show that there are coin values where the greedy strategy of continually giving the biggest coin you can is not always optimal. You must assume that at least one of your coin values is a one cent coin, so that every amount can be made.