1 Introduction

The recent advances in photonics and laser instrumentation, such as pulsed laser, laser interferometry, and infrared cameras, have been creating a favorable environment for thermal-based elastic wave generation techniques and their applications in various fields, such as nondestructive testing and smart structures. The main advantages of thermal and laser based NDE include noncontact evaluation, freedom for complex surface geometry, high spatial and temporal resolution, easy access to cavities, and fast scanning. Two major disadvantages are that the method requires a good physical understanding of thermoelastic wave propagation in solids, which is considerably more complicated than elastic wave propagation, and more complicated instrumentation is needed for data collection.

``In an idealized solid, thermal energy is transported by two different mechanisms: by quantized electronic excitations, which are called free electrons, and the quanta of lattice vibrations, which are called phonons. These quanta undergo collisions of a dissipative nature, giving rise to thermal resistance in the medium. A relaxation time $\tau_0$ is associated with the average communication time between these collisions for the commencement of resistive flow” [1,2].

There are a number of optical methods available for elastic wave generation and detection. The most commonly utilized techniques include interferometric and noninterferometric techniques, optical heterodyning, differential interferometry and time-delay interferometry. Monchalin [3] evaluates a number of possible ways of using these optical techniques in laser-based elastic wave generation for a large spectrum of applications, and discusses advantages and disadvantages of these methods in detail.

The presented formulation has its root at the results reported in [4], which is one of the earlier works on the transient thermoelastic propagation in a layer. They studied the thermoelastic wave propagation in a thermoelastic layer due to a laser beam using the Laplace and Hankel Transforms. Since they have not considered the second sound effect, the arrival times presented some physical inconsistencies in transient responses of the layer which are calculated using the residues theorem to invert the transfer functions.


Based on the generalized dynamical theory of thermoelasticity, a transfer matrix formulation including the second sound effect is developed for longitudinal wave component propagation in a thermoelastic layer. The second sound effect is included to eliminate the thermal wave travelling with infinite velocity as predicted by the diffusion heat transfer model. Using this formulation and the periodic systems framework, the attenuation and propagation properties of one-dimensional thermoelastic waves in both continuum and layered structures are studied. Strong localization of thermal waves predicted by the analysis in the transformed domain is demonstrated in the temporal-spatial domain by an FFT-based transient analysis. Also, a perturbation analysis for identifying leading terms in thermal attenuation is performed, and the role of the thermal elastic coupling term in attenuation is determined. The attenuation factor, defined as the real part of the propagation constant, is obtained in thermoelastic solids. The reflection and transmission coefficients between half-spaces are also calculated to evaluate the potential practical use of the approach in thermal-based nondestructive testing. [S0739-3717(00)00403-7]
or discontinuity. The thermal wave localization predicted by the transformed domain analysis and its extent are demonstrated by a set of transient response results.

A more complete review of works in the generalized thermoelasticity literature can be found in [5,9–11]. An extensive account of the recent periodic systems research has been given in [12].

2 Formulation

The linearized governing equations for the displacement vector $(u)$ and temperature $(T)$ fields for an isotropic medium consist of two coupled partial differential equations, the equation of motion and energy equation:

$$
(\lambda + 2\mu) \nabla (\nabla \cdot u) - \mu \nabla \times (\nabla \times u) - \beta \nabla T + p_0 (f - \ddot{u}) = 0
$$

(1)

$$
- \kappa \nabla^2 T - p_0 h + T_0 \beta \nabla \cdot \mathbf{u} + \gamma (T + \tau_0 \ddot{T}) = 0
$$

(2)

where the pair $(\lambda, \mu)$ are the Lamé constants of the material, $p_0$ is the material density, $\kappa$ is the thermal conductivity, $\gamma$ is the specific heat, $h$ is the internal heat source intensity, $T_0$ is the temperature at the normal state, $\beta = 3\kappa a$ is the thermoelastic coupling ($K$ is the bulk modulus and $a$ is the linear expansion coefficient), $\tau_0$ is the relaxation time and, at an overdot represents differentiation with respect to time. These coupled equations along with the following constitutive relation and Fourier’s law for isotropic materials in the forms of

$$
\sigma_{ij} = -\beta T \delta_{ij} + \lambda u_{\nu,\nu}\delta_{ij} + \mu (u_{ij,\nu} + u_{ji,\nu})
$$

(3)

$$
\dot{q}_i + \tau_0 \ddot{q}_i = -\kappa T
$$

(4)

where $\dot{q}_i$ is the heat flux vector and $\delta_{ij}$ is the Kronecker delta and the index following the comma in the subscript indicates differentiation with respect to the corresponding coordinate. These four equations form the generalized dynamic theory of linear thermoelasticity [8].

Unlike the classical heat equation, which is of parabolic type, in the generalized theory, the energy equation (Eq. (2)) contains a term with $T$, and, consequently, it is a hyperbolic equation as the equation of motion (Eq. (3)). This offsets the problem of infinite velocity thermal wave propagation predicted by the parabolic heat diffusion equation. The propagation of thermal waves (the second sound effect) was first postulated by Maxwell in 1867 [13] on the physical argument that heat pulses cannot propagate with infinite velocity in matter. For the first time, Peskiwosh [14] demonstrated experimentally thermal wave propagation at finite velocity in liquid helium in 1944. However, it appears that the experimental verification in solids has been an open problem for a long time [1,2].

To produce a transfer matrix formulation for a longitudinal wave propagation study, the Laplace transform (with respect to the scaled time $\tau$) of Eqs. (1) and (2) along with Eqs. (3) and (4) in the displacement potential function and temperature are performed in one-dimension. Only one component of the vector potential is needed in one-dimensional analysis, since the shear component in the propagation direction vanishes. The resultant equations in this potential function are given as

$$
\frac{-p^2}{a^2} \Phi + \frac{d^2 \Phi}{d\xi^2} - \frac{H \beta}{c_\perp p_0} \ddot{T} = 0
$$

(5)

$$
p_0 \gamma \frac{c_T}{H} \ddot{T} - p_0 h + T_0 \beta \frac{c_T}{H} p \frac{d^2 \Phi}{d\xi^2} + \frac{k}{H^2} \frac{d^2 T_0}{d\xi^2} = 0
$$

(6)

where $\xi = x/H$ is the scaled coordinate, $\tau = c_T t/\hbar$ is the scaled time $(\tau)$, $\Phi - \Phi/H$, the scaled scalar potential function, $H$ is a characteristic distance (e.g., in the following sections, $H$ will be taken as the thickness of a layer), $p$ is the Laplace variable, the overbar represents a Laplace transformed field variable, and $a = c_\perp/c_T$, $c_L$ and $c_T$ are the propagation speeds of irrotational isothermal and equivoluminal isothermal waves in an isotropic medium, respectively.

The solutions to Eqs. (5) and (6) with no internal heat source $(h = 0)$ are obtained as

$$
\ddot{T} = A_1 C_1(p) e^{-c_T \xi} - A_2 C_1(p) e^{c_T \xi} + A_3 C_2(p) e^{-\xi} + A_4 C_2(p) e^{\xi}
$$

(7)

$$
\ddot{\Phi} = A_1 E_1(p) e^{-c_T \xi} - A_2 E_1(p) e^{c_T \xi} + A_3 E_2(p) e^{-\xi} + A_4 E_2(p) e^{\xi}
$$

(8)

where

$$
c_1 = \frac{1}{\sqrt{2a_3}} \left(-a_1 - a_3 b_1 + a_2 b_2 \right)
$$

$$
c_2 = \frac{1}{\sqrt{2a_3}} \left(-4 a_1 a_3 b_1 + (a_1 + a_3 b_1 - a_2 b_2) a_3 \right)
$$

$$
c_3 = \frac{1}{a_2} \left(1 + \frac{c_T}{H} \frac{p_0}{\gamma} \right) \frac{c_T}{H} \rho \frac{C_1(p)}{b_2} - \frac{1}{b_2} \left(1 + \frac{b_1}{d_1} \right)
$$

(9)

$$
q_{\xi} = \frac{-\kappa}{1 + p \tau_0 H} \ddot{T}
$$

From the constitutive relation (Eq. (3)), the displacement and normal stress component are written as

$$
\frac{\ddot{\Phi}}{\ddot{T}} = \frac{\ddot{T}}{\ddot{\Phi}}
$$

(10)

$$
\frac{\ddot{\Phi}}{\ddot{T}} = \frac{(\lambda + 2\mu)}{H} \frac{d^2 \Phi}{d\xi^2} - \beta \ddot{T}
$$

(11)

Substituting the scalar potential and temperature fields, Eqs. (5) and (6), into Eqs. (9), (10), and (11), a matrix formulation for the integration coefficients $A_1$, $A_2$, $A_3$, and $A_4$ are obtained:

$$
[T(\xi)] [E_1 A_1 E_2 A_2 E_3 A_3 E_4 A_4]^T = [\ddot{u}_i \ddot{\Phi}_{\xi} \ddot{T} \ddot{q}_{\xi}]^T
$$

(12)

where the superscript $T$ stands for the transpose operation and the matrix $[T(\xi)]$ is written as

Transactions of the ASME
where \( C_{e1} = c_1 / E_1 = (c_1 / b_2)(c_1 + b_1 / c_1) \) and \( C_{e2} = c_2 / E_2 = (d_1 / b_2)(d_1 + b_1 / d_1) \). From this formulation, a transfer matrix for a single layer defined in the spatial interval \( \xi \in [0,1] \) is generated by eliminating the integration coefficients by specifying the displacement, stress, temperature and heat flux on both surfaces of the layer:

\[
\begin{bmatrix}
-c_1 e^{-\xi_1} & -c_1 e^{\xi_1} & -d_1 e^{-\xi_1} & d_1 e^{\xi_1} \\
(\lambda + 2 \mu) c_1 - \beta C_{e1} \right) e^{-\xi_1} & -d_1 e^{-\xi_1} & -d_1 e^{\xi_1} & d_1 e^{\xi_1} \\
C_{e1} e^{-\xi_1} & C_{e1} e^{\xi_1} & C_{e2} e^{-\xi_1} & C_{e2} e^{\xi_1} \\
\frac{C_{e1} \kappa c_1}{H(1 + p Z_0)} e^{-\xi_1} & \frac{C_{e1} \kappa c_1}{H(1 + p Z_0)} e^{\xi_1} & \frac{C_{e2} \kappa d_1}{H(1 + p Z_0)} e^{-\xi_1} & \frac{C_{e2} \kappa d_1}{H(1 + p Z_0)} e^{\xi_1}
\end{bmatrix}
\]

(13)

Recognizing from Eq. (13) that the coefficient matrix can be represented in the following decomposed form:

\[
[T(1)] [T(0)]^{-1} \begin{bmatrix} \bar{u}_{\xi} \bar{\sigma}_{\xi\xi} \bar{T} \bar{q}_{\delta}(L) \end{bmatrix}^T
= \begin{bmatrix} \bar{u}_{\xi} \bar{\sigma}_{\xi\xi} \bar{T} \bar{q}_{\delta}(L) \end{bmatrix}^T
\]

(14)

By eliminating the integration constants in these two boundary conditions, one obtains the following matrix equation:

\[
[T(1)] [T(0)]^{-1} [\bar{u}_{\xi} \bar{\sigma}_{\xi\xi} \bar{T} \bar{q}_{\delta}(L)]^T
= \begin{bmatrix} \bar{u}_{\xi} \bar{\sigma}_{\xi\xi} \bar{T} \bar{q}_{\delta}(L) \end{bmatrix}^T
\]

Fig. 1 The directions of the incident P-wave (solid arrow) and the reflected and transmitted P- and thermal waves in half-spaces A and B.
not only this representation simplifies the transfer matrix formulation, but it also provides a useful insight in dealing with inherent numerical instabilities (due to loss of precision) encountered in transfer matrix-based wave propagation analysis. Therefore, the coefficient matrix of the left-hand side of Eq. (14) \([T(1)][T(0)]^{-1}\) can be written as \([T(0)][L][T(0)]^{-1}\) which implies a similarity transformation between the transfer matrix for a layer and the matrix \([L]\).

Since the matrix \([L]\) is diagonal, the entries of this matrix are the eigenvalues of the transfer matrix and the columns of \([T(0)]\) correspond to the eigenvectors of the transfer matrix. Consequently, the condition number for the transfer matrix is the maximum of \(e^{|c_1|}\) or \(e^{|d_1|}\). Similarly, for another layer, i.e., layer B in this work, the similarity transformation matrices become \([T(0)]_B[L][T(0)]_B^{-1}\). The transfer matrix for a bi-periodic set consisting of two layers, e.g. layer A and layer B, is represented as

\[
[T]_{set} = [T]_B[T]_A
\]

\[
=[T(0)]_B[L][T(0)]_B^{-1}[T(0)]_A[L][T(0)]_A^{-1}
\]

As applied in [15], the thermoelastic wave propagation in infinite and finite layered structures can be studied by utilizing this transfer matrix formulation. The transfer functions between various quantities at any point can be generated by imposing boundary conditions.

3 Attenuation and Propagation Characteristics

The attenuation and propagation properties of thermoelastic waves are determined by the terms \(c_1\) and \(d_1\). It is evident from Eqs. (7) and (8) that, while the frequencies resulting in purely imaginary values for \(c_1\) and \(d_1\) propagate, the real parts of these exponents determine the rate of attenuation. It is observed that the real part of the exponent \(c_1\) is very small compared to that of the exponent \(d_1\). The longitudinal wave mode \(P\) corresponds to \(c_1\) and for the purely elastic case, this mode propagates without any attenuation. As the following perturbation analysis indicates, the attenuation stems from the interactions between thermal and elastic effects. For the ranges of the coefficients and variables in System I and System II, the following observations are made about the orders of the coefficients in \(c_1\) and \(d_1\) terms. Note that all the units are in the International System of Units (SI) except the radial frequency \(\omega\), which corresponds to the scaled time \(\tau\). Here the Laplace transform variable \(p\) and the radial frequency \(\omega\) are related as \(p = -i\omega\).

Fig. 2 \(c_1\) and \(d_1\) for (a) System I and (b) System II materials and layer thicknesses
A perturbation analysis is performed by taking $O(10^5) = 1/e^2$, and the following approximations for the terms $c_1$ and $d_1$ are obtained.

$$c_1 = i \left[ \frac{1}{a} \left( \frac{\beta^2 c_1 T_0}{2 \gamma c_1^2 p_0} \right) \omega + \frac{\beta^2 k T_0}{2 a^2} \right] \omega^2 + H.O.T$$

$$d_1 = \frac{1}{2 \sqrt{2}} (1 + i) \left[ \sqrt{\frac{H c T_0}{\gamma k c_1^2 p_0}} + \sqrt{\frac{H c T_0}{\kappa}} \right] \sqrt{\omega^2 + H.O.T}$$

The orders of terms in these expressions are as follows:

<table>
<thead>
<tr>
<th>term</th>
<th>$\tau_0$</th>
<th>$\omega$</th>
<th>$c_1$</th>
<th>$\rho_0 c_L c_T c_0$</th>
<th>$\beta$</th>
<th>$\lambda_1 \mu_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>order</td>
<td>$O(10^{-13})$</td>
<td>$O(10^{-3})$</td>
<td>$O(10^0)$</td>
<td>$O(10^0)$</td>
<td>$O(10^0)$</td>
<td>$O(10^0)$</td>
</tr>
</tbody>
</table>

The largest real term in $c_1$ is the order of $e^2$ and is proportional to $\omega^2$. The coefficient $\beta$ is the only coupling term in the governing equations (Eqs. (1) and (2)). For the decoupled system, $\beta$ vanishes, and $c_1$ becomes the exponent associated with the $P$-waves component. It is also noteworthy that these terms contain no relaxation time in them and thus the relaxation plays no role in the attenuation. The largest real and imaginary parts of $d_1$, corresponding to the thermal wave component, are equal to each other, and their order is $1/e$. Consequently, this wave mode is highly attenuated and travels at a low speed.

### 4 Reflection and Transmission at the Interfaces

The reflection and transmission coefficients defined on the displacement and temperature components of the wave modes are computed for the interfaces of two half-spaces. In general, the harmonic components of the one-dimensional bulk waves are characterized by eight reflection and propagation coefficients for a generic thermoelastic medium: two for each propagating mode in both directions (Fig. 1). In the previous section, the expressions for the harmonic displacement and temperature are obtained as:

$$\vec{u} = A_1 e^{-ci \xi} - A_2 e^{+ci \xi} + A_3 e^{-d \xi} + A_4 e^{+d \xi}$$

$$\vec{T} = A_1 C_1(p) e^{-ci \xi} - A_2 C_1(p) e^{+ci \xi} + A_3 C_2(p) e^{-d \xi} + A_4 C_2(p) e^{+d \xi}$$

$$\bar{u} = A_1 e^{-ci \xi} - A_2 e^{+ci \xi} + A_3 e^{-d \xi} + A_4 e^{+d \xi}$$

$$\bar{T} = A_1 C_1(p) e^{-ci \xi} - A_2 C_1(p) e^{+ci \xi} + A_3 C_2(p) e^{-d \xi} + A_4 C_2(p) e^{+d \xi}$$

The largest real term in $c_1$ is the order of $e^2$ and is proportional to $\omega^2$. The coefficient $\beta$ is the only coupling term in the governing equations (Eqs. (1) and (2)). For the decoupled system, $\beta$ vanishes, and $c_1$ becomes the exponent associated with the $P$-waves component. It is also noteworthy that these terms contain no relaxation time in them and thus the relaxation plays no role in the attenuation. The largest real and imaginary parts of $d_1$, corresponding to the thermal wave component, are equal to each other, and their order is $1/e$. Consequently, this wave mode is highly attenuated and travels at a low speed.
Here the coefficients \( A_1 \) and \( A_3 \) correspond to the harmonic amplitudes of the two displacement wave modes propagating in the positive-direction at two distinct wave speeds corresponding to \( c_1 \) and \( d_1 \), while \( A_2 \) and \( A_4 \) are for the wave propagating in the opposite direction. As indicated in the previous section, the longitudinal (P) wave mode corresponding to \( c_1 \) is lightly attenuated, while the wave components corresponding to \( d_1 \) are highly attenuated.

A study of reflection of transmission coefficients in two bonded half-spaces provides a useful insight about the behavior of the thermoelastic wave in a layered structure. With the transfer matrix formulation at hand, the reflection and transmission coefficients are readily obtained by solving a set of linear algebraic equations stemming from matching boundary conditions at the interface:

\[
[T(0)]_A \{ E_1 A_1 \ E_1 A_2 \ E_2 A_3 \ E_2 A_4 \}^T_A = [T(0)]_B \{ E_1 A_1 \ E_1 A_2 \ E_2 A_3 \ E_2 A_4 \}^B
\]

Specifying a P-wave propagating in the positive \( \xi \) direction incident, namely \( A_1 = 1 \) and \( A_2 = 0 \), for the half-space A, and imposing no radiation condition in half-space B in the negative \( \xi \) direction, namely, \( A_3 = 1 \) and \( A_4 = 0 \) in half-space B, the values of all other harmonic wave amplitudes can be calculated using the above boundary condition for a range of frequencies. The amplitudes and propagation directions for both half-spaces are summarized in Fig. 1. For the materials of System I and II, the reflection and transmission coefficients are calculated for a frequency interval. The results are presented in Figs. 2a,b for the two propagation directions (\( \uparrow \) and \( \downarrow \)), two modes (\( c_1 \) and \( d_1 \)) and two field variables (\( u \) and \( T \)). The values of reflection and transmission coefficients corresponding to the displacement amplitude with the exponent \( +c_1 \) in layer A and \( -c_1 \) in layer B are very close to those given in [16] for purely elastic half-spaces. Note that the coefficients in this work are defined in terms of the displacement amplitudes. The reflection and transmission coefficients for the temperature component in the System I materials are higher than those in the System II materials. It is also noteworthy that no thermal wave is incident in these calculations and the thermal field is generated both in materials and at the interface due to the coupling of the two fields. The temperature generated at the interface of System I is nearly three times larger than that of System II.

At the interface between the two half-spaces, the modes with \( d_1 \) exponent are created even though the P-wave with \( c_1 \) is incident in half-space A. As indicated earlier, the modes travelling with \( d_1 \) are highly attenuated. Therefore these newly generated wave components will be of very short propagation distance from the interface. Consequently, from the harmonic analysis presented, it is reasonable to expect that the temperature gradients in transient propagation will be steeper around the interface.

5 Thermoelastic Transient Response

To study the effects of transient interactions between thermal and mechanical waves in layered structures, the thermoelastic responses of two bi-periodic structures (System I and II) consisting of four sets are considered. The thermoelastic and geometric properties of layer materials (A and B) for System I are as follows: \( E_A = 310.3 \) GPa, \( \rho_{0A} = 3248.8 \) kg/m\(^3\), \( \lambda_A = 124.1 \) GPa, \( \mu_A = 124.1 \) GPa, \( H_A = 12.7 \times 10^{-3} \) m, \( \eta_A = 0.25 \), \( c_{LA} = 10705.2 \) m/s, \( c_{TA} = 6180.6 \) m/s, \( \kappa_A = 403 \) W/m·K, \( \gamma_A = 390 \) J/kg·K, \( \beta_A = 1.04 \times 10^7 \), \( \tau_{0A} = 8.325 \times 10^{-12} \) s, \( E_B = 0.69 \) GPa, \( \rho_{0B} = 1068.6 \) kg/m\(^3\), \( \lambda_B = 5.6 \) GPa, \( \mu_B = 0.23 \) GPa, \( H_B = 0.25 \times 10^{-3} \) m, \( \eta_B = 0.48 \), \( c_{LB} = 2380.5 \) m/s, \( c_{TB} = 466.86 \) m/s, \( \kappa_B = 236 \) W/m·K, \( \gamma_B = 960 \) J/kg·K, \( \beta_B = 3.97 \times 10^7 \), \( \tau_{0B} = 1.217 \times 10^{-10} \) s. The properties of Layer A of System II are the same as those of System I. The thermoelastic and geometric properties of Layer B System I are as follows: \( E_B = 1.38 \) GPa, \( \rho_{0B} = 10686.9 \) kg/m\(^3\), \( \lambda_B = 11.2 \) GPa, \( \mu_B = 0.47 \) GPa, \( H_B = 0.25 \times 10^{-3} \) m, \( \eta_B = 0.48 \), \( c_{LB} = 1064.6 \) m/s, \( c_{TB} = 208.8 \) m/s, \( \kappa_B = 236 \) W/m·K, \( \gamma_B = 960 \) J/kg·K, \( \beta_B = 7.9 \times 10^7 \), \( \tau_{0B} = 6.08 \)
A trapezoidal time-dependent pressure field (Figs. 3a,b) is applied at one end of the layered structures. No thermal loading is specified in this analysis. The time duration of the pulse is about 12 micro-seconds, and its power spectrum covers approximately three propagation zones in the frequency response of the structure for the Nyquist frequency at around 7.5 MHz. Mechanically and thermally free-free boundary conditions are imposed at both ends of the layered structures in calculating transfer functions. The numerical simulations are based on the Fast Fourier Transform algorithm. Various sampling rates from $10^5$ to $10^7$ are used for required resolution in order to obtain physically sensible results. In some cases, a damping term coming from the material is used to minimize the effect of excessive sampling requirements around the resonance frequencies. Common values for material damping were in the 1 percent–5 percent range and it is observed that these values have almost no effect in the short term transient response. As previously noted, in case of thermoelastic wave propagation, the need for material damping is less severe than the purely elastic case since the coupling between the displacement and temperature fields provides mode conversion attenuation.

The thermoelastic transient response of the structures under the time-dependent load (Fig. 3a) is computed at all three interfaces between the bi-periodic sets. In addition, the response at nine points inside the layer B of set 2 and layer A of set 3 is obtained (Fig. 4) to examine the localization of the thermal wave around the interfaces.

Figures 5 and 6 show the transient stress response of the two structures at four interface points. The only notable difference between the response of the two systems is that System II slightly overshoots about 5 percent, while System I undershoots about the same amount compared to the load function with unit amplitude. It is also noteworthy that the response at point 2 (the central point of the layered structures) is clearly the highest after the arrival of the wavefront. However, the spatial attenuation characteristics of the two systems are still rather similar.

In Figs. 7 and 8, the temperature fields generated inside the structures are compared. The temperatures calculated at the interfaces of System I are over three times larger than those of System II in the wavefronts. As with the stress waves, the spatial attenuation characteristics of the thermal waves in both systems are similar, and the temperature at point 2 is always higher than at the other three points, after the arrival of the wavefront. The main
difference between the elastic and thermal response at the interfaces is that the variation in the amplitudes of thermal waves is much larger.

To study the localization behavior of stress and thermal waves, the transient response at nine points (at points 5–13) around interface 2 (Fig. 4) are calculated and compared. In Figs. (9) and (10), the longitudinal stress components at these points are superimposed. Both plots indicate that the stress components from points 5 to 13 are changing rather smoothly in the wave fronts, and the overshoot in System II is still visible. However, the situation is rather different for the thermal waves. Figs. (11) and (12) reveal that the temperature maximum in the wavefronts vary a lot with the sensor location. It is observed that the thermal transient responses are grouped into three zones. The temperature response

Fig. 8 Thermal response of System II at the interfaces

Fig. 9 Normal stress of System I at points 5–13

Fig. 10 Normal stress of System II at points 5–13. Some response curves for sensor points are close to one another. Numbers in a box indicate the sensor point numbers in a descending order.

Fig. 11 Thermal response of System I at points 5–13. Some response curves for sensor points are close to one another. Numbers in a box indicate the sensor point numbers in a descending order.

Fig. 12 Thermal response of System II at points 5–13. Some response curves for sensor points are close to one another. Numbers in a box indicate the sensor point numbers in a descending order.
at the two interfaces (points 5 and 9) are very close to each other as the temperature profiles in Layer B (points 6, 7, and 8) and in Layer A (points 10, 11, 12, and 13) are grouped together. This is especially true for System II. This behavior indicates a large temperature gradient and, consequently, a large heat flux around the interfaces, which attests thermal wave localization as predicted by the harmonic wave propagation solutions in the previous section by the localization properties of the thermal waves. In System I’s points 5 to 8, the temperature gradient is less steep than that of System II at the same sensor locations.

6 Conclusions and Remarks

A transfer matrix formulation based on the generalized dynamical theory of thermoelasticity is developed for longitudinal wave propagation in a thermoelastic layer. The second sound effect is included to eliminate the thermal wave travelling with infinite velocity as predicted by the diffusion heat transfer model. Using this formulation and the periodic systems framework, the attenuation and propagation properties of one-dimensional thermoelastic wave propagation in both homogeneous and layered structures are studied. The reflection and transmission coefficients at the interface of two half-spaces are calculated in a frequency interval. From the highly attenuative nature of the thermal wave mode, a strong localization of this mode at around the interfaces is concluded.

A perturbation analysis is carried out to study the dispersive behavior of the propagating bulk wave modes. The attenuation factor, defined as the real part of the propagation constant, is obtained in thermoelastic solids. This strong localization makes experimental observation of propagating thermal waves rather difficult. By obtaining the leading terms in an expansion, it is shown that the relaxation time plays no role in the propagation and attenuation properties of the thermoelastic waves in the one-dimensional problem. The reflection and transmission coefficients between half-spaces are also calculated, and the strength of the thermal wave generated due to the mode conversion at the interface is demonstrated. The generation of a highly attenuative thermal wave leads to the accumulation of heat along the interface. Such heat localization can be utilized in a thermal-based nondestructive evaluation of defects.

An FFT-based transient analysis is carried out for two layered structures. Strong localization of thermal waves predicted by the reflection and transmission analysis in the transformed domain is demonstrated in the time-space domain. It is also shown that the variation in the temperature field in layered structures is much greater than that in the stress fields. Since, in general, the greater the variation is in a field, the easier its detection is. This finding along with the heat localization might be useful in the development of an infrared imaging-based nondestructive testing method.

References